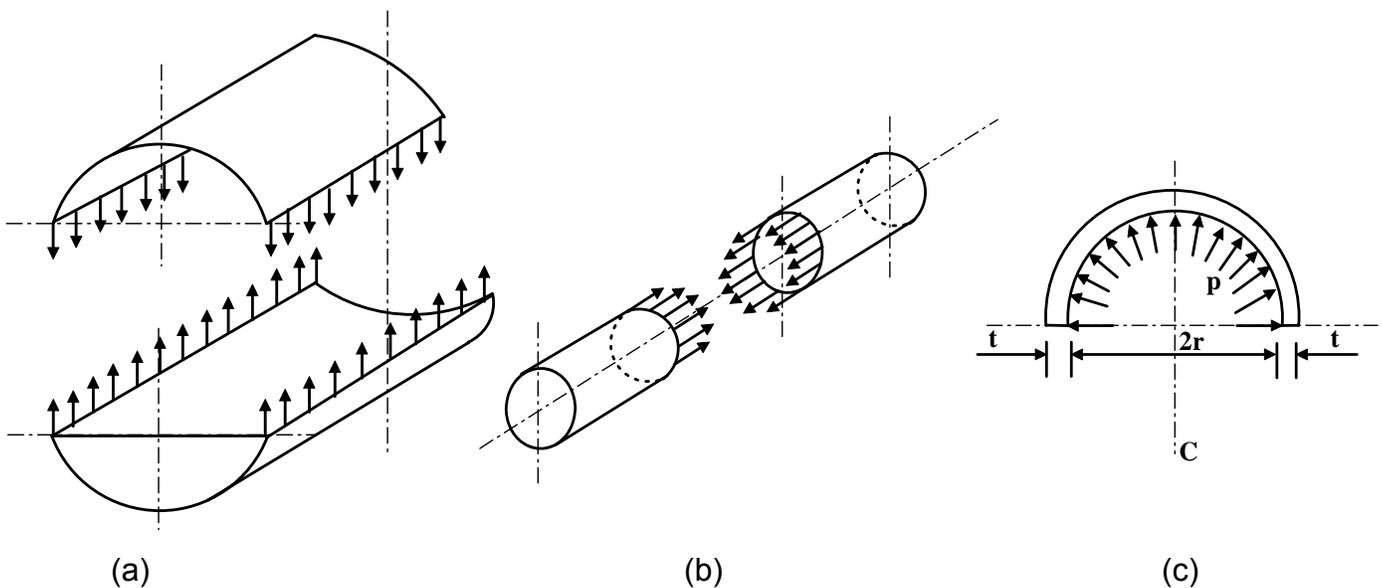# Thin Cylinders

Instructional Objectives:

At the end of this lesson, the students should have the knowledge of:

- Stresses developed in thin cylinders.
- Formulations for circumferential and longitudinal stresses in thin cylinders.
- Basic design principles.
- Joint design; Welded or Riveted.

9.1.1 Stresses in thin cylinders

If the wall thickness is less than about 7% of the inner diameter then the cylinder may be treated as a thin one. Thin walled cylinders are used as boiler shells, pressure tanks, pipes and in other low pressure processing equipments. In general three types of stresses are developed in pressure cylinders viz. circumferential or hoop stress, longitudinal stress in closed end cylinders and radial stresses. These stresses are demonstrated in **figure-9.1.1.1**.

