Structural Synthesis of 6 Bar Mechanisms as Mechanically Constrained 3R Chains

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Abstract - In this paper, we consider the planar robot formed by 3R chain. To mechanically constrain the relative movement of the joints so that the end-effector reaches a specified set of task positions, two additional links are added to the 3R chain. Graphical representation is presented for enumeration process. Reverse transformation technique is developed to reconstruct 6-bar mechanisms from their corresponding graphs. Structural synthesis process yields designs for seven different forms of a six-bar linkage the Watt I and Stephenson I, II, and III six-bar linkages.

Keywords - *structural synthesis, six-bar linkages, 3R chain, graphical enumeration.*

I. INTRODUCTION

Planar four-bar mechanisms have been, and remain widely implemented in mechanical systems. Due primarily to the joint type, joint axes orientations and overall planar kinematics of this mechanism, it is often very practical to analyze, design, fabricate and implement. In this paper, we consider the planar robot formed by a serial chain constructed from three revolute joints, the planar 3R chain. Our goal is to mechanically constrain the relative movement of the joints so the end-effector reaches a specified set of task positions. This work is inspired by Krovi et al. (2002) [4], who derived synthesis equations for planar nR planar serial chains in which the n joints are constrained by a cable drive. They obtained a “single degree-of-freedom coupled serial chain” that they use to design an assistive device. Rather than use cables to constrain the relative joint angles, we add two RR chains. The planar 3R robot consists of four bars, if we include its base. Therefore, the appropriate attachment of two RR chains results in a planar six-bar with seven joints forming a one degree-of-freedom system. Kinematic analysis of planar mechanisms are presented by Bottema , Roth 1990 [1]. Tsai (2001) [2] produce a systematic methodology for the creation and classification of mechanisms using analytical approach. Some of the functional requirements of a desired mechanism can be transformed into structural characteristics that can be employed for systematic enumeration of mechanisms. McCarthy et al.(2006) [9] discuss the problem of synthesis of a mechanically constrained 3R chain designs two RR chains to reach five task positions. The 3R chain has been sized by McCarthy et al.(2008) [12] to reach the five task positions, then two RR chains may be attached to obtain seven different forms of a six-bar linkage. This procedure results in as many as 63 candidate designs. The isomorphism of mechanism kinematics chain has been identified by comparing the eigenvalues and corresponding eigenvectors of adjacent matrices by Yuxin Wang (2002) [3] . Zhen Huang (2009) [14] discuss Isomorphism identification of graphs by finding a unique representation of graphs to solve the problem of isomorphism in mechanism synthesis process.

II. GRAPHICAL REPRESENTATION FOR 3R CHAIN

An illustration of the ways that a 3R serial chain can be constrained to obtain a one degree-of- freedom system using graphical techniques. Originally the 3R chain is represented graphically by identifying each link as a vertex, and each joint as an edge as shown in Fig.1. The linkage graph is constructed by identifying each link as a vertex, and each joint as an edge.

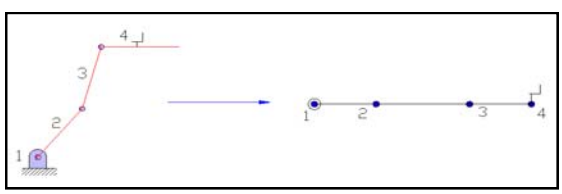

Fig. 1 : A schematic of a planar 3R robot and its corresponding graph representation

III. SYNTHESIS OF MECHANICALLY CONSTRAINED 3R CHAINS

The planar 3R robot consists of four bars, if we include its base. Therefore, the appropriate attachment of two RR chains results in a planar six-bar with seven joints forming a one degree-of-freedom system. The introduction of an RR chain adds a vertex and two edges to the graph. Open 4-bar mechanism or 3-R chain can be mechanically constrained by addition of 2 RR chains i.e. two links and four revolute joints to maintain its degree of freedom. This means that two vertices and four edges will be added to the initial graph that represents the 3R chain. This will transform the open four bar chain into planar mechanically constrained mechanism having 6 links, 7 revolute joints and one degree of freedom. Our synthesis of a mechanically constrained 3R chain proceeds in two main steps. The first is to enumerate all possible 6-link graph representations. Then these graph representations are reverse transformed into schematic planar 6-bar mechanisms.

