# Modelling Of Multiple Effect Evaporators With Boiling Point Rise

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System that exhibit boiling point elevations generally possess liquid enthalpies that depend on both temperature and composition. In the following analysis, the general case is considered wherein both variations are involved.

**Forward Feed :**

Equation of energy balance ,heat transfer rate and boiling point rise of $i^{th}$effect evaporator for forward feed arrangement is as follows.

Solute material balance:

$FX_f = L_i Xi$ ...24.1

Overall material balance:

$L_{i-1} = V_i + L_i$ ...24.2

Energy balance:

$L_{i-1}h(Ŧ_{i-1}, X_{i-1}) + V_{i-1}H(P_{i-1}) = L_i h(Ŧ_i X_i) + V_i H(P_i Ŧ_i) + L'_{i-1}h(P_{i-1})$

Since $L'_{i-1} = V_{i-1}$

$L_{i-1}h(Ŧ_{i-1}, X_{i-1}) + V_{i-1}\lambda(P_{i-1}) = L_i h(Ŧ_i, X_i) + V_i H(P_i, Ŧ_i)$ ...24.3

Heat transfer rate:

$V_{i-1}H(P_{i-1}) - L'_{i-1}h(P_{i-1}) = U_i A_i(T_{i-1} - Ŧ_i)$

Since $L'_{i-1} = V_{i-1}$

$V_{i-1}\lambda (P_{i-1}) = U_i A_i(T_{i-1} - Ŧ_i)$ ...24.4

Boiling point rise:

$Ŧ_i = T_i + (\gamma + nT_i)X_i + \alpha X_i^2 + \beta X_I^3$ ...24.5

Substituting equation (24.1),(24.2),(24.3) in (24.4) we get-:

$L_{i-1}h(Ŧ_{i-1}, X_{i-1}) + (L_{i-2} - L_{i-1})\lambda(P_{i-1}) = L_i\, h(Ŧ_i, X_i) + (L_{i-1} - L_i)H(P_i, Ŧ_i)$ . ...24.6

$(L_{i-2} - L_{i-1})\lambda(P_{i-1}) = UiA_i(T_{i-1} - Ŧ_i)$ ...24.7

Equations (24.1),(24.5),( 24.6),( 24.7) represent the model for forward feed arrangement in an N-effect evaporator. Systems. However, it cannot be used in its present form as it contains quantities like enthalpy of liquid , h; enthalpy of vapour, H; latent heat of vaporization of steam, λ; and overall heat transfer coefficient , U whose direct measurement is difficult. The functional relationship of these quantities with temperature and concentration of the liquid streams and the pressure of the steam fed to the evaporator have been given by various equations.

The models for aqueous **sugar solution** in an N-effect evaporator is-:

For sugar solution correlation for h, H , λ ,U are-:

$h = (4.182-2.2403X_{i-1})( Ŧ_{i-1}-T_r)$

$H = 4.154(T_i-T_r) + 2.0125*10^{-4}(T_i^2-T_r^2)+1.62(Ŧ_i-T_r)+2.0285*10^{-4}(Ŧ_i^2-T_i^2)-0.3747*10^{-7}(Ŧ_i^3-T_i^3)+\lambda$

$\lambda = -80.345T_{i-1}-21035.87/T_{i-1}+2049.123T_{i-1}^{.5} – 4213.519\ln T_{i-1}+0.0918T_{i-1}^2- 1.04*10^{-4}T_{i-1}^3+8597.953$

$U = 18.083(Ŧ_i/X_i)$

Solute material balance:

$FX_f = L_iXi$ ...24.8

Energy Balance:

$L_{i-1}(4.182-2.2403X_{i-1})( Ŧ_{i-1}-T_r) + (L_{i-2} – L_{i-1})[-80.345T_{i-1}-21035.87/T_{i-1}+2049.123T_{i-1}^{.5} – 4213.519\ln T_{i-1} +0.0918T_{i-1}^2-1.04*10^{-4}T_{i-1}^3+8597.953] = L_i(4.182-2.2403X_{i-1})(Ŧ_i-T_r) + (L_{i-1}-L_i)[4.154(T_i-T_r) + 2.0125*10^{-4}(T_i^2-T_r^2)+1.62(Ŧ_i-T_r)+2.0285*10^{-4}(Ŧ_i^2-T_i^2)-0.3747*10^{-7}(Ŧ_i^3-T_i^3)-80.345T_i-21035.87/T_i+2049.123T_i^{.5} – 4213.519\ln T_i+0.0918T_i^2- 1.04*10^{-4}T_i^3+8597.953]$ ...24.9

Heat transfer rate:

$(L_{i-2}–L_{i-1})[-80.345T_{i-1}-21035.87/T_{i-1}+2049.123T_{i-1}^{.5}-4213.519\ln T_{i-1}+0.0918T_{i-1}^2-1.04*10^{-4}T_{i-1}^3 +8597.953] = 18.083(Ŧ_i/X_i) A_i(T_{i-1} - Ŧ_i)$ ...24.10

Boiling point rise:

$Ŧ_i= T_i + 7.2X_i - 11X_i^2 + 29.50X_I^{\,3}$ ...24.11

The solution of these equation can be done as explained in previous lecture.