(a)   (b)   (c)

9.1.1.1F- (a) Circumferential stress (b) Longitudinal stress and (c) Radial stress developed in thin cylinders.
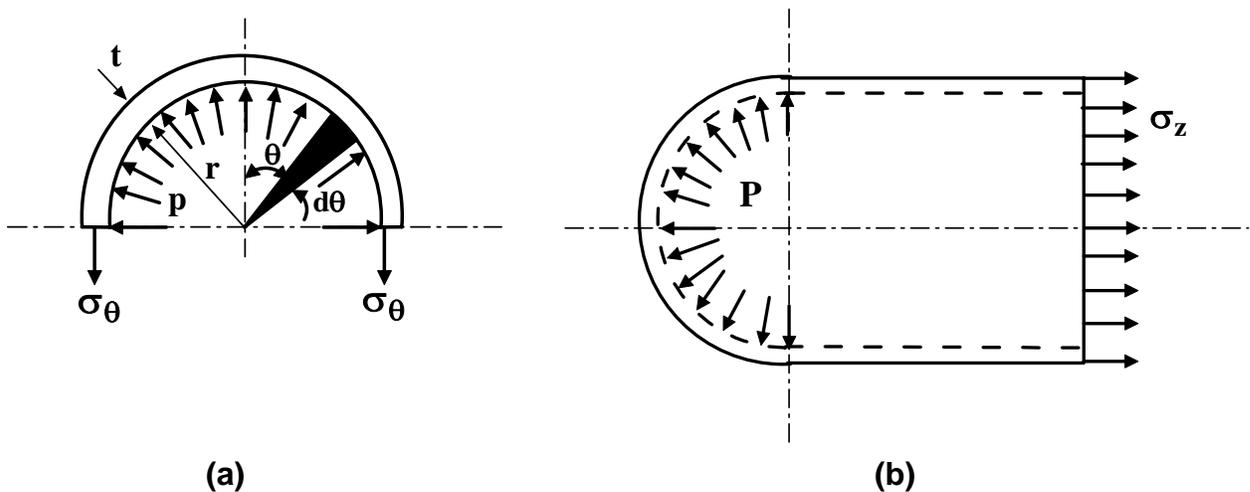
In a thin walled cylinder the circumferential stresses may be assumed to be constant over the wall thickness and stress in the radial direction may be neglected for the analysis. Considering the equilibrium of a cut out section the circumferential stress σ_θ and longitudinal stress σ_z can be found. Consider a section of thin cylinder of radius r, wall thickness t and length L and subjected to an internal pressure p as shown in **figure-9.1.1.2(a)**. Consider now an element of included angle $d\theta$ at an angle of θ from vertical. For equilibrium we may write

$$2\int_{0}^{\frac{\pi}{2}} prd\theta L\cos\theta = 2\sigma_\theta tL$$

This gives $\sigma_\theta = \dfrac{pr}{t}$

Considering a section along the longitudinal axis as shown in **figure-9.1.1.2 (b)** we may write $p\pi r^2 = \sigma_z \pi \left(r_o^2 - r_i^2\right)$

where r_i and r_o are internal and external radii of the vessel and since $r_i \approx r_o = r$

(say) and $\qquad r_o - r_i = t$ we have $\sigma_z = $ ——

(a)   (b)

9.1.1.2F- (a) Circumferential stress in a thin cylinder (b) Longitudinal stress in a thin cylinder

Thin walled spheres are also sometimes used. Consider a sphere of internal radius r subjected to an internal pressure p as shown in **figure-9.1.1.3**. The circumferential and longitudinal stresses developed on an element of the surface of the sphere are equal in magnitude and in the absence of any shear stress due to symmetry both the stresses are principal stresses. From the equilibrium condition in a cut section we have

$\sigma_1 = \sigma_2 = $ ——
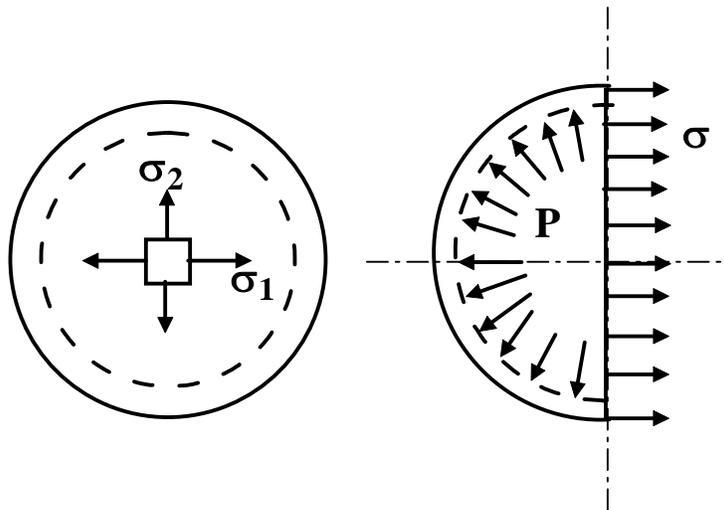
9.1.1.3 F- Stresses in a spherical shell

