Introduction to Elasticity in Biological Materials and Hookean Elasticity

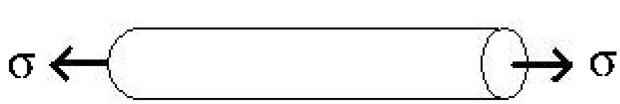
As you will already know, many engineering materials, such as metals, show Hookean elasticity in which the tensile stress applied to a sample is directly proportional to the resultant strain. Within the range of Hookean elasticity, the stress-strain curves on loading and unloading are identical. Such linear elasticity is the usual assumption in engineering design. However, the elasticity of most materials in living systems is much more complicated.

In this TLP, you will learn that some biomaterials exhibit non-linear stress-strain curves. Mammalian skin, for instance, exhibits a *J-shaped stress-strain curve*, as do healthy arterial walls. Materials with J-shaped curves are usually tough, and have other advantages. For example, materials with different elastic behaviour such as S-shaped stress-strain curves are prone to elastic instabilities such as aneurysms when used for tubes under pressure.

This TLP also discusses viscoelasticity. Many biomaterials show time-dependent stress-strain curves. Associated with this, the loading and unloading curves do not superimpose on each other. Although deformation is elastic (i.e. recoverable), energy is absorbed during the deformation. This is particularly important in spider silk: when a fly hits the web its energy should be absorbed by the deformation of the web. If spider silk showed elasticity without energy absorption, then the web instead would act as a trampoline!

Hookean elasticity

For many materials loaded in uniaxial tension, the tensile stress on the material, σ, is directly proportional to the tensile strain , ε.



A sample loaded in uniaxial tension

The linear relationship between stress and strain is known as Hooke's Law,

$$\sigma \propto \varepsilon$$

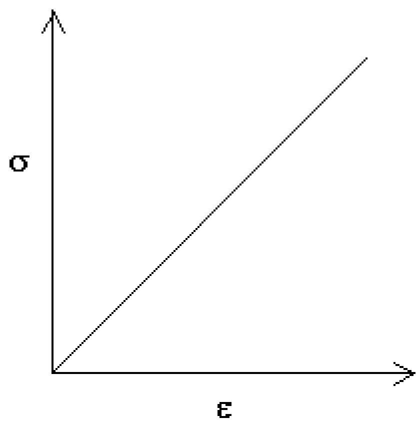
The constant of proportionality in this equation for simple tension is the Young Modulus of the material, *E*:

$$E = \frac{\sigma}{\varepsilon}$$

The Young Modulus of a material has values ranging from approx. 0.01 GPa for rubbers to approx. 1000 GPa for diamond.

Hooke's Law further states that the stress response of a material is independent of time and that the strain of a material disappears completely on removal of the applied stress (i.e. a Hookean material shows elastic deformation).

This leads to a linear stress-strain curve with a gradient of *E*. Loading and unloading occur along the same curve.



A stress-strain curve for a Hookean material

Most materials are Hookean only at small strains (typically less than 1%). Metals, for which fully elastic behaviour is only for very small strains (typically <0.2%), show Hookean behaviour. In this region, the extension is usually both linear and recoverable. At larger strains, extension is non-Hookean (i.e. either non-recoverable, or non-linear, or both).

Although many materials used in engineering applications show Hookean behaviour, only a few biomaterials approximate to it (wood and bone being the two most common). Many biomaterials exhibit a J-shaped stress-strain curve, but firstly, we shall consider the S-shaped stress-strain curve seen in rubbery materials.

Source:http://www.doitpoms.ac.uk/tlplib/bioelasticity/hookean-elasticity.php