The First Law of Thermodynamics for Closed Systems Part II

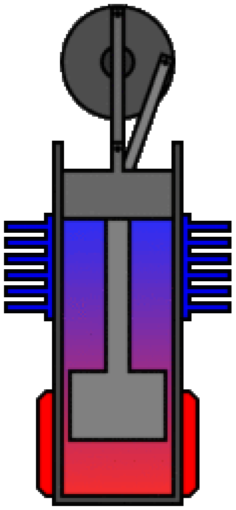
Ideal Stirling Cycle Machines (Engines / Coolers)

The Stirling Cycle Engine

Conceptually the Stirling engine is the simplest of all heat engines. It has no valves, and includes an externally heated space and an externally cooled space. It was invented by Robert Stirling, and an interesting website by **Bob Sier** includes a photograph of Robert Stirling, his original patent drawing of 1816, and an animated model of Stirling's original engine.

In its original single cylinder form the working gas (typically air or helium) is sealed within its cylinders by the piston and shuttled between the hot and cold spaces by a displacer. The linkage driving the piston and displacer will move them such that the gas will compress while it is mainly in the cool compression space and expand while in the hot expansion space. This is clearly illustrated in the adjacent animation which was produced by Richard Wheeler (**Zephyris**) of **Wikipedia**.

Refer also to the animation produced by **Matt Keveney** in his **Stirling engine animation** website. Since the gas is at a higher temperature, and therefore pressure, during its expansion than during its compression, more power is produced during expansion than is reabsorbed during compression, and this net excess power is the useful output of the engine. Note that there are no valves or intermittent combustion, which is the major source of noise in an internal combustion engine. The same working gas is used over and over again, making the Stirling engine a sealed, closed cycle system. All that is added to the system is steady high temperature heat, and all that is removed from the system is low temperature (waste) heat and mechanical power.



Athens, Ohio, is a hotbed of Stirling cycle machine activity, both engines and coolers, and includes R&D and manufacturing companies as well as internationally recognized consultants in the area of Stirling cycle computer analysis. The parent company of this activity is **Sunpower, Inc**. It was formed by William Beale in the early 1970's, mainly based on his invention of the free-piston Stirling engine which we describe below. **Update (Jan. 2013)**: Sunpower was recently acquired by **AMETEK, Inc** in Pensylvania, however continues doing Stirling cycle machine development in Athens, Ohio. **Update (Nov. 2013)**: Sunpower has recently introduced a **1 kW Stirling Developers Kit** based on a free piston Stirling engine fired by Propane or natural gas.

