Quantum Mechanics_control volume

In fluid mechanics and Thermodynamics, a **control volume** is a mathematical abstraction employed in the process of creatingmathematical models of physical processes. In an inertial frame of reference, it is a volume fixed in space or moving with constant velocity through which the fluid (gas or liquid) flows. The surface enclosing the control volume is referred to as the **control surface**.[1]

At steady state, a control volume can be thought of as an arbitrary volume in which the mass of the fluid remains constant. As fluid moves through the control volume, the mass entering the control volume is equal to the mass leaving the control volume. At steady state, and in the absence of work and heat transfer, the energy within the control volume remains constant. It is analogous to the classical mechanics concept of the free body diagram.

Contents

Overview

Typically, to understand how a given physical law applies to the system under consideration, one first begins by considering how it applies to a small, control volume, or "representative volume". There is nothing special about a particular control volume, it simply represents a small part of the system to which physical laws can be easily applied. This gives rise to what is termed a volumetric, or volume-wise formulation of the mathematical model.

One can then argue that since the physical laws behave in a certain way on a particular control volume, they behave the same way on all such volumes, since that particular control volume was not special in any way. In this way, the corresponding point-wise formulation of the mathematical model can be developed so it can describe the physical behaviour of an entire (and maybe more complex) system.

In fluid mechanics the conservation equations (for instance, the Navier-Stokes equations) are in integral form. They therefore apply on volumes. Finding forms of the equation that are *independent* of the control volumes allows simplification of the integral signs.

Substantive derivative

Main article: Material derivative

Computations in fluid mechanics often require that the regular time derivationoperator d/dt is replaced by the substantive derivative operator D/Dt. This can be seen as follows.

Consider a bug that is moving through a volume where there is some scalar, e.g.Pressure, that varies with time and position: $p = p(t, x, y, z)$.

If the bug during the time interval from t to $t + dt$ moves from (x, y, z) to $(x + dx, y + dy, z + dz)$, then the bug experiences a change dp in the scalar value,

$$dp = \frac{\partial p}{\partial t}dt + \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz$$

(the total differential). If the bug is moving with velocity $\mathbf{v} = (v_x, v_y, v_z)$, the change in position is $\mathbf{v}dt = (v_x dt, v_y dt, v_z dt)$, and we may write

$$\begin{aligned} dp &= \frac{\partial p}{\partial t}dt + \frac{\partial p}{\partial x}v_x dt + \frac{\partial p}{\partial y}v_y dt + \frac{\partial p}{\partial z}v_z dt \\ &= \left(\frac{\partial p}{\partial t} + \frac{\partial p}{\partial x}v_x + \frac{\partial p}{\partial y}v_y + \frac{\partial p}{\partial z}v_z\right) dt \\ &= \left(\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p\right) dt. \end{aligned}$$

where ∇p is the gradient of the scalar field *p*. If the bug is just a fluid particle moving with the fluid's velocity field, the same formula applies, but now the velocity vector is that of the fluid. The last parenthesized expression is the substantive derivative of the scalar pressure. Since the pressure p in this computation is an arbitrary scalar field, we may abstract it and write the substantive derivative operator as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla.$$

References

- James R. Welty, Charles E. Wicks, Robert E. Wilson & Gregory Rorrer*Fundamentals of Momentum, Heat, and Mass Transfer* ISBN 0-471-38149-7