DAMPING CROSS-REFERENCE

By Tom Irvine
Email: tomirvine@aol.com

August 9, 1999

There are at least eleven parameters commonly used to express damping. Cross-reference formulas are given in Tables 1A through 1C. The formulas are taken from Reference 1.

Let ω_n be the natural frequency in units of radians per second. Note that $\omega_n = 2\pi f_n$, where f_n is in units of Hertz.

Table 1A. Damping Reference

Parameter	3 dB Bandwidth $\Delta\omega$ (rad/sec)	3 dB Bandwidth Δf (Hz)	Damping Frequency fd (Hz)	Loss Factor η
3 dB Bandwidth $\Delta\omega$ (rad/sec)	–	$\Delta\omega = 2\pi\Delta f$	$\Delta\omega = 4\pi fd$	$\Delta\omega = \omega_n \eta$
3 dB Bandwidth Δf (Hz)	$\Delta f = \frac{\Delta\omega}{2\pi}$	–	$\Delta f = 2fd$	$\Delta f = \frac{\omega_n \eta}{2\pi}$
Damping Frequency fd (Hz)	$fd = \frac{\Delta\omega}{4\pi}$	$fd = \frac{\Delta f}{2}$	–	$fd = \frac{\omega_n \eta}{4\pi}$
Loss Factor η	$\eta = \frac{\Delta\omega}{\omega_n}$	$\eta = \frac{2\pi\Delta f}{\omega_n}$	$\eta = \frac{4\pi fd}{\omega_n}$	–
Fraction of Critical Damping ζ	$\zeta = \frac{\Delta\omega}{2\omega_n}$	$\zeta = \frac{\pi\Delta f}{\omega_n}$	$\zeta = \frac{2\pi fd}{\omega_n}$	$\zeta = \frac{\eta}{2}$
Quality Factor Q	$Q = \frac{\omega_n}{\Delta\omega}$	$Q = \frac{\omega_n}{2\pi\Delta f}$	$Q = \frac{\omega_n}{4\pi fd}$	$Q = \frac{1}{\eta}$
Decay Constant σ (1/sec)	$\sigma = \frac{\Delta\omega}{2}$	$\sigma = \pi\Delta f$	$\sigma = 2\pi fd$	$\sigma = \frac{\omega_n \eta}{2}$
Time Constant τ (sec)	$\tau = \frac{2}{\Delta\omega}$	$\tau = \frac{1}{\pi\Delta f}$	$\tau = \frac{1}{2\pi fd}$	$\tau = \frac{2}{\omega_n \eta}$
Reverberation Time RT_{60} (sec)	$RT_{60} = \frac{13.8}{\Delta\omega}$	$RT_{60} = \frac{2.2}{\Delta f}$	$RT_{60} = \frac{1.1}{fd}$	$RT_{60} = \frac{13.8}{\omega_n \eta}$
Decay Rate D (dB/sec)	$D = 4.34\Delta\omega$	$D = 27.3\Delta f$	$D = 54.6 fd$	$D = 4.34\omega_n \eta$
Logarithmic Decrement δ	$\delta = \frac{\pi\Delta\omega}{\omega_n}$	$\delta = \frac{2\pi^2 \Delta f}{\omega_n}$	$\delta = \frac{4\pi^2 fd}{\omega_n}$	$\delta = \pi\eta$

