WHY LAWS OF MOTION ARE POSSIBLE

The first relativity principle: Galilean relativity

In the history of physics, recognition of the central position of inertia has proceeded in stages. With the benefit of the current knowledge the Copernican revolution can be recognized as the first step to making inertia the prime organizing principle of dynamics understanding. (Here, I take 'Copernican revolution' in its widest possible sense: the revolution in thinking about motion that was completed by Newton's work.)

Galilei pointed out why it is possible to home in on laws of motion. Imagine you are in a cabin of a boat that is in motion over perfectly smooth water. I will call this particular boat the test boat. The test boat does not accelerate in any way, it is in uniform motion. You are on the test boat, juggling, or you are throwing darts, or something like that. (Galilei used other examples.) Galilei argued: no matter the velocity of the test boat, the skill for juggling the balls or throwing the darts is the same. There is a large range of circumstances that are different (boats having a velocity relative to each other), where as far as juggling is concerned things are the same for each separate boat. To become a skilful juggler is the acquirement of implicit knowledge (motoric knowledge) of how to work with the properties of motion.

The grid of the dartboard serves as a reference, the darter's aim is with respect to that reference system. One layer of 'the same' is that it does not matter where in space the dartboard is positioned, or in what direction the dartboard is facing; the properties of motion are the same. A second layer is that given a state of unaccelerated motion of the test boat its velocity relative to other boats does not matter either, the properties of motion as experienced on the test boat are always the same.

Symmetry of inertia

If you exert a force on an object, causing it to accelerate, then for all directions the same force results in the same acceleration; the isotropy of inertia. (Isotropy = same in all directions). For all locations and orientations in space, inertia is symmetrical. Likewise, for all uniform velocities, inertia is symmetrical. Imagine the opposite: imagine that inertia would be erratic, changing from place to place and from one instant to another - motion would be lawless then. But inertia is extremely symmetric. Formulating laws of motion is a fruitful undertaking because of the extensive symmetries of inertia.

A theory of motion describes the properties of inertia; in order to formulate a theory of motion it is necessary to have a way of embodying the symmetries of inertia. When a mathematical theory of motion was formulated, culminating in the work of Isaac Newton, embodying these symmetries was straightforward: euclidean space has the same symmetries as inertial space.

In retrospect we can recognize that the Copernican revolution changed the role that geometry played in physics. For the ancients the geometry was an instrument for describing shapes, the trajectory of a planet was thought of as a shape, and this shape was described by geometry. But in newtonian dynamics the euclidean geometry plays a much more profound role. In Newtonian dynamics inertia is the prime organizing principle and euclidean geometry is appropriate because it perfectly models the *symmetries of inertia*. In retrospect we can recognize that the act of using euclidean geometry as instrument for describing motion is actually the adoption of a *physics*theory.

Equivalence class of coordinate systems

The euclidean space is represented with a coordinate system (a coordinate grid). A coordinate system has a zero point and it has orientation. The comprehensive representation of the symmetries of inertial space must be specified as an *equivalence class of coordinate systems*.

Linear transformations

The symmetry demands narrow down the transformations to the following:

- Transformations between coordinate systems that are at an angle with respect to each other
- Transformation between coordinate systems that are translated with respect to each other
- Transformations between coordinate systems that have a uniform velocity relative to each other

For any object that is in inertial motion there is a coordinate system that is co-moving with that object. A coordinate system that is co-moving with an object in inertial motion is an inertial coordinate system. What that means is that the law that describes velocity addition of pairs of objects and the law that describes the transformation between inertial coordinate systems are one and the same law.

Source: http://www.cleonis.nl/physics/phys256/special.php#section_1