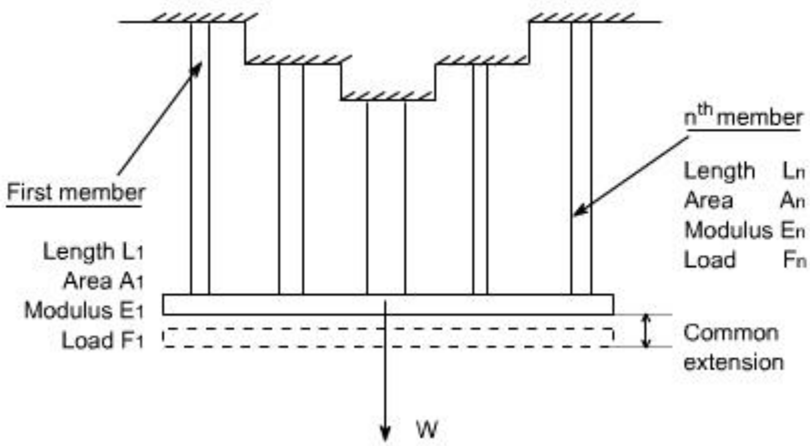
Thermal stresses, Bars subjected to tension and Compression

Compound bar: In certain application it is necessary to use a combination of elements or bars made from different materials, each material performing a different function. In over head electric cables or Transmission Lines for example it is often convenient to carry the current in a set of copper wires surrounding steel wires. The later being designed to support the weight of the cable over large spans. Such a combination of materials is generally termed compound bars.

Consider therefore, a compound bar consisting of n members, each having a different length and cross sectional area and each being of a different material. Let all member have a common extension 'x' i.e. the load is positioned to produce the same extension in each member.



For the 'n' the members

$$\frac{\text{stress}}{\text{strain}} = E_n = \frac{F_n / A_n}{x_n / L_n}$$

$$= \frac{F_n . L_n}{A_n . x_n}$$

or $$F_n = \frac{E_n . A_n . x_n}{L_n} = \frac{E_n . A_n . x}{L_n} \qquad .....(1)$$

Where F_n is the force in the nth member and A_n and L_n are its cross - sectional area and length.

Let W be the total load, the total load carried will be the sum of all loads for all the members.

$$W = \sum \frac{E_n . A_n . x}{L_n}$$

$$= x . \sum \frac{E_n . A_n}{L_n} \qquad ....(2)$$

From equation (1), force in member 1 is given as

$$F_1 = \frac{E_1 . A_1 . x}{L_1}$$

from equation (2)

$$x = \frac{W}{\sum \frac{E_n . A_n}{L_n}}$$

$$\text{Thus, } F_1 = \frac{E_1 . A_1}{L_1} . \frac{W}{\sum \left(\frac{E_n . A_n}{L_n} \right)}$$

Therefore, each member carries a portion of the total load W proportional of EA / L value.

The above expression may be writen as

$$F_1 = \frac{\frac{E_1 . A_1}{L_1}}{\sum \frac{E_n . A_n}{L_n}} . W$$

if the length of each individual member in same then, we may write

$$F_1 = \frac{E_1 . A_1}{\sum E . A} . W$$

Thus, the stress in member '1' may be determined as $\sigma_1 = F_1 / A_1$

Determination of common extension of compound bars: In order to determine the common extension of a compound bar it is convenient to consider it as a single bar of an imaginary material with an equivalent or combined modulus E_c.

Assumption: Here it is necessary to assume that both the extension and original lengths of the individual members of the compound bar are the same, the strains in all members will than be equal.