Table 1B. Damping Reference				
Parameter	Fraction of Critical Damping ζ	Quality Factor Q	Decay Constant σ (1/sec)	Time Constant τ (sec)
3 dB Bandwidth $\Delta\omega$ (rad/sec)	$\Delta\omega = 2\omega_n \zeta$	$\Delta\omega = \frac{\omega_n}{Q}$	$\Delta\omega = 2\sigma$	$\Delta\omega = \frac{2}{\tau}$
3 dB Bandwidth Δf (Hz)	$\Delta f = \frac{\omega_n \zeta}{\pi}$	$\Delta f = \frac{\omega_n}{2\pi Q}$	$\Delta f = \frac{\sigma}{\pi}$	$\Delta f = \frac{1}{\pi\tau}$
Damping Frequency fd (Hz)	$fd = \frac{\omega_n \zeta}{2\pi}$	$fd = \frac{\omega_n}{4\pi Q}$	$fd = \frac{\sigma}{2\pi}$	$fd = \frac{1}{2\pi\tau}$
Loss Factor η	$\eta = 2\zeta$	$\eta = \frac{1}{Q}$	$\eta = \frac{2\sigma}{\omega_n}$	$\eta = \frac{2}{\omega_n \tau}$
Fraction of Critical Damping ζ	–	$\zeta = \frac{1}{2Q}$	$\zeta = \frac{\sigma}{\omega_n}$	$\zeta = \frac{1}{\omega_n \tau}$
Quality Factor Q	$Q = \frac{1}{2\zeta}$	–	$Q = \frac{\omega_n}{2\sigma}$	$Q = \frac{\omega_n \tau}{2}$
Decay Constant σ (1/sec)	$\sigma = \omega_n \zeta$	$\sigma = \frac{\omega_n}{2Q}$	–	$\sigma = \frac{1}{\tau}$
Time Constant τ (sec)	$\tau = \frac{1}{\omega_n \zeta}$	$\tau = \frac{2Q}{\omega_n}$	$\tau = \frac{1}{\sigma}$	–
Reverberation Time RT_{60} (sec)	$RT_{60} = \frac{6.9}{\omega_n \zeta}$	$RT_{60} = \frac{13.8Q}{\omega_n}$	$RT_{60} = \frac{6.9}{2\sigma}$	$RT_{60} = 6.9\tau$
Decay Rate D (dB/sec)	$D = 8.68\omega_n \zeta$	$D = \frac{4.34\omega_n}{Q}$	$D = 8.68\sigma$	$D = \frac{8.68}{\tau}$
Logarithmic Decrement δ	$\delta = 2\pi\zeta$	$\delta = \frac{\pi\Delta\omega}{Q}$	$\delta = \frac{2\pi\sigma}{\omega_n}$	$\delta = \frac{2\pi}{\omega_n \tau}$

Table 1C. Damping Reference			
Parameter	Reverberation Time RT_{60} (sec)	Decay Rate D (dB/sec)	Logarithmic Decrement δ
3 dB Bandwidth $\Delta\omega$ (rad/sec)	$\Delta\omega = \frac{13.8}{RT_{60}}$	$\Delta\omega = \frac{D}{4.34}$	$\Delta\omega = \frac{\omega_n \delta}{\pi}$
3 dB Bandwidth Δf (Hz)	$\Delta f = \frac{2.2}{RT_{60}}$	$\Delta f = \frac{D}{27.3}$	$\Delta f = \frac{\omega_n \delta}{2\pi^2}$
Damping Frequency fd (Hz)	$fd = \frac{1.1}{RT_{60}}$	$fd = \frac{D}{54.5}$	$fd = \frac{\omega_n \delta}{4\pi^2}$
Loss Factor η	$\eta = \frac{13.8}{\omega_n RT_{60}}$	$\eta = \frac{D}{4.34\omega_n}$	$\eta = \frac{\delta}{\pi}$
Fraction of Critical Damping ζ	$\zeta = \frac{6.90}{\omega_n RT_{60}}$	$\zeta = \frac{D}{8.68\omega_n}$	$\zeta = \frac{\delta}{2\pi}$
Quality Factor Q	$Q = \frac{\omega_n RT_{60}}{13.8}$	$Q = \frac{4.34\omega_n}{D}$	$Q = \frac{\pi\omega_n}{\delta}$
Decay Constant σ (1/sec)	$\sigma = \frac{6.90}{RT_{60}}$	$\sigma = \frac{D}{8.68}$	$\sigma = \frac{\omega_n \delta}{2\pi}$
Time Constant τ (sec)	$\tau = \frac{RT_{60}}{6.90}$	$\tau = \frac{8.68}{D}$	$\tau = \frac{2\pi}{\omega_n \delta}$
Reverberation Time RT_{60} (sec)	–	$RT_{60} = \frac{60}{D}$	$RT_{60} = \frac{43.4}{\omega_n \delta}$
Decay Rate D (dB/sec)	$D = \frac{60}{RT_{60}}$	–	$D = 1.38\omega_n \delta$
Logarithmic Decrement δ	$\delta = \frac{43.4}{\omega_n RT_{60}}$	$\delta = \frac{\pi D}{4.34\omega_n}$	–

<u>Reference</u>

1. Svend Gade and Henrik Herlufsen, "Digital Filter versus FFT Techniques for Damping Measurement," Sound and Vibration, Bay Village, Ohio, March 1990.