Metals: the Drude model of electrical conduction

Due to the quantum mechanical nature of electrons, a full simulation of electron movement in a solid (i.e. conduction) would require consideration of not only all the positive ion cores interacting with each electron, *but also each electron with every other electron.* Even with advanced models, this rapidly becomes far too complicated to model adequately for a material of macroscopic scale.

The Drude model simplifies things considerably by using classical mechanics and treats the solid as a fixed array of nuclei in a 'sea' of unbound electrons. Additionally, the electrons move in straight lines, do not interact with each other, and are scattered randomly by nuclei.

Rather than model the whole lattice, two statistically derived numbers are used: τ, the average time between collisions (the **scattering time**), and l, the average distance traveled between collisions (the **mean free path**)

Under the application of a field, E, electrons experience a force $-e\,E$, and thus an acceleration from $F = m\,a$

For an electron emerging from a collision with velocity v_0, the velocity after time t is given by:

$$v = v_0 - \frac{eEt}{m}$$

Of course, if the electrons are scattered randomly by each collision, v_0 will be zero. If we also consider the time $t = \tau$, an equation for the **drift velocity** is given:

$$v = -\frac{eE\tau}{m}$$

For **n** free electrons per unit volume, the current density J is: $J = -n\,e\,v$

Substituting v for the drift velocity:

$$J = \frac{ne^2\tau E}{m}$$

The conductivity $\sigma = n\,e\,\mu$, where μ is the **mobility**, and is defined as

$$\sigma = \frac{|v|}{E} = \frac{eE\tau}{mE} = \frac{e\tau}{m}$$

The net result of all this maths is a reasonable approximation of the conductivity of a number of monovalent metals. At room temperature, by using the kinetic theory of gases to estimate the

drift velocity, the Drude model gives $\sigma \sim 10^6 \; \Omega^{-1} \; m^{-1}$. This is about the right order of magnitude for many monovalent metals, such as sodium ($\sigma \sim 2.13 \times 10^5 \; \Omega^{-1} \; m^{-1}$).
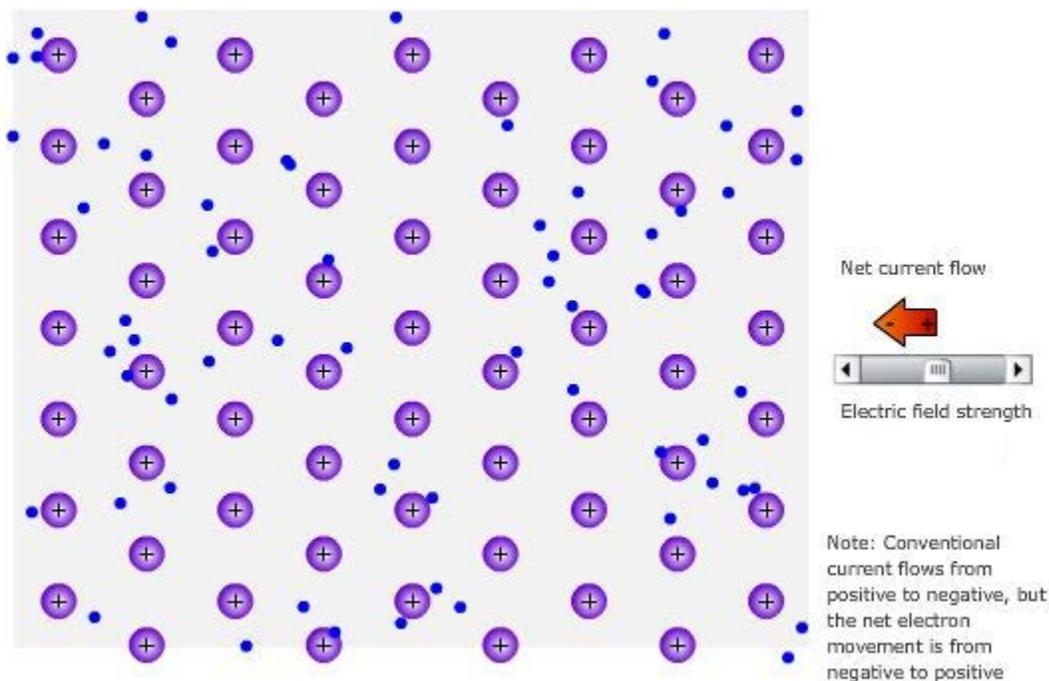
The Drude model can be visualised using the following simulation. With no applied field, it can be seen that the electrons move around randomly. Use the slider to apply a field, to see its effect on the movement of the electrons.