Total load on compound bar = $F_1 + F_2 + F_3 + \ldots\ldots + F_n$

where F_1 , F_2 ,….,etc are the loads in members 1,2 etc

But force = stress . area,therefore

$\sigma (A_1 + A_2 + \ldots\ldots + A_n) = \sigma_1 A_1 + \sigma_2 A_2 + \ldots\ldots + \sigma_n A_n$

Where σ is the stress in the equivalent single bar

Dividing throughout by the common strain $\in$.

$$\frac{\sigma}{\in}(A_1 + A_2 + \ldots + A_n) = \frac{\sigma_1}{\in}A_1 + \frac{\sigma_2}{\in}A_2 + \ldots \frac{\sigma_n}{\in}A_n$$

$$\text{i.e } E_c(A_1 + A_2 + \ldots + A_n) = E_1A_1 + E_2A_2 + \ldots E_n.A_n$$

$$\text{or } E_c = \frac{E_1A_1 + E_2A_2 + \ldots E_n.A_n}{A_1 + A_2 + \ldots + A_n}$$

$$\text{or } E_c = \frac{\sum EA}{\sum A}$$

with an external load W applied stress in the equivalent bar may be computed as

$$\text{stress} = \frac{W}{\sum A}$$

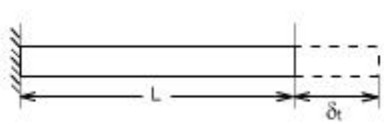
$$\text{strain inthe equivalent bar} = \frac{x}{L} = \frac{W}{\sum AE_c}$$

$$\text{hence commen extension } x = \frac{W.L}{E_c \sum A}$$

Compound bars subjected to Temp. Change : Ordinary materials expand when heated and contract when cooled, hence , an increase in temperature produce a positive thermal strain. Thermal strains usually are reversible in a sense that the member returns to its original shape when the temperature return to its original value. However, there here are some materials which do not behave in this manner. These metals differs from ordinary materials in a sence that the strains are related non linearly to temperature and some times are irreversible .when a material is subjected to a change in temp. is a length will change by an amount.

$$\delta_t = \alpha .L.t$$

or $\in_t = \alpha L.t$ or $\sigma_t = E .\alpha.t$



α = coefficient of linear expansoin for the material

L = original Length

t = temp. change

Thus an increase in temperature produces an increase in length and a decrease in temperature results in a decrease in length except in very special cases of materials with zero or negative coefficients of expansion which need not to be considered here.

If however, the free expansion of the material is prevented by some external force, then a stress is set up in the material. They stress is equal in magnitude to that which would be produced in the bar by initially allowing the bar to its free length and then applying sufficient force to return the bar to its original length.

Change in Length = α L t

Therefore, strain = α L t / L

= α t

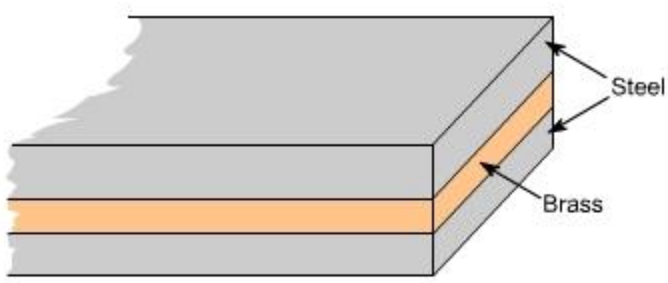
Therefore ,the stress generated in the material by the application of sufficient force to remove this strain

= strain x E

or Stress = E α t

Consider now a compound bar constructed from two different materials rigidly joined together, for simplicity.

Let us consider that the materials in this case are steel and brass.