Equation for N- effect for other type of feed arrangements are as follows-:

**Backward feed:**

For backward feed arrangement, equations of material balance, energy balance, heat transfer rate, and boling point rise about $i^{th}$ effect are as follows:

Overall material balance:

$L_{i+1}=V_i+L_i$ ...24.12

Energy balance:

$L_{i+1}h(Ŧ_{i+1} ,X_{i+1}) + V_{i-1} H(P_{i-1}) = L_ih(Ŧ_i ,X_i) + V_i h(P_i ,Ŧi) + L'_{i-1} h(P_{i-1})$

Since $L'_{i-1} = V_{i-1}$

$L_{i+1}h(Ŧ_{i+1} , X_{i+1}) + V_{i-1} \lambda(P_{i-1}) = L_i h(Ŧ_i ,X_i) + V_i H(P_i ,Ŧ_i)$ ...24.13

Heat transfer rate

$V_{i-1}H(P_{i-1}) – L'_{i-1} h(P_{i-1}) = U_i A_i(T_{i-1} –Ŧ_i)$

Since $L'_{i-1} = V_{i-1}$

$V_{i-1}\lambda(P_{i-1}) = U_i A_i(T_i-Ŧ_i)$ ...24.14

Boiling point rise:

$Ŧ_i = T_i + (\gamma + nT_i)X_i + \alpha X_i^2 + \beta X_i^3$ ...24.15

Substituting equation (1) in (2) and(3) we get-:

$L_{i+1}h(Ŧ_{i+1} ,X_{i+1}) + (L_i –L_{i-1})\lambda(P_{i-1}) = L_i h(Ŧ_i ,X_i) + (L_{i+1} –L_i)H(P_i ,Ŧ_i)$ ...24.16

$(L_i –L_{i-1})\lambda(P_{i-1}) = U_i A_i(T_{i-1}-Ŧ_i)$ ...24.17

Equations (24.4), (24.5) and (24.6) represent a model for the evaporation of aqueous solution in an N-effect evaporator having backward feed arrangement. The terms h, H,λ, and U contained in equations (24.5) and (24.6) are substituted by their respective equations the resulting equations describe the model of N-effect evaporator under backward feed arrangement.

**Mixed feed:**

In this type of feed arrangement the liquid stream entering to the $i^{th}$ effect has been taken as $L_k$.the equations of material balance , energy balance, heat transfer rate, and boiling point rise about $i^{th}$ effect are as follows:

Overall material balance:

$L_K = V_i + L_i$ ...24.18

Energy balance:

$L_kh (Ŧ_k,X_k) + V_{i-1}H(P_{i-1}) = L_ih(Ŧ_i ,X_i) + V_iH(P_i ,Ŧ_i) + L'_{i-1}h(P_{i-1})$

Since $L'i_{-1} = V_{i-1}$

$L_kh(Ŧ_k,X_k)+V_{i-1}\lambda(P_{i-1})=L_ih(Ŧ_i,X_i)+V_iH(P_i,Ŧ_i)$ ...24.19

Heat transfer rate:

$V_{i-1}H(P_{i-1},T_{i-1}) – L'_{i-1}h(P_{i-1}) = U_iA_i(T_{i-1} –Ŧ_i)$

Since $L'i_{-1} = V_{i-1}$

$V_{i-1}\lambda (P_{i-1}) = U_iA_i( T_{i-1}-Ŧ_i)$ ...24.20

Boiling point rise:

$Ŧ_i = T_i + (\gamma +nT_i) Xi+ \alpha X_i^2 + \beta X_i^3$ ...24.21

Substituting equation (1) to (2) and(3) we get-:

$L_k h(\text{Ŧ}_k, X_k) + (L_{k-1} – L_{i-1})(P_{i-1}) = L_i h(\text{Ŧ}_i, X_i) + (L_k – L_i) H(P_i, \text{Ŧ}_i)$ ...24.22

$(L_{k-i} – L_{i-1})(P_{i-1}) = U_i A_i (T_{i-1} - \text{Ŧ}_i)$ ...24.23

It is important to mention here that when $i^{th}$ effect corresponds to the feed-introduction effect, $L_k = F$. for the case when $i^{th}$ effect is located after the feed-introduction effect, $L_k = L_{i-1}$; whereas $L_k = L_N$ when $i^{th}$ effect is just before the feed-introduction effect; and for all other situations, $L_k = L_{i+1}$.

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