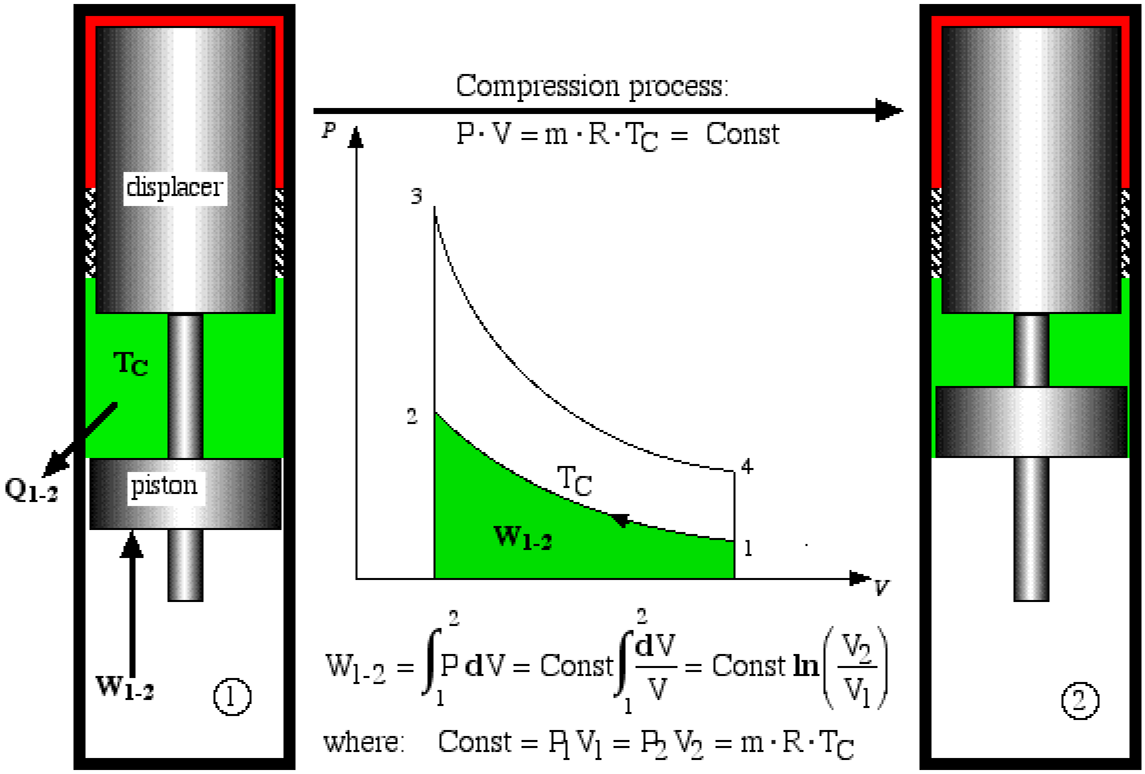
Some examples of single cylinder Stirling engines: Stirling Technology Inc. is a spinoff of Sunpower, and was formed in order to continue the development and manufacture of the 5 kW ST-5 Air engine. This large single cylinder engine burns biomass fuel (such as sawdust pellets or rice husks) and can function as a cogeneration unit in rural areas. It is not a free-piston engine, and uses a bell crank mechanism to obtain the correct displacer phasing. Another important early Stirling engine is Lehmann's machine on which Gusav Schmidt did the first reasonable analysis of Stirling engines in 1871. Andy Ross of Columbus, Ohio built a small working replica of the **Lehmann machine**, as well as a **model air engine**.

Solar Heat and Power Cogeneration: With the current energy and global warming crises, there is renewed interest in renewable energy systems, such as wind and solar energy, and distributed heat and power cogeneration systems. **Cool Energy, Inc** of Boulder, Colorado, is currently in advanced stages of developing a complete solar heat and power cogeneration system for home usage incorporating Stirling engine technology for electricity generation. This unique application includes **evacuated tube solar thermal collectors**, thermal storage, hot water and space heaters, and a Stirling engine/generator.