Introduction to the Drude model

The Drude model is a simplistic model for conduction.

It uses classical mechanics, and treats the solid as a fixed array of ions, with electrons not bound. The electrons do not interact with each other, and are scattered by the ions.

When a field is applied, there is a net movement of charge through the lattice, a current flow.

Net current flow

Electric field strength

Note: Conventional current flows from positive to negative, but the net electron movement is from negative to positive

However, it is important to note that for non-metals, multivalent metals, and semiconductors, the Drude model fails miserably. To be able to predict the conductivity of these materials more accurately, quantum mechanical models such as the Nearly Free Electron Model are required. These are beyond the scope of this TLP

Superconductors are also not explained by such simple models, though more information can be found at the Superconductivity TLP.

Factors affecting electrical conduction

Electrical conduction in most metallic conductors (not semiconductors!) is straightforward to approximate. There are three important cases:
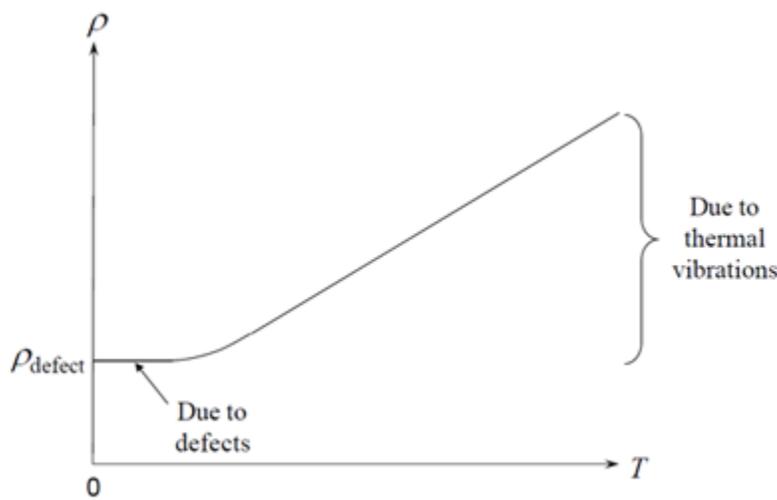
Pure and nearly pure metals

For pure metals at around room temperature, the resistivity depends linearly on temperature.

$$\rho_2 = \rho_1[1 + \alpha(T_2 - T_1)]$$

However, at low temperatures, the conductivity ceases to be linear (superconductors are dealt with separately), and resistivity is related to temperature by Matthiesen's rule:

$$\rho(T) = \rho_{defect} + \rho_{thermal}$$

The low temperature resistivity (ρdefect)depends on the concentration of lattice defects, such as dislocations, grain boundaries, vacancies, and interstitial atoms. Consequently, it is lower in annealed, large crystal metal samples, and higher in alloys and work hardened metals. You might think that at higher temperatures the electrons would have more energy to be able to move through the material, so perhaps it is rather surprising that resistivity increases (and conductivity therefore decreases) as temperature increases. The reason for this is that as temperature increases, the electrons are scattered more frequently by lattice vibrations, or phonons, which causes the resistivity to increase. This contribution to the resistivity is described by $\rho_{\mathbf{thermal}}$.