9.1.2 Design Principles

Pressure vessels are generally manufactured from curved sheets joined by welding. Mostly V– butt welded joints are used. The riveted joints may also be used but since the plates are weakened at the joint due to the rivet holes the plate thickness should be enhanced by taking into account the joint efficiency. It is probably more instructive to follow the design procedure of a pressure vessel. We consider a mild steel vessel of 1m diameter comprising a 2.5 m long cylindrical section with hemispherical ends to sustain an internal pressure of (say) 2MPa.

The plate thickness is given by $t \geq \dfrac{pr}{\sigma_{yt}}$ where σ_{yt} is the tensile yield stress. The minimum plate thickness should conform to the "Boiler code" as given in **table-9.1.2.1**.

9.1.2.1T- Minimum plate thickness

| Boiler diameter(m)	≤ 0.90	0.94 to 1.37	1.4 to 1.80	> 1.80
Plate thickness (mm)	6.35	8.00	9.525	12.70

The factor of safety should be at least 5 and the minimum ultimate stresses of the plates should be 385 MPa in the tension, 665 MPa in compression and 308 MPa in shear.

This gives $t_c \geq \dfrac{2 \times 10^6 \times 0.5}{(385 \times 10^6 / 5)}$, i.e., 13 mm. Since this value is more than the value prescribed in the code the plate thickness is acceptable. However for better safety we take t_c =15mm. Thickness t_s of the hemispherical end is usually taken as half of this value and we take $t_s \approx$ 8mm.

Welded Joint

The circumferential stress developed in the cylinder $\sigma_\theta = \dfrac{pr}{t_c}$. With p=2MPa , r=0.5m and t_c = 15 mm, σ_θ =67 MPa and since this is well below the allowable stress of 100 MPa (assumed) the butt welded joint without cover plate would be adequate.

Consider now a butt joint with 10mm cover plates on both sides, as shown in **figure- 9.1.2.1**.
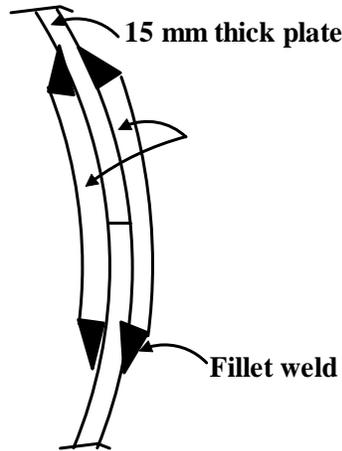
9.1.2.1F- Longitudinal welded joint with cover plates.

The stress induced in the weld σ_w is given by $F_c = 2\sigma_w L t_c \sin 45^0$

where L is the weld length. We may now write $F_c = \sigma_\theta\, t.L$ and therefore σ_w is

given by $\sigma_w = \sigma_\theta \dfrac{t}{t_c\, 2\sin 45^0} = 67\text{x}\dfrac{15}{10\text{x}2\text{x}\sin 45^o}$ which gives σ_w = 71 MPa which