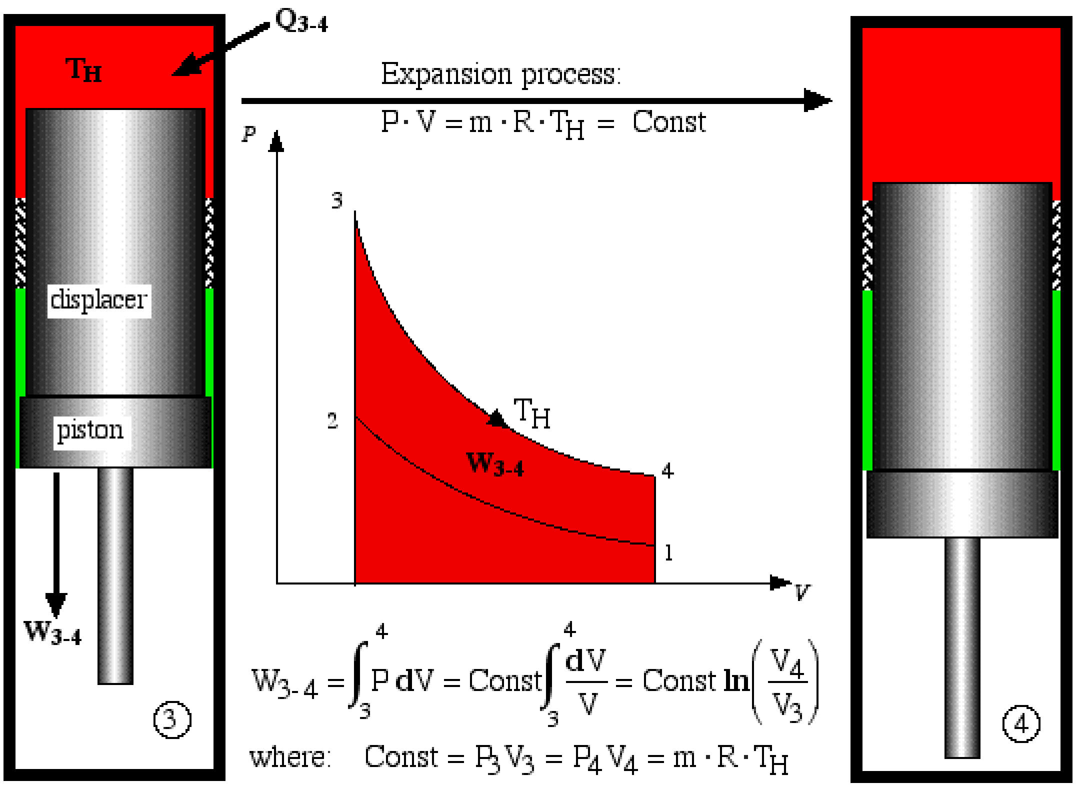
Ideal Analysis: Please note that the following analysis of Stirling cycle engines is ideal, and is intended only as an example of **First Law Analysis** of closed systems. In the real world we cannot expect actual machines to perform any better than 40 - 50% of the ideal machine. The analysis of actual Stirling cycle machines is extremely complex and requires sophisticated computer analysis (see for example the course notes on: **Stirling Cycle Machine Analysis**.)

The free-piston Stirling engine developed by Sunpower, Inc is unique in that there is no mechanical connection between the piston and the displacer, thus the correct phasing between them occurs by use of gas pressure and spring forces. Electrical power is removed from the engine by permanent magnets attached to the piston driving a linear alternator. Basically the ideal Stirling engine undergoes 4 distinct processes, each one of which can be separately analysed, as shown in the *P-V* diagram below. We consider first the work done during all four processes.

- Process 1-2 is the compression process in which the gas is compressed by the piston while the displacer is at the top of the cylinder. Thus during this process the gas is cooled in order to maintain a constant temperature T_C. Work W_{1-2} required to compress the gas is shown as the area under the *P-V* curve, and is evaluated as follows.



- Process 2-3 is a constant volume displacement process in which the gas is displaced from the cold space to the hot expansion space. No work is done, however as we shall see below, a significant amount of heat Q_R is absorbed by the gas from the regenerator matrix.
- Process 3-4 is the isothermal expansion process. Work $W_{3\text{-}4}$ is done by the system and is shown as the area under the *P-V* diagram, while heat $Q_{3\text{-}4}$ is added to the system from the heat source, maintaining the gas at a constant temperature T_H.



$$P \cdot V = m \cdot R \cdot T_H = \text{Const}$$

$$W_{3-4} = \int_3^4 P\,\mathbf{d}V = \text{Const} \int_3^4 \frac{\mathbf{d}V}{V} = \text{Const}\ \mathbf{ln}\left(\frac{V_4}{V_3}\right)$$