The temperature dependence of the conductivity of pure metals is illustrated schematically in the following simulation. Use the slider to vary the temperature, to see how the movement of the electrons through the lattice is affected. You can also introduce interstitial atoms by clicking within the lattice.
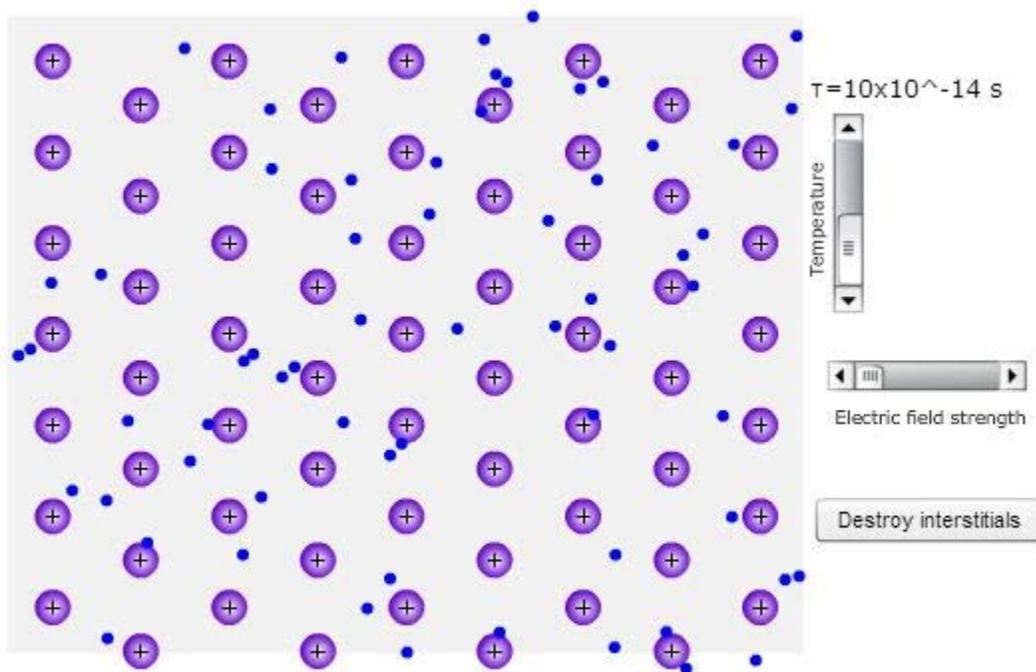
Alloys - Solid solution

As before, adding an impurity (in this case another element) decreases the conductivity. For a solid solution, the variation of resistivity with composition is given by Nordheim's rule:

$\rho = \chi_\alpha \rho_\alpha + \chi_\beta \rho_\beta + C \chi_\alpha \chi_\beta$

where C is a constant and C_A and C_B are the atomic fractions of the metals A and B, whose resistivities are ρ_A and ρ_B respectively.

Further, the difference in valency between the bulk lattice and the impurity atoms is proportional to the difference in resistivity - Linde's rule.

$\Delta\rho \propto (\Delta Z)^2$

where ΔZ is the difference in valence between the solute and the solvent.

Thus, solute atoms with a higher (or lower) charge than the lattice will have a greater effect on the resistivity.