again is adequate. For increased safety we may choose the butt joint with 10mm thick cover plates. The welding arrangement of the vessel is shown in **figure-9.1.2.2.**
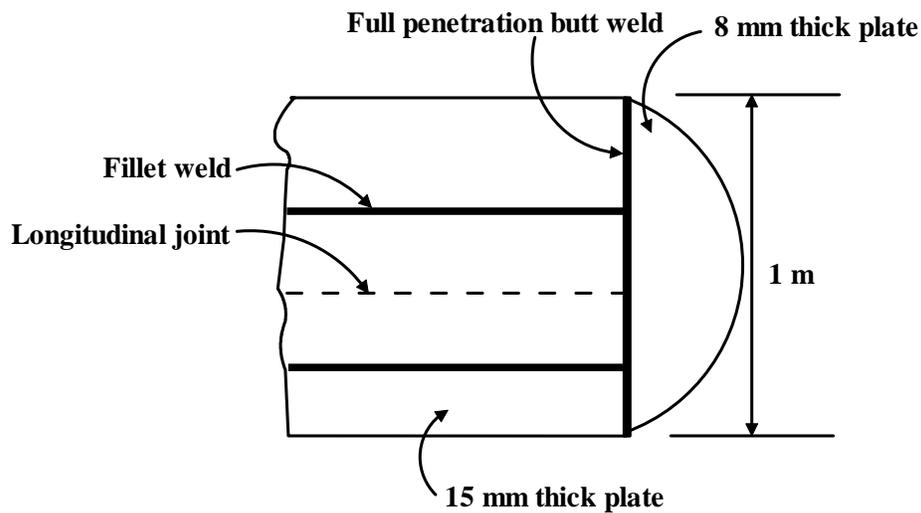
9.1.2.2F- The welding arrangement of the joint.

Riveted Joint

The joints may also be riveted in some situations but the design must be checked for safety. The required plate thickness must take account the joint efficiency η.

This gives $t_c = \dfrac{pr}{\eta\sigma_{ty}}$ Substituting p = 2MPa, r = 0.5 m, η = 70 % and σ_{ty} = (385/5)

MPa we have t_c = 18.5 mm. Let us use mild steel plate of 20 mm thickness for the cylinder body and 10mm thick plate for the hemispherical end cover. The cover plate thickness may be taken as 0.625t_c i.e. 12.5 mm. The hoop stress is

now given by $\sigma_\theta = \dfrac{pr}{t_c} = 50\text{MPa}$ and therefore the rivets must withstand $\sigma_\theta t_c$ i.e. 1
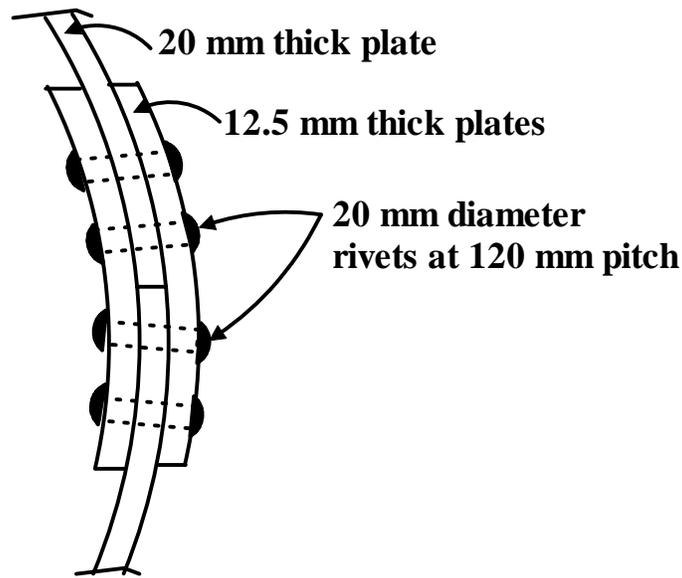
MN per meter.