IV. GRAPHICAL ENUMERATION FOR 6-BAR LINKAGES

In this section, we consider how two RR chains are added to a 3R chain to mechanically constrain its movement to one degree-of-freedom using graphical technique, then how to enumerate all possible connections for these revolute joints. The two additional links may be dependent in their connection if there is a revolute connection will connect them. They may be independent if there is no revolute joint will connect them as shown in Fig. 2.

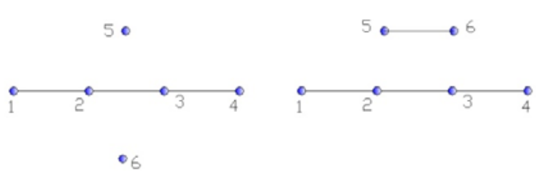

Fig. 2. a- independent link connection.
b- dependent link connection

4.1 Enumeration of independent link connection

We can summarize enumeration procedures in the following steps:

1. The two additional links, 5 and 6, are represented.
2. Each additional link can be connected to the original four links by 4 revolute joints in such a way that both of them should be connected to only one original link by one revolute joint. In other words each additional link should have at least one revolute joint, this will enumerate 16 available candidates shown in Fig. 3.

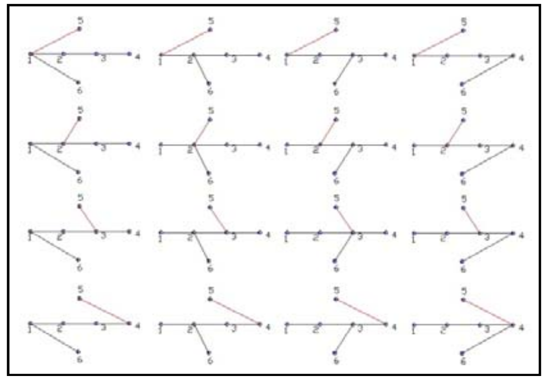

Fig.3 : Available candidates after adding the first two revolute joints for independent link connections

3. Another two revolute joints are added for each candidate such that each additional link has one revolute joint. This will produce 15 graphs for each candidate. Each of additional links 5,6 must has two revolute joints with the original links 1,2,3 and 4. This will exclude 6 rejected graphs in which one of the additional link has three revolute joints with the original links. This produces 9 available constructions.
4. Therefore, there will be 144 available chains from this enumeration process. Some of these results are shown in Fig.4**.**

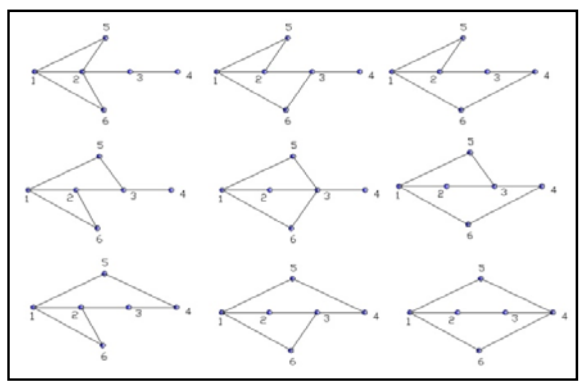

Fig. 4 : Some results of Enumeration of independent link connection

4.1.1 Detect open loop chains

Link 4 that represents the end effector should has at least 2 revolute joints with other links unless this mechanism is considered an open loop mechanism and will not be mechanically constrained. All open loop mechanisms are detected and should be excluded from enumeration process as shown in Fig. 5.

