

General Relativity

G.7 General Relativity

G.7.1 Explain the difference between the terms gravitational mass and inertial mass

The concept of mass turns up in two separate contexts. The first is in Newton's 2nd Law and in Newton's Law of Universal Gravitation. Newton's 2nd law talks about how hard it is to accelerate an object, while Universal Gravitation talks about the forces between two masses. Up to this point we have assumed the two masses are the same, but that assumption is not necessarily true. The mass in Newton's 2nd law is inertial mass and the mass in universal gravitation is gravitational mass. Very precise experiments have been conducted to measure if there is a difference between the two masses, there has never been an experiment that has shown a difference that was larger than the experimental uncertainty.

Under free fall an object is accelerated by gravity...

(1)

$$G \frac{M_E m_g}{r^2} = m_i a$$

Where m_g is the gravitational mass and m_i is the inertial mass. If we rearrange the equation to solve for a , the acceleration of the object:

(2)

$$a = \left(G \frac{M_E}{r^2} \right) \frac{m_g}{m_i}$$

Since all objects experience the same acceleration while undergoing free fall and $GMr^2 = \text{constant}$, therefore $mgmi = \text{constant}$. This is not a complete proof, but strongly suggests that the two masses are equivalent. Ernst Mach was the first to suggest that the two masses are equal.

G.7.2 Describe and discuss Einstein's principle of equivalence

In every day life motion is relative only if there is no acceleration. It is impossible to tell whether a car is moving or whether the earth is moving. However if the car accelerates the passengers feel a force and we know that the car was accelerating not the earth. Einstein didn't like this...

Einstein said that there was no difference between uniform acceleration and acceleration due to gravity! He said that if a person was in a elevator with the door shut and accelerating upwards at 9.8 ms^{-2} the person could do no experiment to prove the elevator was accelerating or if the elevator was sitting on the surface of the earth.

If the person in the accelerating elevator dropped a ball, the person would see the ball accelerate towards the floor at a rate of 9.8 ms^{-2} . However an outside observer would see the ball staying still (traveling at constant velocity) and the floor of the elevator accelerate upwards into the ball.

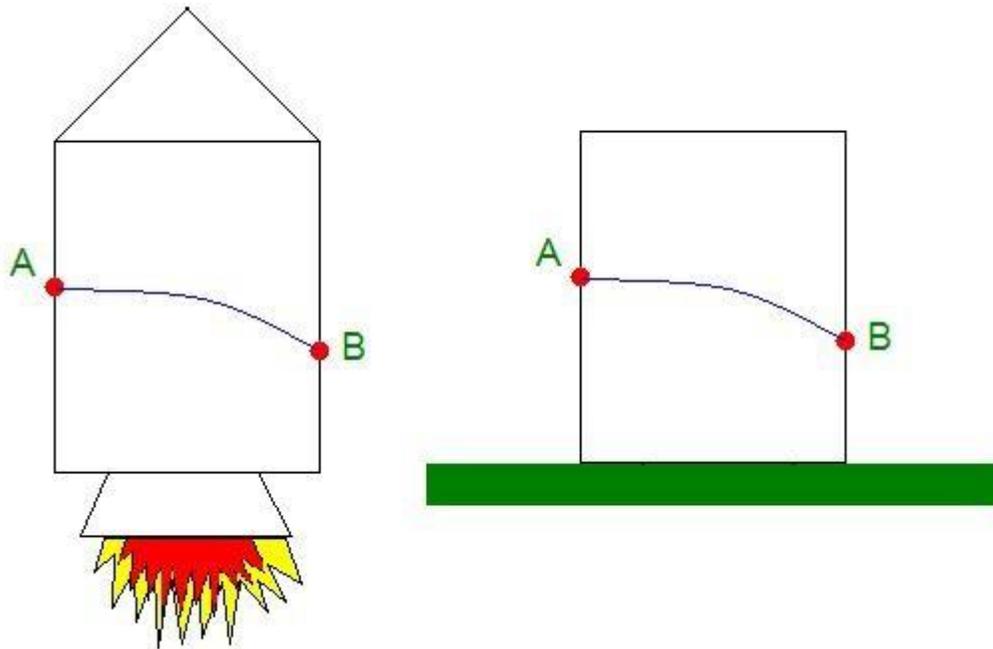
If the elevator was sitting on the surface of the earth and the person inside dropped the ball it would be seen to accelerate towards the floor at 9.8 ms^{-2} . An outside observer would see the same thing, they would not see the earth (and the floor) accelerate towards the ball.

Acceleration is relative in the sense that, observers can see different objects accelerate under the same circumstance.

G.7.3 Deduce that the principle of equivalence predicts bending of light rays in a gravitational field

Being relativity we will bring light back into the story...

If a rocket ship is undergoing constant acceleration and a flash light is shown on one side of the rocket ship towards the other side, the light will not hit the opposite side of the ship at the same height as the window. The speed of light is not infinite, by the time the light reaches the opposite side the ship will have accelerated and the ship will have moved, the light must travel in a straight line...



