Population Balance Models

Keywords: Models, Population Balance

All the previous models discussed in last lectures have been based on the basic mass, momentum and energy balances. The equations that involve dispersion are most representative of a process when the mixing is on a relatively small scale. For mixing in stirred tanks (in turbulence), dispersion models are not effective. Certain categories of models which cannot successfully be treated within the framework of the models based on transport phenomena can be treated by the population balance concept.

The application of population -balance principles to the modelling of flow and mixing characteristics in vessels was formally organized by Danckwert's. He defined certain distribution functions for the residence time of the fluid elements in a process vessel.RTD functions give information about the fraction of the fluid that spends a certain time in a process vessel. RTD models do not give point- to – point variation of the dependent variables. Danckwert's description was based on macroscopic lumped-population-balance. Population balance models represents macro-mixing that are sufficient ti give adequate estimates of the behavior of the process.

Description of Flow Pattern's in Process Vessel's

There are two types of ideal flow patterns in our process vessels;

1. Plug flow pattern – Piston Flow
2. Bochmix Flow Pattern – Perfectly mixed

Plug flow occurs when the fluid velocity is uniform over the entire cross-section of the vessel.

Perfect mixing assumes that the vessel contents are completely homogenous down to a molecular scale.

Between these two extreme's lie flow patterns in actual process.

In channeling (known as by-passing) some elements of fluid slip on pass through the vessel considerably faster than others do. Channeling may be found in flow through purely packed vessels, through vessels of small length-to-diameter ratios or through heat exchanger's with proper baffling.

Stagnant packets (dead space) may occur in header's, at the base of pressure gauges, or in the odd-shaped corner's, represents regions with extremely pure contacting.

AGE DISTRIBUTION FUNCTIONS

The residence time of a fluid element is the time that elapses from the time the element enters the vessel to the time it leaves it. The age of a fluid element at a given instant of time is the time that elapses between the element's entrance into the vessel and the given instant , and is of course , less than or equal to the residence time. The age is equal to the residence time for those molecules that are just leaving the vessel.

INTERNAL AGE DISTRIBUTION OF A FLUID IN A CLOSED VESSEL

The functional notation I(t) will be used for the internal age distribution frequency of fluid elements in a vessel. I(t) have the unit- fraction of ages per unit time.

$$\int_0^{\infty} I(t)\, dt = 1 \qquad \text{…12.1}$$

The time t=0 refers to an arbitrary initial time and not the start of the flow of fluid into the vessel. In physical terms, eq. (12.1) states that all fluid has an age between 0 and ∞.

As a consequence of the above, the fraction of vessel contents younger than a specified age t is,

$$\int_0^{t} I(t')\, dt' = 1 \qquad \text{…12.2}$$

While the fraction older than t is ,

$$\int_t^{\infty} I(t')\, dt' = 1 - \int_0^{t} I(t')\, dt' \qquad \text{…12.3}$$

Age Distribution of the Exit Stream; The Residence Distribution of fluid in a closed vessel, E(t)

The function E(t) is the age distribution frequency of the fluid elements leaving the vessel and has the units of fraction of ages per unit time. The fraction of exit ages itself is E(t)Δt. The function is normalized to;

$$\int_0^{\infty} E(t)\, dt = 1 \qquad \text{...12.4}$$

The fraction of fluid in the exit stream younger than age t is

$$\int_0^{t} E(t')\, dt' = 1 \qquad \text{...12.5}$$

while the fraction of material older than t is

$$\int_t^{\infty} E(t')\, dt' = 1 - \int_0^{t} E(t')\, dt' = 1 \qquad \text{...12.6}$$

The mean RTD is found from the first moment

$$\bar{t} = \int_0^{\infty} tE(t)\, dt = \frac{V}{Q} \qquad \text{...12.7}$$

$$\frac{V}{Q} = \bar{t} = \boldsymbol{\tau}$$

In a similar fashion the mean age of fluid elements inside the vessel is,

$$\bar{t_1} = \int_0^{\infty} tI(t)\, dt \qquad \text{...12.8}$$

Intensity Function, Λ(t)

It is defined as fraction of fluid in the vessel of age t that will leave at time between t and t+Δt. The intensity function is useful in detecting the existence of dead space and by passing.

Relations between Age Distribution Functions:

The three age distribution frequency functions described above are related through the unsteady state macroscopic age population balance. The balance is made in the units of time. The input age distribution is zero. The general macroscopic population balances can be used to relate E(t) and I(t), but a simpler alternate method is used here.

Consider a constant volume vessel, V, with constant flow rate Q and call all fluid entering the vessel at t>0 new fluid. The existing contents at t=0 are the old fluid. At some time t, eq. 12.2 gives the function of new fluid.

Amount of new fluid in vessel = $V\int_0^t I(t')\,dt'$

Eq. 12.6 gives the fraction of out flowing fluid at any instant of time, that has an age greater than t; the amount of old fluid that has left the vessel during all times from 0 to t is

Amount of old fluid gone from the vessel = $\int_0^t Q\,dt' \int_{t'}^{\infty} E(t'')\,dt''$

Then by a simple balance, the old fluid left must have been replaced by the new fluid,

$$V\int_0^t I(t')\,dt' = \int_0^t Q\,dt' \int_{t'}^{\infty} E(t'')\,dt'' \quad \text{…12.9}$$

Differentiation of both sides of eq. 12.9 with respect to time with the introduction of eq.12.7 gives,

$$\bar{t}I(t) = \int_t^{\infty} E(t')\,dt' = 1 - \int_0^t E(t')\,dt' \quad \text{…12.10}$$

Differentiating once more,

$$E(t) = -\bar{t}\frac{dI(t)}{dt} \quad \text{…12.11}$$

The intensity function can also be related to the E(t) and I(t) functions from

{Amount of fluid leaving between t and t+Δt} = {Amount not leaving before time t} [Fraction of age t that will leave between times t, t+Δt]

$$QE(t)dt = [VI(t)][\Lambda(t)dt] \quad \text{…12.12}$$

$$\Lambda(t) = \frac{1}{\bar{t}}\frac{E(t)}{I(t)} = -\frac{d[ln\bar{t}I(t)]}{dt} \quad \text{…12.13}$$

Each of the age distribution functions can be expressed in dimensionless form.

$$\theta = \frac{t}{\bar{t}} \quad \text{…12.14}$$

Thus, $E(\theta)d\theta = E(t)dt, I(\theta)d\theta = I(t)dt$

$$E(\theta) = \bar{t}E(t) \quad \text{…12.15}$$

$$I(\theta) = \bar{t}I(t) \quad \text{…12.16}$$

$$\Lambda(\theta) = \bar{t}\Lambda(t) \quad \text{…12.17}$$

$$E(\theta) = -\frac{d\,I(\theta)}{d\theta}\,I(\theta) \quad \text{…12.18}$$

$$\Lambda(\theta) = \frac{E(\theta)}{I(\theta)} = -\frac{\ln(\theta)}{d\theta} \quad \text{…12.19}$$

Source:
http://nptel.ac.in/courses/103107096/12