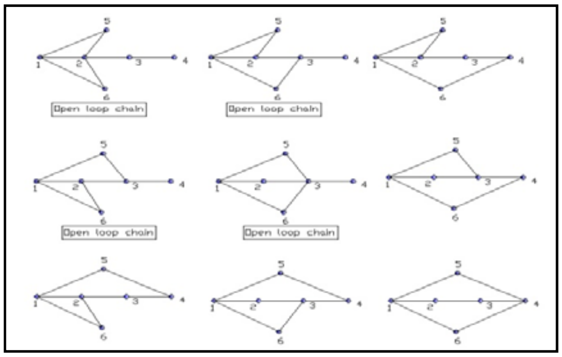

Fig. 5 : Detect open loop graphs

4.1.2 *Isomorphism*

Isomorphic graphs are detected graphically. Isomorphic graphs are specified into groups such that each graph from 144 resulted graphs will has a label number referring to the number of its isomorphic group as shown in Fig 6. Each group will be represented only once by only one graph in the enumeration results. Therefore there will be 15 non-isomorphic graphs shown in Fig 7.

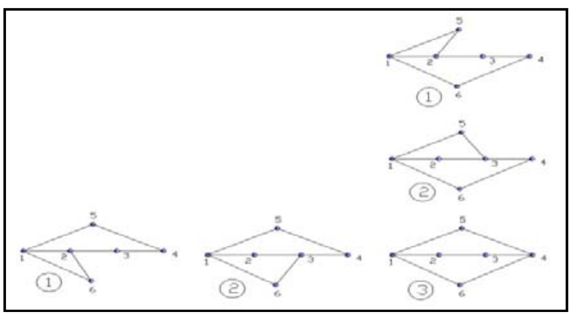

Fig. 6 : Detect isomorphic graphs

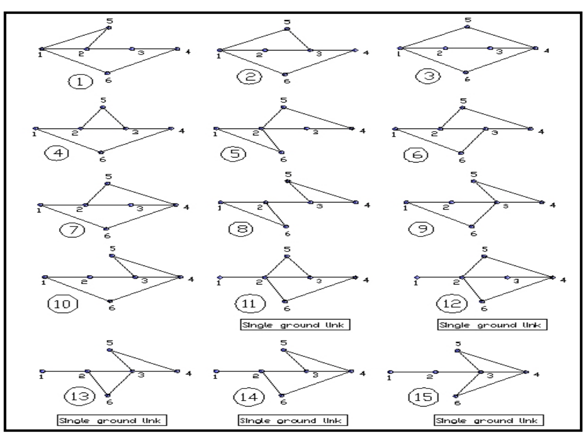

Fig.7 : Non- isomorphic graphs for independent link connection

4.1.3 Detect graphs with single ground link

After adding the 2 RR chains, at least one of the two additional links 5,6 must has one revolute joint with the ground link unless this produces an open loop chain which cannot be mechanically constrained. Referring to Fig.7, graphs with single ground link are detected and then eliminated from enumeration process. This produces 10 graphs having no single ground link as shown in Fig. 8.

4.1.4 Reverse Transformation

As referred before in graphical representation links are denoted as vertices and revolute joints are denoted as thin edges. Each graph from the ten final graphs will be transformed back to its original mechanism or its linkage graph representation.

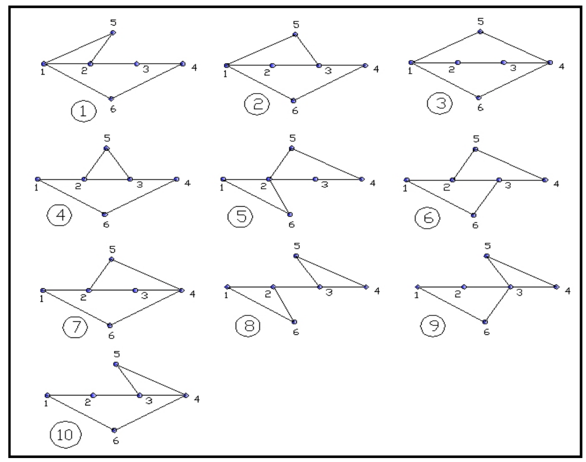

Fig. 8 : Final enumeration results for independent link connection

Number of common edges at the same vertex indicates the type of link connection. Fig. 9 illustrate the first graph representation and its corresponding mechanism. For example vertex 2 has three thin edges with vertices 1, 4, 5. So that, link 2 must be trinary link with three revolute joints with links 1,4,5. Also vertix 5 has only two thin edges with vertices 1, 2. So that link 5 must be binary link with two revolute joints with links 1,2.