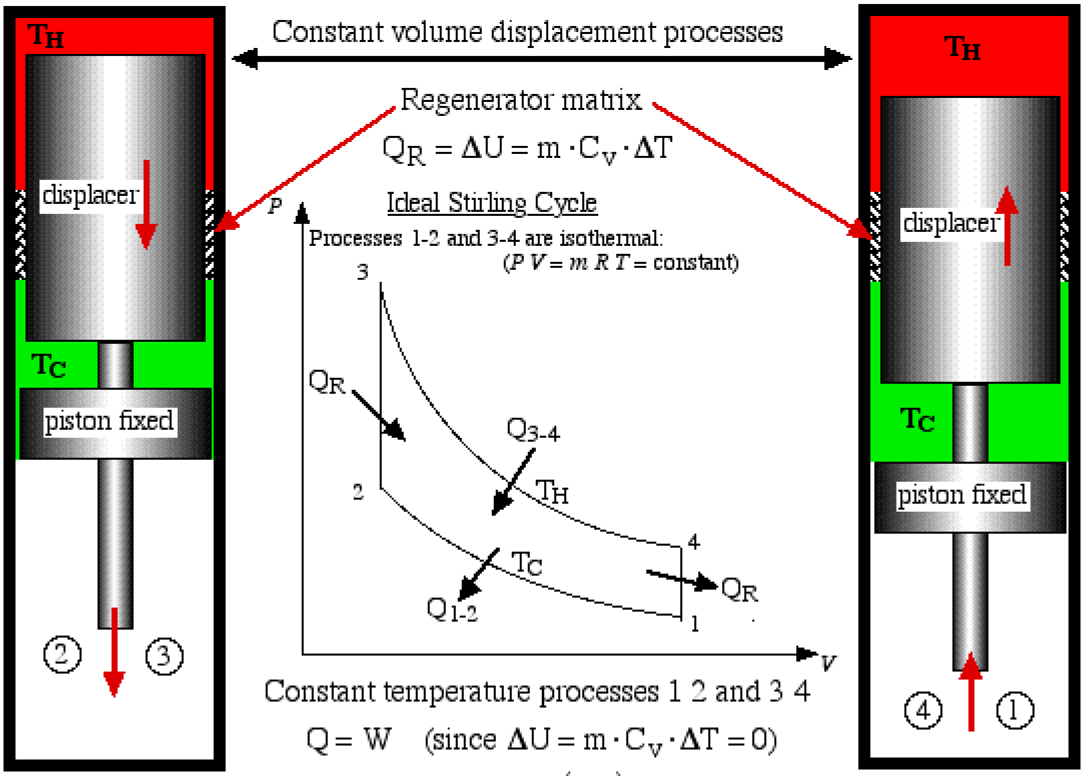
where: $\text{Const} = P_3 V_3 = P_4 V_4 = m \cdot R \cdot T_H$

- Finally, process 4-1 is a constant volume displacement process which completes the cycle. Once again we will see below that heat Q_R is rejected by the working gas to the regenerator matrix.

The net work W_{net} done over the cycle is given by: $W_{net} = (W_{3-4} + W_{1-2})$, where the compression work W_{1-2} is negative (work done *on* the system).

We now consider the heat transferred during all four processes, which will allow us to evaluate the thermal efficiency of the ideal Stirling engine. Recall from the previous section that in order to do a First Law analysis of an ideal gas to determine the heat transferred we needed to develop equations to determine the internal energy change Δu in terms of the **Specific Heat Capacities of an Ideal Gas**

The two constant volume processes are formed by holding the piston in a fixed position, and shuttling the gas between the hot and cold spaces by means of the displacer. During process 4-1 the hot gas gives up its heat Q_R by passing through a regenerator matrix, which is subsequently completely recovered during the process 2-3.



thus: $-Q_{out} = Q_{1\text{-}2} = W_{1\text{-}2} = m \cdot R \cdot T_C \,\mathbf{ln}\left(\frac{V_2}{V_1}\right)$ (Compression process)

and: $Q_{in} = Q_{3\text{-}4} = W_{3\text{-}4} = m \cdot R \cdot T_H \,\mathbf{ln}\left(\frac{V_4}{V_3}\right)$ (Expansion process)

Now from the First Law for a cycle:

$$\boxed{W_{net} = W_{1\text{-}2} + W_{3\text{-}4} = Q_{in} - Q_{out}}$$

Thus thermal efficiency: $\boxed{\eta_{th} = \frac{W_{net}}{Q_{in}} = \left(1 - \frac{Q_{out}}{Q_{in}}\right)}$

Note that: $\mathbf{ln}\left(\frac{V_2}{V_1}\right) = -\mathbf{ln}\left(\frac{V_4}{V_3}\right)$ thus we find that: $\boxed{\eta_{th} = \left(1 - \frac{T_C}{T_H}\right)}$

We will find in **Chapter 5** that this is the maximum theoretical efficiency that is achievable from a heat engine, and usually referred to as the **Carnot** efficiency.