Alloys- many phases

For an alloy where there are two or more distinct phases, the contributions simply contribute linearly to the total resistivity (though the effect of many grain boundaries increases resistivity slightly).
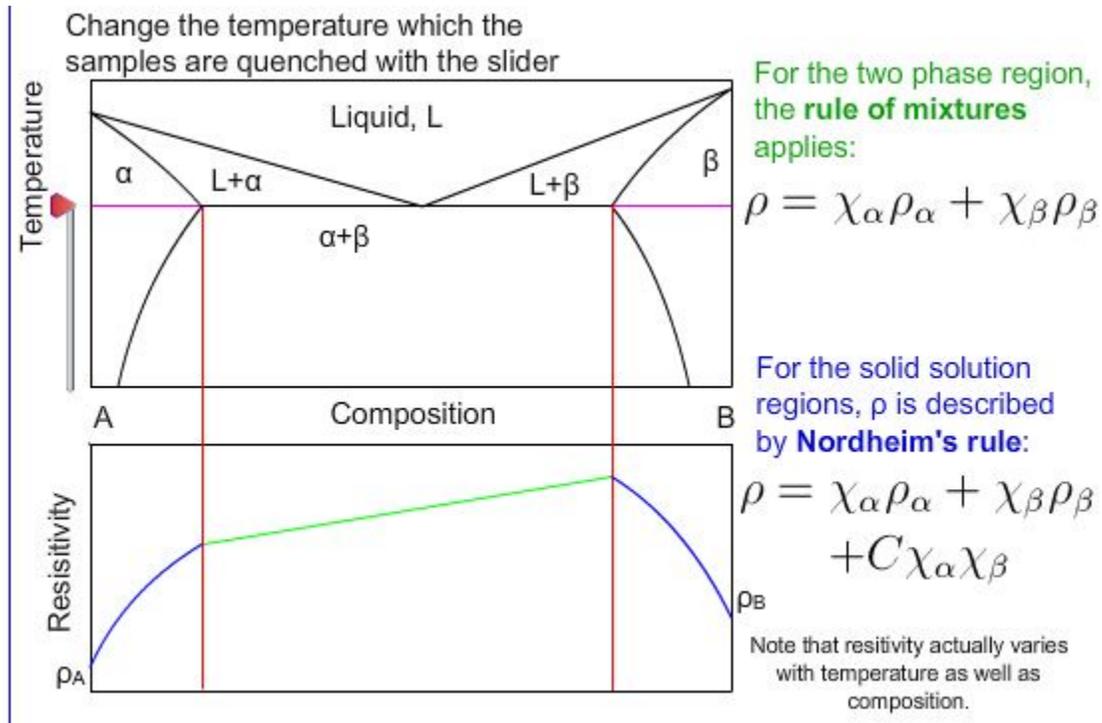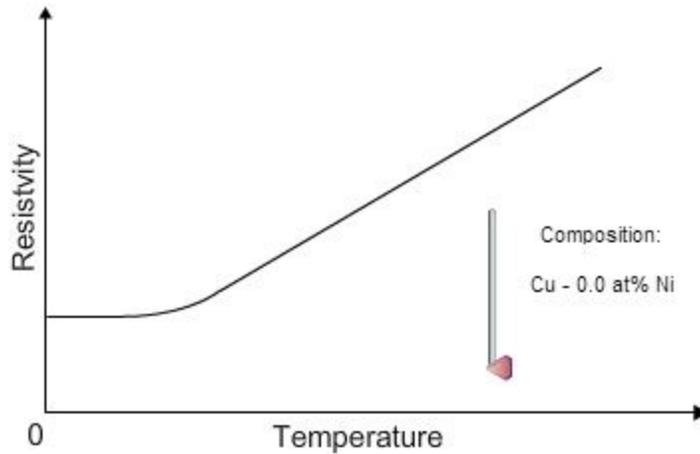
$\rho = \chi_\alpha \rho_\alpha + \chi_\beta \rho_\beta$

The following animation illustrates Mattheisen's rule, Nordheim's rule and the mixture rule.

For low concentrations of solute, and pure metals, the low temperature dependence of resistivity on temperature is described by Matthiesen's rule: $\rho(T) = \rho_{defect} + \rho_{thermal}(T)$

Adding more defects, such as solute atoms, increases ρ_{defect}.

Change the compostion and see the effect on the resistivity.

Resistivty

Composition:

Cu - 0.0 at% Ni

0   Temperature

Change the temperature which the samples are quenched with the slider

Temperature

Liquid, L

α   L+α   L+β   β

α+β

A   Composition   B

Resisitivity

ρ_A

For the two phase region, the **rule of mixtures** applies:

$$\rho = \chi_\alpha \rho_\alpha + \chi_\beta \rho_\beta$$

For the solid solution regions, ρ is described by **Nordheim's rule**:

$$\rho = \chi_\alpha \rho_\alpha + \chi_\beta \rho_\beta + C \chi_\alpha \chi_\beta$$

ρ_B

Note that resitivity actually varies with temperature as well as composition.