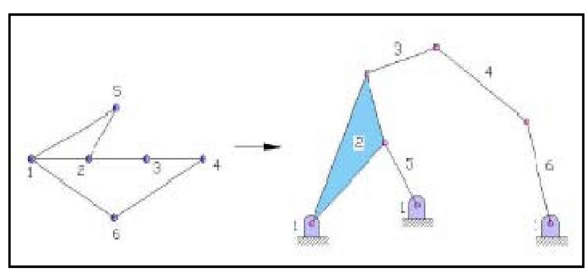

Fig. 9 : Reverse transformation process

The same procedures are performed to have all ten linkage graphs. Fig.10 shows the corresponding ten mechanisms resulting from transformation process of final ten graphs resulting from enumeration process shown in Fig. 8.

4.1.5 Check for kinematic characteristics

The end effector ,link 4, should be floating link. It should be connected to the ground link through maximum 3 revolute joints. Also it should have only binary joints with other links. Applying these rules to the resulting ten mechanisms, seven mechanisms will be excluded and finally we have three available 6-bar mechanisms, Stephenson IIIa, Stephenson IIIb and Stephenson IIb. Fig. 11 illustrates these results.

4.2 Enumeration for dependent link connection

We can summarize enumeration procedures in the following steps:

1. The two additional links, 5 and 6, are represented with their revolute connection.

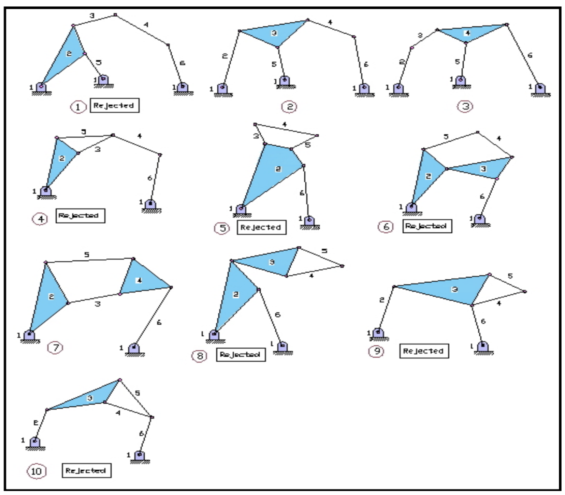

Fig. 10 : Corresponding linkage graphs for final enumeration results

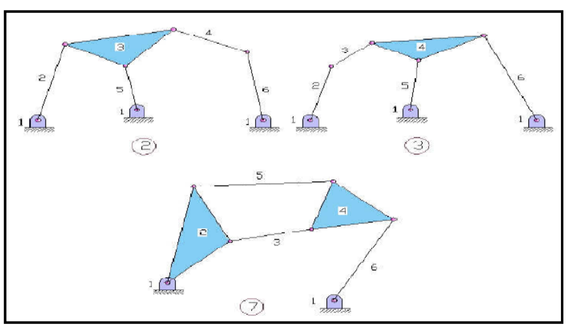

Fig. 11 : Available 6-bar mechanisms for independent link connection

2. There will be three remaining revolute joints to be added which produces 56 candidate graphs. Both of links 5, 6 must have at least one revolute joint with the original links 1, 2, 3 and 4. This will exclude eight graphs from enumeration process and produces 48 available candidates.

