STRESS, STRAIN AND DEFORMATION

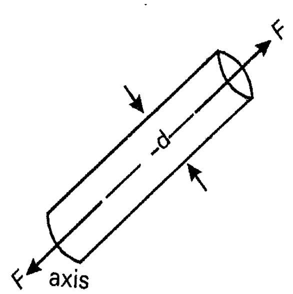
What is stress?

Stress is defined as the load acting on a material per unit area of the cross section.

It is the intensity of the load per unit area. When the load is acting normal to the area of cross section then the stress acting on the area is referred to as “Normal Stress”. As the load on the cross section is increased, the value of stress also increases.

Then stress

$$\sigma = \frac{F}{Area} = \frac{F}{\left(\pi/4\right) d^2} = \frac{4F}{d^2}$$



A Cylindrical bar with axial force

What is Strain?

Strain is defined as the ratio of change in dimension to its original dimension. It signifies how much change in dimensions h as occurred under the influence of external load. The strain in the material under the influence of a normal load is referred to as ”Normal Strain”.

As the load on the material is increased the value of strain also increases.

What is Deformation?

When an external load is applied on the material, it will undergo changes in the dimensions and change in shape will take place. As a result strain will be induced in the material. The change in dimensions or the shape is referred to as "deformation".

Concept of Stress

As defined earlier stress is defined as the force per unit area of the body. The stress is assumed to be distributed uniformly over the cross section.

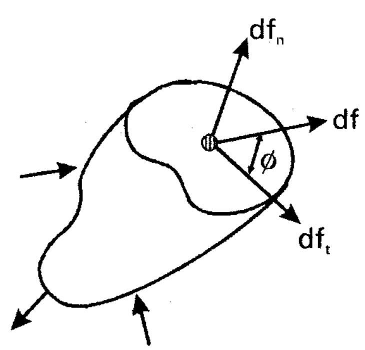
Consider a circular bar of uniform cross section

Of diameter 'd' is subjected to tensile force 'F' acting along the axis of the bar, then the average stress "σ" induced across any transverse cross section(C.S) perpendicular to the axis of the bar as shown:

In general a structural member or an element will not possess uniform geometry and the loads acting will be complex. In such cases one has to introduce the concept of state of stress at a point.

Consider a point 'O' within the body. Let a section plane pass through it and cut the body.
Let 'df' be the force acting on an area 'dA' of the cut section.
Now the stress acting at a point is defined as

$$\underset{dA \to o}{Lt} \frac{df}{dA} = \sigma$$



Resolution of a force

Concept of convention stress and conventional strain, true stress and true strain and their relationship.

Conventional stress: It is defined as the ratio of external load to the original area of cross section. It is also referred to as Engineering stress.

Conventional strain: It is defined as the ratio of change in length to its original length. However, the area of cross section and the length keeps on changing continuously under the influence of the external load. Hence, new terms true stress and true strain are defined as follows.

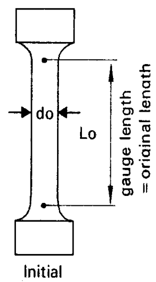
True stress: It is defined as the ratio of the external load to the instantaneous area of cross section.

True strain: It is defined as the ratio of change in length to the instantaneous previous length. For many of the engineering applications conventional stress and conventional strain data is sufficient. For generating true stress and true strain very sophisticated instruments are required, takes lot of time and very tedious.

Consider a bar subjected to tensile test. Let the initial diameter be d_0 and initial length be L_0. Let it be loaded gradually from P_1 , P_2 ,P_3 ………P_n. Now the bar will undergo changes in the diameter and length. The diameter will reduce and the length will increase gradually. Let the change in diameters be d_1,d_2,d_3,….d_n and the corresponding change in lengths be

L_1 ,L_2 ,L_3 ……L_n . and the corresponding areas be

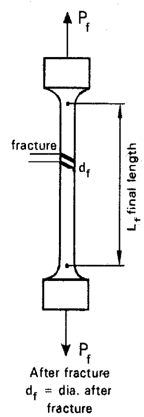
A_1, A_2 ,A_3 …….A_n . Now one can calculate the conventional stress, conventional strain, true stress, true strain and their relationships.



Specimen before testing

$$A_0 = \frac{\pi}{4} d_0^2$$

$$A_f = \frac{\pi}{4} d_f^2$$



Specimen after testing

Conventional Stress

$$\sigma_{C_1} = \frac{P_1}{A_0}$$

$$\sigma_{C_2} = \frac{P_2}{A_0}$$

$$\sigma_{C_3} = \frac{P_3}{A_0}$$

Conventional Strain

$$\epsilon_{C_1} = \frac{L_1 - L_0}{L_0}$$

$$\epsilon_{C_2} = \frac{L_2 - L_0}{L_0}$$

$$\epsilon_{C_3} = \frac{L_3 - L_0}{L_0}$$

True Stress

$$\sigma_{t_1} = \frac{P_1}{A_1}$$

$$\sigma_{t_2} = \frac{P_2}{A_2}$$

$$\sigma_{t_3} = \frac{P_3}{A_3}$$

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True Strain

$$\epsilon_{t_1} = \frac{L_1 - L_0}{L_0}$$

$$\epsilon_{t_2} = \frac{L_2 - L_1}{L_1}$$

$$\epsilon_{t_3} = \frac{L_3 - L_2}{L_2}$$

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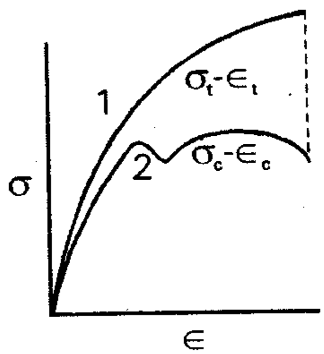

$\sigma_t - \in_t$ and $\sigma_c - \in_c$ curves

$$\text{Conv Stress } \sigma_c = \frac{\text{Load}}{\text{original area of CS}} = \frac{P}{A_0}$$

$$\text{Conventional Strain } \in_c = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{Lo} = \frac{L_f - L_0}{L_0}$$

$$\text{or } \frac{L_f}{L_o} = (1+\in_c)$$

Volume of the bar is constant

$$V = A_0L_0 = A_1L_1 = A_2L_2 \ldots\ldots\ldots A_fL_f$$

$$\frac{A_o}{A_f} = \frac{L_f}{L_o}$$

From the above equations

$$\frac{A_o}{A_f} = 1 + \in_C$$

$$\text{True Stress } \sigma_t = \frac{\text{Load}}{\text{Instantaneous area of C.S}} = \frac{P}{A_i}$$

$$\text{Can be written as } \sigma_t = \frac{P}{A_o} \times \frac{A_o}{A_i}$$

$$\text{but } P/A_o = \sigma_c$$

$$\therefore \sigma_t = \sigma_c (1 + \in_c)$$

$$\text{True strain } \in_t = \frac{\text{Change in length}}{\text{Instantaneous length}} = \int_{L_o}^{L_f} \frac{\Delta L}{L_i}$$

$$= \text{In} \left(\frac{L_f}{L_o}\right)$$

$$\therefore \in_t = \text{In} \left(\frac{A_o}{A_f}\right) \text{ or In } (1 + \in_c)$$

$$\text{ie., } \in_t = \text{In } (1 + \in_c)$$

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