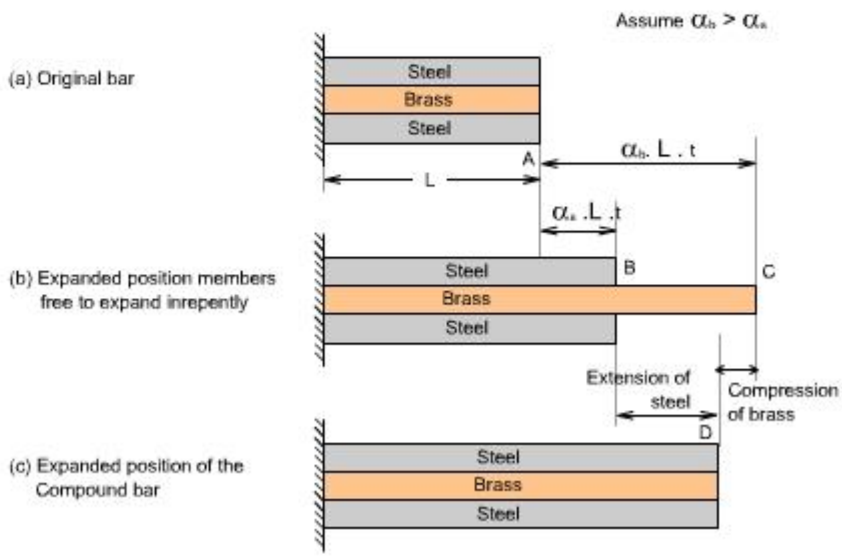


If we have both applied stresses and a temp. change, thermal strains may be added to those given by generalized hook's law equation –e.g.

$$\in_x = \frac{1}{E}\left[\sigma_x - \gamma(\sigma_y + \sigma_z)\right] + \alpha\Delta t$$
$$\in_x = \frac{1}{E}\left[\sigma_y - \gamma(\sigma_x + \sigma_z)\right] + \alpha\Delta t$$
$$\in_x = \frac{1}{E}\left[\sigma_z - \gamma(\sigma_x + \sigma_y)\right] + \alpha\Delta t$$

While the normal strains a body are affected by changes in temperatures, shear strains are not. Because if the temp. of any block or element changes, then its size changes not its shape therefore shear strains do not change.

In general, the coefficients of expansion of the two materials forming the compound bar will be different so that as the temp. rises each material will attempt to expand by different amounts. Figure below shows the positions to which the individual materials will expand if they are completely free to expand (i.e not joined rigidly together as a compound bar). The extension of any Length L is given by α L t



In general, changes in lengths due to thermal strains may be calculated form equation $\delta_t = \alpha$ Lt, provided that the members are able to expand or contract freely, a situation that exists in statically determinates structures. As a consequence no stresses are generated in a statically determinate structure when one or more members undergo a uniform temperature change. If in a structure (or a compound bar), the free expansion or contraction is not allowed then the member becomes s statically indeterminate, which is just being discussed as an example of the compound bar and thermal stresses would be generated.

Thus the difference of free expansion lengths or so called free lengths

$= \alpha_B.L.\ t - \alpha_s\ .L\ .t$

$= (\ \alpha_B - \alpha_s\ ).L\ .t$

Since in this case the coefficient of expansion of the brass α_B is greater then that for the steel α_s. the initial lengths L of the two materials are assumed equal.

If the two materials are now rigidly joined as a compound bar and subjected to the same temp. rise, each materials will attempt to expand to its free length position but each will be affected by the movement of the other. The higher coefficient of expansion material (brass) will therefore, seek to pull the steel up to its free length position and conversely, the lower coefficient of expansion martial (steel) will try to hold the brass back. In practice a compromised is reached, the compound bar extending to the position shown in fig (c), resulting in an effective compression of the brass from its free length position and an effective extension of steel from its free length position.

Therefore, from the diagrams,we may conclude thefollowing

Conclusion 1.

Extension of steel + compression brass = difference in “ free” length

Applying Newton 's law of equal action and reaction the following second Conclusion also holds good.

Conclusion 2.

The tensile force applied to the short member by the long member is equal in magnitude to the compressive force applied to long member by the short member.

Thus in this case

Tensile force in steel = compressive force in brass

These conclusions may be written in the form of mathematical equations as given below:

for conclusion 1

$$\frac{\sigma_s . L}{E_s} + \frac{\sigma_B . L}{E_B} = (\alpha_B - \alpha_s) L . t$$

for conclusion 2

$$\sigma_s . A_s = \sigma_B . A_B$$

Using these two equations, the magnitude of the stresses may be determined.

Source: http://nptel.ac.in/courses/Webcourse-contents/IIT-ROORKEE/strength%20of%20materials/homepage.htm