The light will leave point A traveling at the speed of light in a purely horizontal direction. When it reaches the other wall it will not reach the same height. According to Einstein's principle of equivalence light will also bend in a gravitational field...

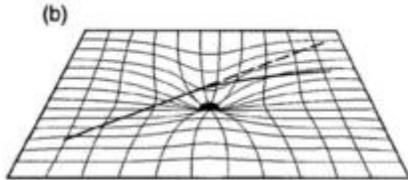
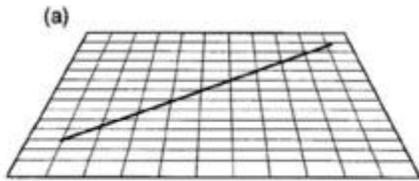
G.7.4 Describe the concept of spacetime

The concept of spacetime is that space (physical distance) and time are linked. They can not be separated. Objects move through space and time (thus spacetime), not just space or time. If we stop moving in space then we are moving only through time. No matter what objects always travel at the speed of light, but we travel through spacetime not just space. Since there are 3 space dimensions (that we observe) and 1 time dimension it is impossible to draw or even imagine what it would look like. To simplify the situation we can look at just one space dimension...

G.7.5 State that moving objects take the shortest path between two points in spacetime

Moving objects take the shortest path between two points in spacetime. The shortest point in space is a straight line. If the space is curved we have to redefine what a straight line is... sort of.

G.7.6 Explain Gravitational attraction in terms of the warping of spacetime by matter



Gravitation can be explained by the curvature of spacetime. As an object travels in straight line in curved space its path will curve towards a massive object. No force is needed to explain the change of path, the curvature of spacetime is enough to explain the motion. Pretty clever.

G.7.7 Describe the concept of gravitational red shift

When light travels away from a massive object the light is red-shifted, or in other words its frequency is reduced and the wavelength increased. It is called a red-shifted because red light has the lowest frequency of visible light.

This can be explained in several different ways. First if we go back to Einstein's equivalence principle, the accelerating rocket and a gravitational field... If a person at the back of a the rocket ship shines a light towards the front the rocket where there is a detector to measure the frequency. When the light is emitted both the source and the detector are going at velocity v , when the light is detected the detector is going at velocity $v+at$, where a is the acceleration of the rocket and t is the time for the light to travel. The time for the light to travel is: $t=\Delta h/c$, where Δh is the length of the rocket. Now if we fudge it just a bit and simplify the math, by assuming that the velocity at the time the light is emitted is $v = 0$, we can use the classical Doppler shift equations to arrive at the equation for the gravitational red-shift:

(3)

$$f' = f \left(1 \pm \frac{v_0}{v} \right)$$

$$f' = f \left(1 - \frac{a \left(\frac{\Delta h}{c} \right)}{c} \right) = f \left(1 - \frac{a \Delta h}{c^2} \right) \quad (4)$$

Doing a bit of algebra:

(5)

$$f' - f = f a \Delta h / c^2$$

(6)

$$\Delta f / f = a \Delta h / c^2$$

If the acceleration $a = g$, then the formula becomes:

(7)

$$\Delta f / f = g \Delta h / c^2$$

This formula appears in your IB formula handbook.

Where Δf is the change in the frequency, f is the frequency of the light measured at the source, g is the acceleration of gravity, Δh is the vertical distance between the source and the detector, and c is the speed of light in a vacuum.

G.7.9 Describe black holes

G.7.10 Define term Schwarzschild radius

G.7.11 Apply the calculation for calculating the Schwarzschild radius

Stars and planets form by mass attracting other mass and coming together, as the masses get closer they pull harder. Gravity pulls the masses together and forms a star or a planet. Making a long story short, if the mass of the object is great enough then the gravitational forces will be large enough to create a black hole. So what is a black

hole? A black hole is simply a massive object that generates such an intense gravitational field that not even light can escape, that's it, nothing fancy...

One way of looking at a black hole is to say that the escape velocity is equal to or greater than the speed of light, so therefore nothing, not even light can escape. Any object could form a black hole, IF it was small enough so that it generated a very strong gravitational field at its surface. For an object of given mass there is a minimum radius for that object in order for it to be a black hole, this minimum radius is called the Schwarzschild radius. If we set the escape velocity to be c and solve for the radius (This is not the correct way to do it, but we arrive at the same result):

(8)

$$c = \sqrt{\frac{Gm}{R}}$$

$$R_{sch} = \frac{2GM}{c^2} \quad (9)$$

This is the expression for the Schwarzschild radius and is given in your formula handbook.

G.8 Evidence to Support General Relativity

G.8.1 Outline the experimental evidence for bending of star light rays by the Sun

Light is bent by gravity. During a solar eclipse when the sunlight is largely blocked, observations of the positions of stars very near to the sun (as seen from the surface of the earth) can be easily made. When this is done, it is found that the stars do not appear to be in the correct place (as measured when they were not close to the sun, i.e. a different time of year). Why? Because the light was bent by the sun, when the light was bent it appeared that the stars were not in the correct position.

Source: <http://ibphysicsstuff.wikidot.com/general-relativity>