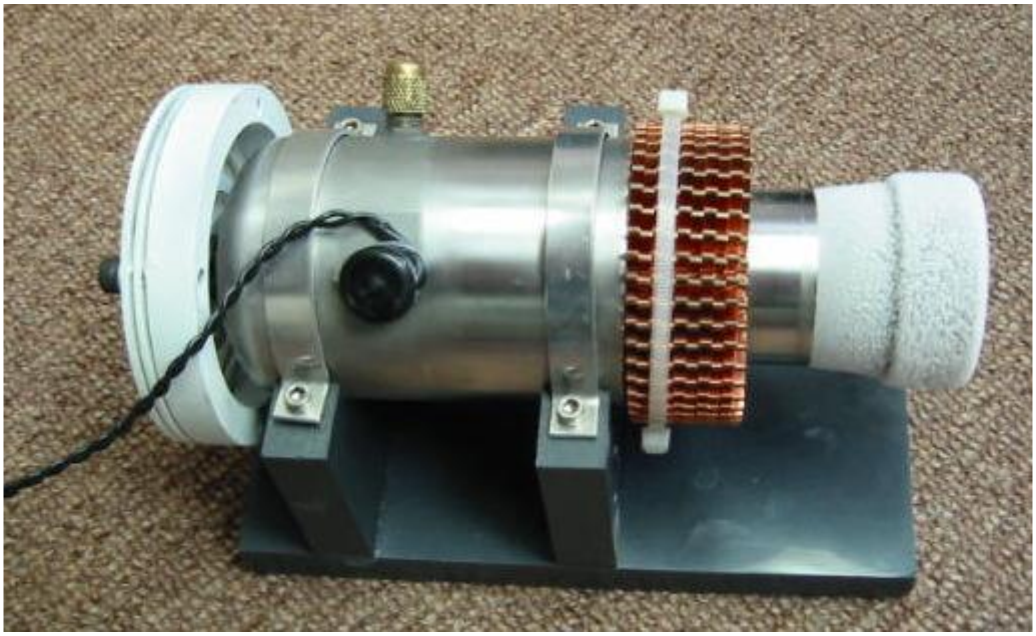
Note that if no regenerator is present the heat Q_R must be supplied by the heater. Thus the efficiency will be significantly reduced to $\eta_{th} = W_{net} / (Q_{in} + Q_R)$. Furthermore the cooler will then have to reject the heat that is normally absorbed by the regenerator, thus the cooling load will be increased to $Q_{out} + Q_R$. Recall that $Q_{2-3} = Q_R = -Q_{4-1}$.

Note that the practical Stirling cycle has many losses associated with it and does not really involve isothermal processes, nor ideal regeneration. Furthermore since the Free-Piston Stirling cycle machines involve sinusoidal motion, the *P-V* diagram has an oval shape, rather than the sharp edges defined in the above diagrams. Nevertheless we use the ideal Stirling cycle to get an initial understanding and appreciation of the cycle performance.