This enumeration process can be performed by another technique to avoid the presence of rejected structures:

1. The two additional links, 5 and 6, are represented with their revolute connection.
2. Add the second revolute joint only by all 8 deferent ways as shown in Fig.12.
3. The last two revolute joints are added to each one of 8 structures. These two joints connect the original links with the link which has no joints with original links. This will produce six available candidates for each structure.
4. Therefore, there will be 48 available chains from this enumeration process. Some of these results are shown in Fig.13**.**

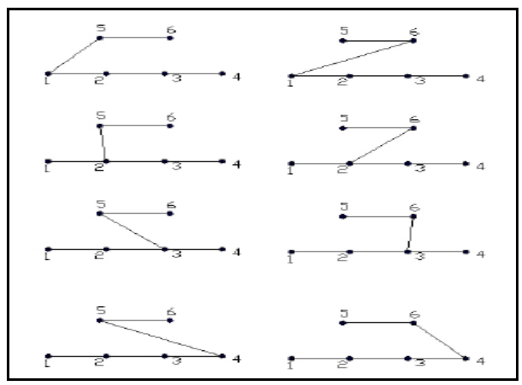

Fig. 12 : Available candidates after adding the first two revolute joints for independent link connections

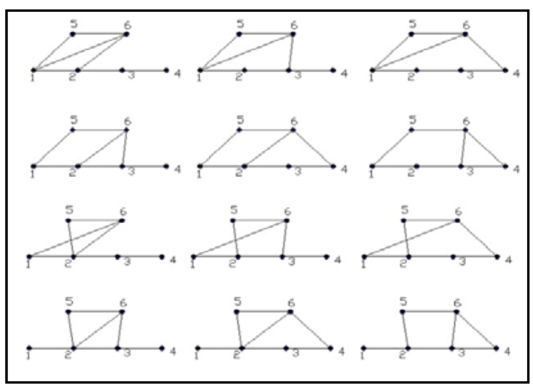

Fig. 13 : Some results of Enumeration of dependent link connection

4.2.1 Detect open loop chains

Link 4 that represents the end effector should has at least two revolute joints with other links unless this mechanism is considered an open loop mechanism. All open loop mechanisms should be detected and excluded from enumeration process as shown in Fig. 14.

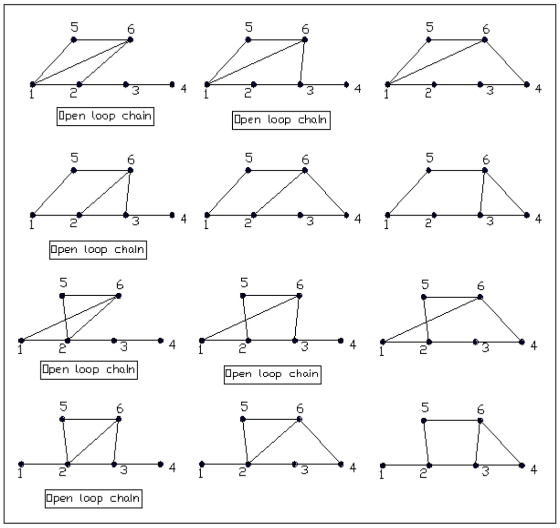

Fig. 14 : Detect open loop graphs

4.2.2 Detect graphs with single ground link

At least one of the two additional links 5,6 must has one revolute joint with the ground link. Referring to Fig. 14, graphs with single ground link are detected and then eliminated from enumeration process as shown in Fig. 15.

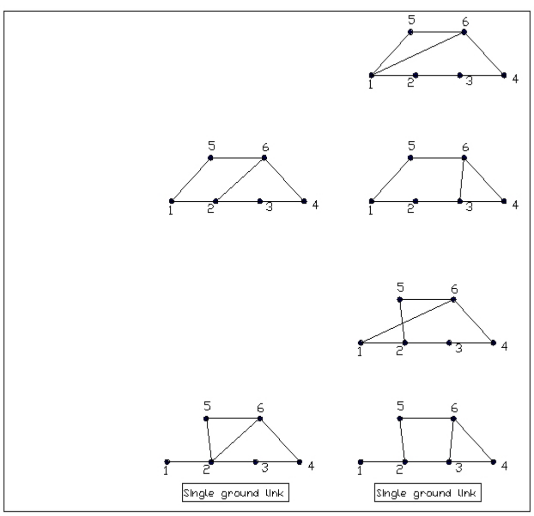

Fig. 15 : Detect graphs with single ground link

4.2.3 Isomorphism

Using the same technique in independent link connection, there will be eight non isomorphic graphs can be detected as shown in Fig. 16.

