THE EÖTVÖS EFFECT

In the early 1900s a German team from the Institute of Geodesy in Potsdam carried out gravity measurements on moving ships in the Atlantic, Indian and Pacific Oceans. While studying their results the Hungarian nobleman and physicist Lorand Eötvös noticed that the readings were lower when the boat moved eastwards, higher when it moved westward. He identified this as primarily a consequence of the rotation of the Earth. In 1908 new measurements were made in the Black Sea on two ships, one moving eastward and one westward. The results substantiated Eötvös' claim. Since then geodesists use the following formula to correct for velocity relative to the Earth during a measurement run.

$$a_r = 2\Omega u \cos\phi + \frac{u^2 + v^2}{R}$$

a_r correction when moving relative to the Earth
Ω rotation rate of the Earth
u velocity in latitudinal direction (east-west)
φ latitude where the measurements are taken.
v velocity in longitudinal direction (north-south)
R radius of the Earth

Physical explanation

The most common design for a gravimeter for field work is a spring-based design; a spring that suspends an internal weight. The suspending force provided by the spring counteracts the gravitational force. A well manufactured spring has the property that the amount of force that the spring exerts is proportional to the amount of stretch. The stronger the effective gravity at a particular location, the more the spring is extended; the spring extends to a length at which the internal weight is sustained. Also, the moving parts of the gravimeter will be dampened, to make it less susceptible to outside influences such as vibration.

For the calculations I'll assume the internal weight in the gravimeter has a mass of 10 kilogram, 10,000 grams. I assume that for surveying a method of transportation is used that gives good speed while moving very smoothly: an airship. Let the cruising velocity of the airship be 25 meters per second (90 km/h, 55 miles/h).

To calculate what it takes for the internal weight to be neutrally suspended when it is *stationary* with respect to the Earth the fact that the Earth rotates must be taken into account. At the equator the velocity of Earth's surface is about 465 meters per second. The amount of centripetal force required to cause an object to move along a circular path with a radius of 6378 kilometer (the Earth's equatorial radius), at 465 m/s, is about 0.034 newton per kilogram of mass. For the 10,000 gram internal weight that amounts to about 0.34 newtons. The amount of suspension force required is the mass of the internal weight (multiplied with the gravitational acceleration), minus those 0.34 newtons. In other words: any object co-rotating with the Earth at the equator has its measured weight reduced by 0.34 percent, thanks to the Earth's rotation.

When cruising at 25 m/s due east, the total velocity becomes 465 + 25 = 490 m/s, which requires a centripetal force of about 0.375 newtons. Cruising at 25 m/s due West the total velocity is 465 - 25 = 440 m/s, requiring about 0.305 newtons. So if the internal weight is neutrally suspended while cruising due east, it will not be neutrally suspended anymore after a U-turn; after the U-turn, the weight of the 10,000 gram internal weight has increased by about 7 grams; the spring of the gravimeter must extend some more to accommodate the larger weight. On the other hand: on a non-rotating planet, making the same U-turn would not result in a change of gravimetric reading.

Derivation of the formula for motion along the Equator

A convenient coordinate system in this situation is the inertial coordinate system that is co-moving with the center of mass of the Earth. Then the following is valid: objects that are at rest on the surface of the Earth, co-rotating with the Earth, are circling the Earth's axis, so they are in centripetal acceleration with respect to that inertial coordinate system.

What is sought is the difference in centripetal acceleration of the surveying airship between being stationary with respect to the Earth and having a velocity with respect to the Earth. The following derivation is exclusively for motion in east-west or west-east direction.

Notation:

a_u required centripetal acceleration when moving at velocity u

a_s required centripetal acceleration when stationary with respect to the Earth.

Ω angular velocity of the Earth: one revolution per Sidereal day.

ω_r angular velocity of the airship relative to the angular velocity of the Earth.

u velocity with respect to the Earth.

R radius of the earth.

$$\begin{aligned} a_r &= a_u - a_s \\ &= (\Omega + \omega_r)^2 R - \Omega^2 R \\ &= \Omega^2 R + 2\Omega\omega_r R + \omega_r^2 R - \Omega^2 R \\ &= 2\Omega\omega_r R + \omega_r^2 R \\ &= 2\Omega u + u^2/R \end{aligned}$$

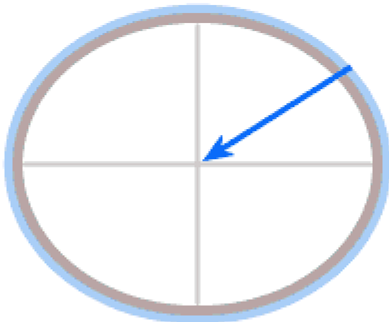
It can readily be seen that in the case of motion along the equator the formula for any latitude simplifies into the formula above.

$$a_r = 2\Omega u \cos\phi + \frac{u^2 + v^2}{R}$$

The second term represents the required centripetal acceleration for the internal weight to follow the curvature of the earth. It is independent of both the earth's rotation and the direction of motion. For example, when an aeroplane carrying gravimetric reading instruments cruises over one of the poles at constant altitude, the aeroplane's trajectory follows the curvature of the earth. The first term in the formula is zero then, due to the cosine of the angle being zero, and the second term then represents the centripetal acceleration to follow the curvature of the Earth's surface.