We may begin with 20mm diameter rivets with the allowable shear and bearing stresses of 100 MPa and 300 MPa respectively. This gives bearing load on a single rivet F_b = 300x10^{6}x0.02x0.02 = 120 kN. Assuming double shear
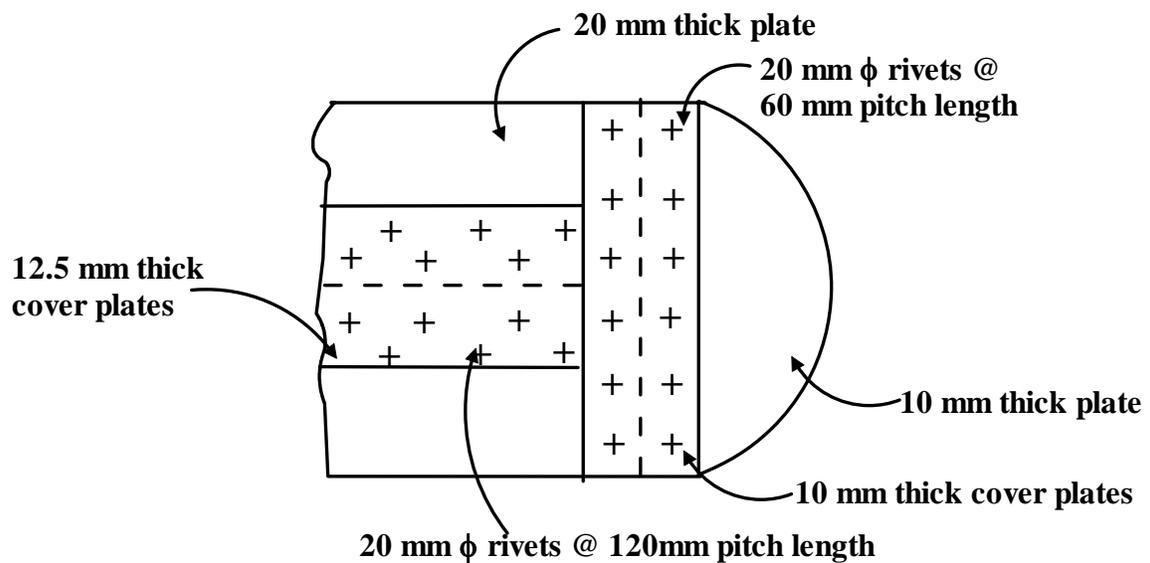
the shearing load on a single rivet F_S = 100x10^{6}x2x$\dfrac{\pi}{4}(0.02)^2 = 62.8\text{kN}$.

The rivet pitch based on bearing load is therefore (120 kN/ 1MN per meter) i.e. 0.12m and based on shearing load is (62.8 kN/ 1MN per meter) i.e. 0.063m. We may therefore consider a minimum allowable pitch of 60mm. This gives approximately 17 rivets of 20 mm diameter per meter. If two rows are used the pitch is doubled to 120mm. For the hemispherical shaped end cover the bearing load is 60 kN and therefore the rivet pitch is again approximately 60 mm.

The maximum tensile stress developed in the plate section is σ_t = 1x10^6/[(1-17x0.02)x0.02] = 75.76 MPa which is a safe value considering the allowable tensile stress of 385 MPa with a factor of safety of 5. A longitudinal riveted joint with cover plates is shown in **figure–9.1.2.3** and the whole riveting arrangement is shown in **figure-9.1.2.4.**

20 mm thick plate

12.5 mm thick plates

**20 mm diameter
rivets at 120 mm pitch**

9.1.2.3F- A longitudinal joint with two cover plates

20 mm thick plate

**20 mm ϕ rivets @
60 mm pitch length**

**12.5 mm thick
cover plates**

10 mm thick plate

10 mm thick cover plates

20 mm ϕ rivets @ 120mm pitch length

9.1.2.4F- General riveting arrangement of the pressure vessel.

9.1.3 Summary of this Lesson

Stresses developed in thin cylinders are first discussed in general and then the circumferential (σ_θ) and longitudinal stresses (σ_z) are expressed in terms of internal pressure, radius and the shell thickness. Stresses in a spherical shell are also discussed. Basic design principle of thin cylinders are considered. Design of both welded and riveted joints for the shells are discussed.