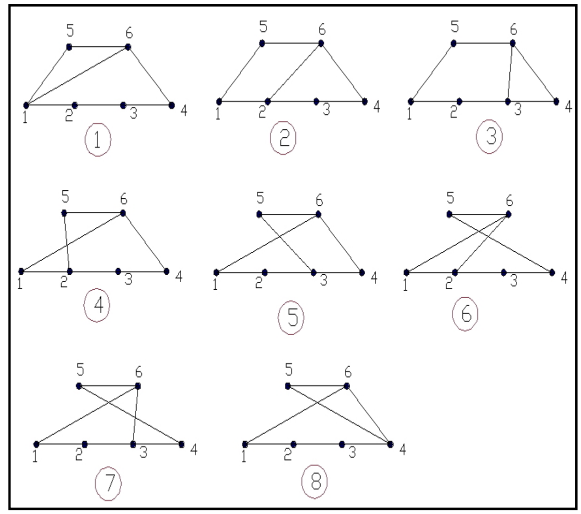

Fig. 16 : Final enumeration results for dependent link connection

4.2.4 Reverse Transformation

Each graph from the eight final graphs will be transformed back to its original schematic mechanism representation. The corresponding linkage mechanisms for these eight structures are shown in Fig. 17.

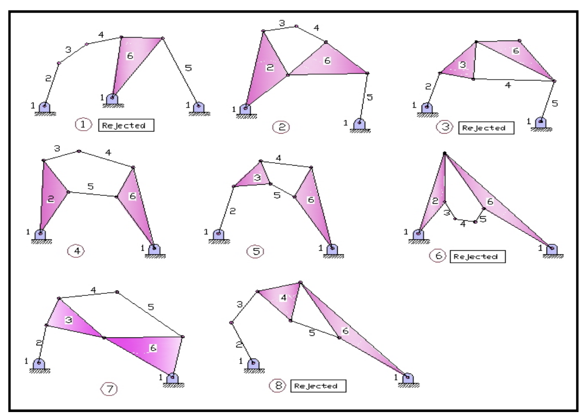

Fig. 17 : Corresponding linkage graphs for final enumeration results

4.2.5 Check for kinematic characteristics

Applying the same rules of independent link connection, four mechanisms will be rejected and excluded from enumeration process. Finally we have four available 6-bar mechanisms: Watt Ib, Stephenson I, Stephenson IIa and Watt Ia as shown in Fig. 18.

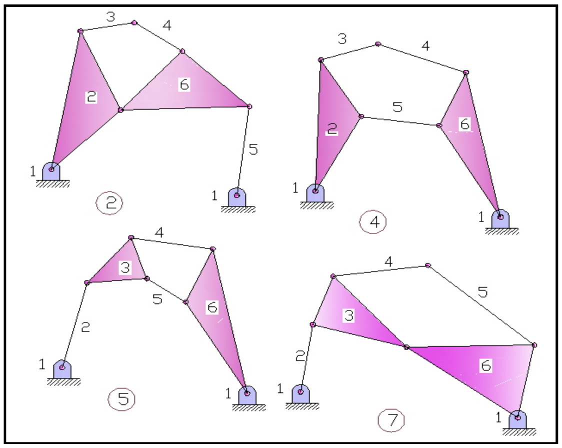

Fig. 18 : Available 6-bar mechanisms for dependent link connection

V. RESULTS

To control the movement of 3R chain, 2 RR chains can be used such that the system has one degree of freedom. Graphical representation and enumeration techniques are used in synthesis process. Structural synthesis results in three 6- bar mechanisms from independent link connection seen in Fig. 11. Four 6-bar mechanisms have been synthesized from dependent link connections as shown in Fig. 18. This approach does not apply to the Watt II because its floating link is not connected to the ground frame by a 3R chain.

VI. CONCLUSION

In this paper graphical representation and enumeration are presented to synthesis 6-bar mechanisms constraining the movement of the 3R chain. Adding the 2 RR chains produces seven non-isomorphic 6-bar structures. A new systematic technique has been presented for reverse transformation process to transform these graphs into linkage graphs.

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