Explanation of the cosine in the first term

The mathematical derivation for the Eötvös effect for motion along the Equator explains the factor 2 in the first term of the Eötvös correction formula.


Picture 1. Image
The force of gravity and the normal force. The resultant force acts as the required centripetal force.

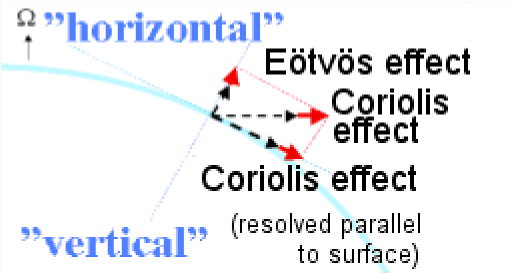
Because of its rotation, the Earth is not spherical in shape, there is an equatorial bulge. The force of gravity is directed towards the center of the Earth. The normal force is perpendicular to the local surface.

On the poles and on the equator the force of gravity and the normal force are exactly in opposite direction. At every other latitude the two are not exactly opposite, so there is a resultant force, that acts towards the Earth's axis. At every latitude there is precisely the amount of centripetal force that is necessary to maintain an even thickness of the atmospheric layer. (The solid Earth is ductile. Whenever the shape of the solid Earth is not entirely in equilibrium with its rate of rotation, then the shear stress deforms the solid Earth over a period of millions of years until the shear stress is resolved.)

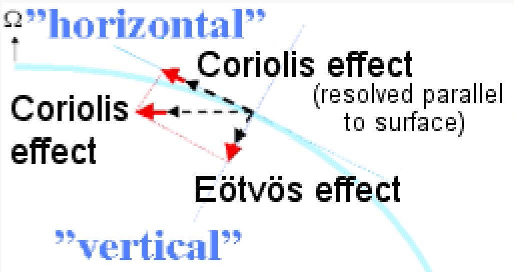
Again the example of an airship is convenient for discussing the forces that are at work. When the airship has a velocity relative to the Earth in latitudinal direction then the weight of the airship is not the same as when the airship is stationary with respect to the Earth. If an airship has an eastward velocity, then the airship is in a sense "speeding". The situation is comparable to a racecar on a banked circuit with an extremely slippery road surface. If the racecar is going too fast then the car will tend to drift wide. For an airship in flight that means a reduction of the weight, compared to the weight when stationary with respect to the Earth. If the airship has a westward velocity then the situation is like that of a racecar on a banked circuit going too slow: on a slippery surface the car will slump down. For an airship that means an increase of the weight.

The Eötvös effect is proportional to the component of the required centripetal force perpendicular to the local Earth surface, and is thus described by a cosine law: the closer to the Equator, the stronger the effect.

Motion along 60 degrees latitude



Picture 2. Image
The Eötvös effect for an object moving eastward along 60 degrees latitude. The object tends to move away from the Earth's axis.



Picture 3. Image
The Eötvös effect for an object moving westward along 60 degrees latitude. The object tends to be pulled towards the Earth's axis.

An object located at 60 degrees latitude, co-moving with the Earth, is following a circular trajectory, with a radius of about 3190 kilometer, and a velocity of about 233 m/s. That circular trajectory requires a centripetal force of about 0.017 newton for every kilogram of mass; 0.17 newtons for the 10,000 gram internal weight. At 60 degrees latitude the component that is perpendicular to the local surface (the local vertical) is half the total force. Hence, at 60 degrees latitude, any object co-moving with the Earth has its weight reduced by about 0.08 percent, thanks to the Earth's rotation.

When the surveying airship is cruising at 25 m/s towards the east the total velocity becomes 233 + 25 = 258, which requires a centripetal force of about 0.208 newtons for the gravimeter's internal weight; local vertical component about 0.104 newton. Cruising at 25 m/s towards the west the total velocity becomes 233 - 25 = 208 m/s, which requires a centripetal force of about 0.135 newtons; local vertical component about 0.68 newtons. Hence at 60 degrees latitude the difference before and after the U-turn of the 10,000 gram internal weight is a difference of 4 gram in measured weight.

The diagrams also show the component in the direction parallel to the local surface. In Meteorology and in Oceanography it is customary to refer to the effects of the component parallel to the local surface as the Coriolis effect.

To my knowledge the first scientist who recognized that the Eötvös effect and the meteorological Coriolis effect are interconnected was the meteorologist Anders Persson, who has published about it in several articles, starting around the year 2000.

Source : http://www.cleonis.nl/physics/phys256/eotvos.php