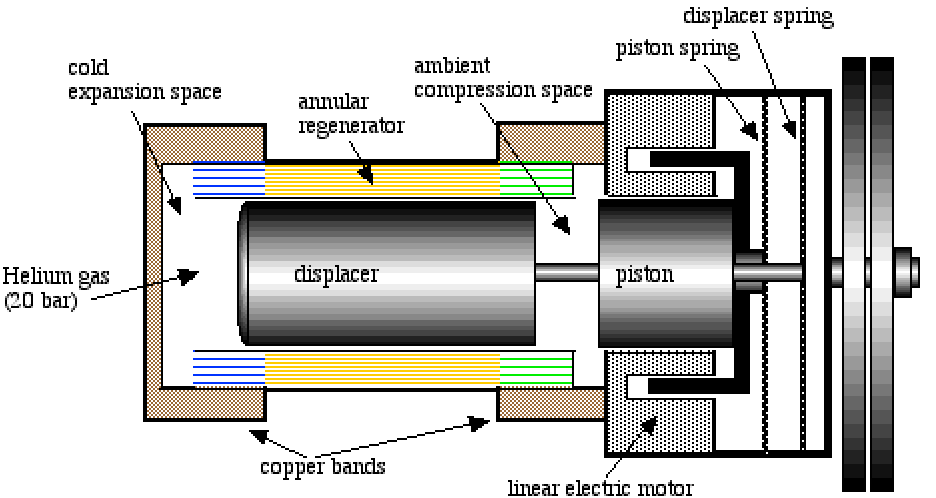
The Stirling Cycle Cooler

One important aspect of Stirling cycle machines that we need to consider is that the cycle can be reversed - if we put net work into the cycle then it can be used to pump heat from a low temperature source to a high temperature sink. **Sunpower, Inc**. has been actively involved in the deveplopment of Stirling cycle refrigeration systems and produces Stirling cycle croygenic coolers for liquifying oxygen. In 1984 Sunpower developed a free piston **Duplex Stirling Machine** having only three moving parts including one piston and two displacers, in which a gas fired Stirling cycle engine powered a Stirling cycle cooler. **Global Cooling, Inc** was established in 1995 as a spinoff of Sunpower, and was formed mainly in order to develop free-piston Stirling cycle coolers for home refrigerator applications. These systems, apart from being significantly more efficient than regular vapor-compression refrigerators, have the added advantage of being compact, portable units using helium as the working fluid (and not the HFC refrigerants such as R134a, having a Global Warming Potential of 1,300). More recently Global Cooling decided to concentrate their development efforts on systems in which there are virtually no competitive systems - cooling between -40°C and -80°C, and they established a new company name: **Stirling Ultracold**.

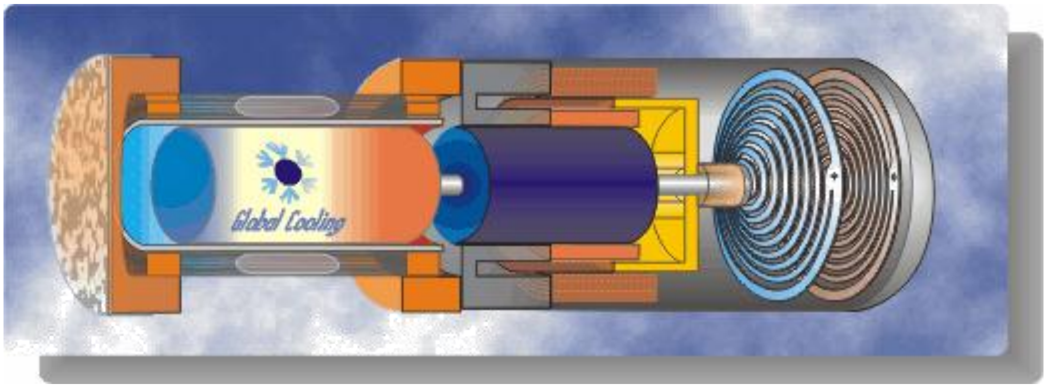
We are fortunate to have obtained two original M100B coolers from Global Cooling. The one is used as a demonstrator unit, and is shown in operation in the following photograph. The second unit is set up as a ME **Senior Lab project** in which we evaluate the actual performance of the machine under various specified loads and temperatures.



A schematic diagram followed by an animated schematic of the cooler (both courtesy of **Global Cooling**) are shown below



M100B Stirling Cooler Schematic



Conceptually the cooler is an extremely simple device, consisting essentially of only two moving parts - a piston and a displacer. The displacer shuttles the working gas (helium) between the compression and expansion spaces. The phasing between the piston and displacer is such that when the most of the gas is in the ambient compression space then the piston compresses the gas while rejecting heat to the ambient. The displacer then displaces the gas through the regenerator to the cold expansion space, and then both displacer and piston allow the gas to expand in this space while absorbing heat at a low temperature.

Source: http://www.ohio.edu/mechanical/thermo/Intro/Chapt.1_6/Chapter3b.html