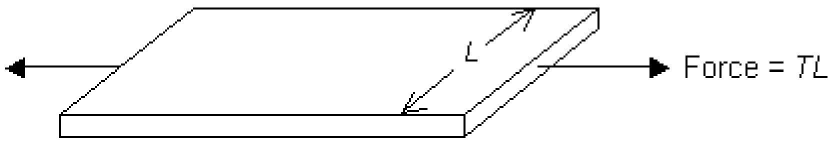
Elasticity in Biological Materials : **Aneurysms**

Hardened and weakened arteries can show elastic instabilities such as aneurysms. A hardened artery can be modelled as a long cylindrical balloon, with radius r and internal pressure P. Consider the tension, T, in the polymer sheet as a function of the extension ratio, λ. The advantage of using T is that it allows us to ignore changes in the thickness of the polymer sheet, which may be considerable at large λ. The units of T are force/length.

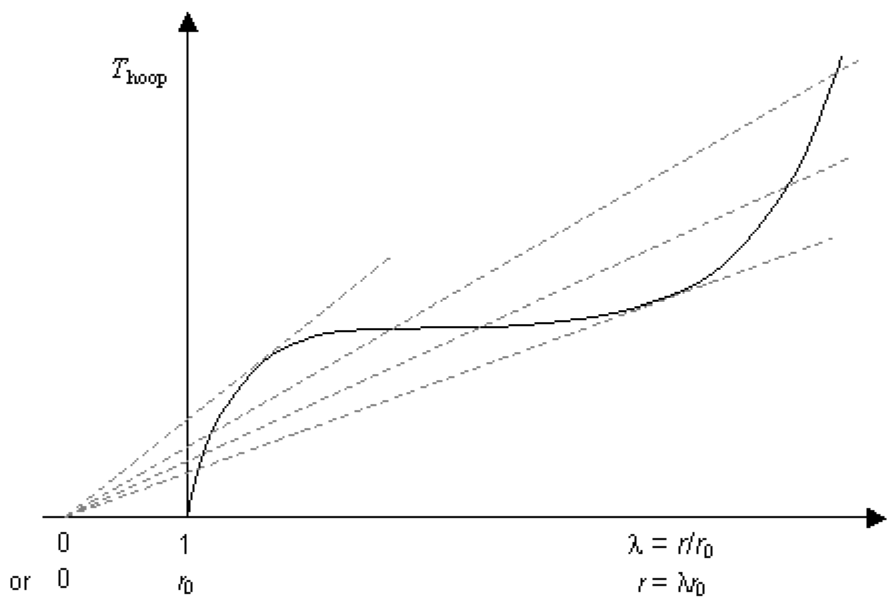


For a cylinder of radius r, the pressure is related to the tension in the hoop direction, T_{hoop}, by:

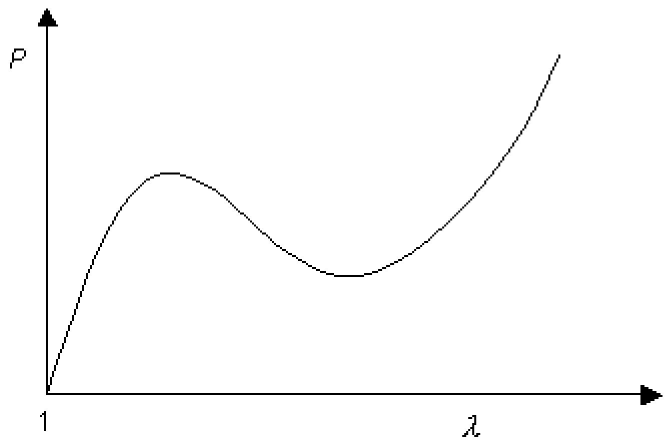
$$P = \frac{T_{hoop}}{r}$$

An increased hoop tension for a given radius, or a decreased radius for a given hoop tension gives an increased pressure.

A plot of T_{hoop} against λ for the balloon rubber would show a characteristic S-shaped curve, similar (but not identical) in shape to the stress - strain (σ — ε) curve:



The gradients of the dashed lines are proportional to the pressure inside the balloon at the values of extension ratio (or radius) at which they cross the curve. This allows us to obtain the curve showing the variation of pressure with extension ratio:



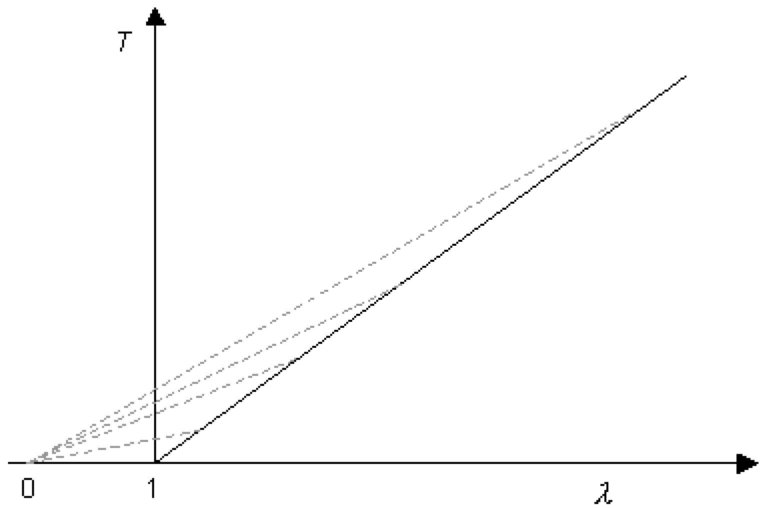
Although the tension in the balloon only ever rises with λ, it can be seen that this is not the case with pressure. The fact that there are regions of the graph for which

$$dP/d\lambda < 0$$

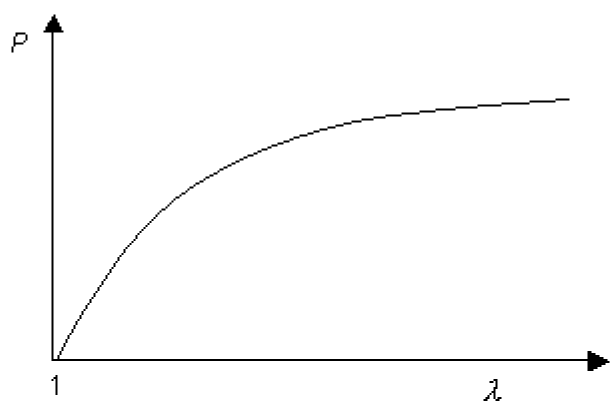
allows the occurrence of different radii at equal pressures, which leads to the formation of aneurysms.

This behaviour can be observed in modelling balloons, as shown in the next section.

Now consider Hookean behaviour:



This then gives:



In this case, there is no region of

$$\frac{dP}{d\lambda} < 0$$

(although the gradient does tend to zero at large extension ratios). Thus if an artery exhibited Hookean elasticity, it should theoretically be stable against aneurysms, but only just, as the pressure inside the tubes is almost independent of its radius. However, since a normal artery exhibits natural variations along its length, Hookean behaviour in the arterial wall would not provide a strong safeguard against aneurysms. In fact, arteries exhibit J-shaped stress-strain curves as described later. For J-shaped curves $dP/d\lambda$ is again always greater than zero but now does not tend to zero at large extension ratios. This provides greater stability against aneurysms.

Source : http://www.doitpoms.ac.uk/tlplib/bioelasticity/aneurysms.php