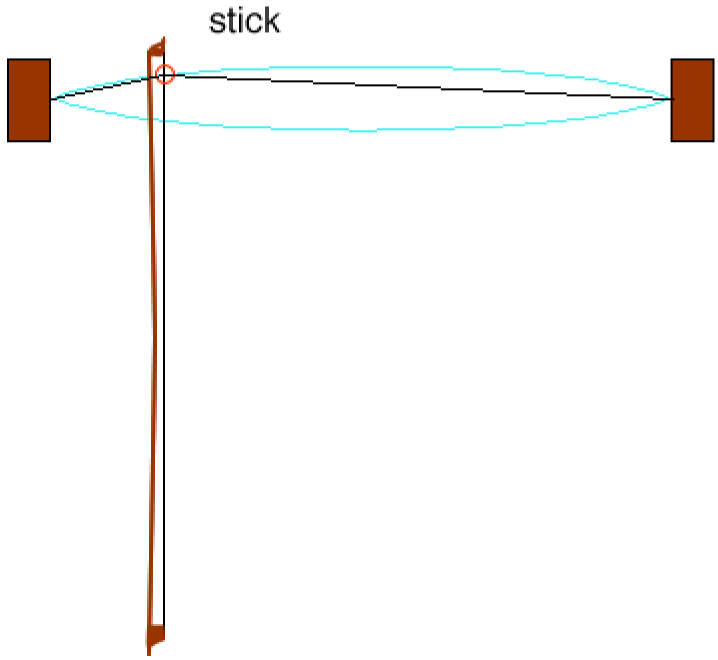
Standing Waves: Bows and strings

Many of the most useful features of the orchestral string instruments result from their use of bows. Compared with *pizzicato* (plucking the string), the bow allows the player to continuously input energy and so to maintain a note. This is important to the timbre, too: after a pluck, the high harmonics fade away quickly, leaving only the fundamental and some weak lower harmonics. Bowing maintains the rich harmonic spectrum.

The part of the bow that touches the strings is made of horsehair (or a synthetic substitute). In the contemporary bow, the wooden support for the hair is bent towards the hair - not away from it like an archery bow, or like the violin bows of the eighteenth century.

This difference in bow shape makes a big difference to playing and sound. With the old bow, which curved away from the strings, pushing harder on a string made the bow more curved, so the ends were closer together. So the force exerted on the string doesn't increase rapidly as you push the bow down. With the modern bow, when the player presses hard on the strings, the wood straightens, which helps to pull the hair tighter. Another effect of the modern (Tourte) bow is the presence of the 'hatchet" head, which distributes moreweight to the tip, allowing a more uniform application of force during the stroke. These effects give the modern bow a greater range of loudness.

The action of the bow which drives the strings is a regular cycle of stick-slip-stick-slip. This involves some interesting properties of friction, the force that makes things difficult to slide. If you have ever tried to slide a heavy object such as a piece of furniture, you will know that it is easier to keep it moving than it is to get it moving in the first place. In this case static friction (sticking) is greater than kinetic friction (sliding). This is true for most dry surfaces. It is also true of the string and the bow, and the player puts rosin on the bow to give a big difference between the two conditions: the coefficient of static friction is high, while that of sliding friction is very low. (Note that, in the animation below, the vertical scale of the string's motion has been greatly magnified, and the wave speed in the string has been reduced by the same factor. There are also many complicating factors, including one due to torsional waves.)

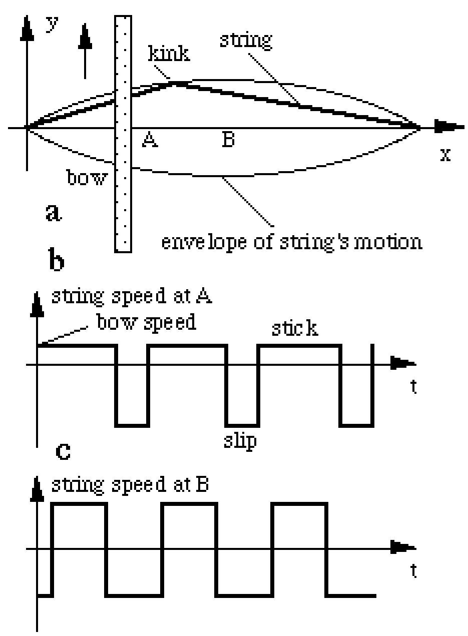


If your browser doesn't support this animation, see the series of graphics at the foot of the page.

With high static friction, the bow tends to stick to the string ("stick" in the animation) and for a while it drags the string along with it. Meanwhile, the kink in the string travels along the string and reflects at the fixed end. When the kink returns to meet the contact point, the tension in the string now acts to pull it off the bow. Under appropriate bowing conditions (that are not easy to learn!), it breaks free of the bow and then slides past it easily with very little friction, thanks to the low kinetic friction. This is the "slip" phase in the animation. The string doesn't stop when it gets to the straight position because its momentum carries it on until eventually it stops and reverses direction. Meanwhile, the kink has travelled to the near end of the string and reflected back. At the end of the "slip" phase, it is going at about the same speed and in the same direction as the bow. At this point, it catches on the bow again, static friction reigns, and the cycle begins again. Usually the vibration of the string governs the cycle of stick-slip: while the vibration is in the same direction as the bow travel, it sticks and moves with it, when it reverses it slips. Thus *the cycle of stick and slip on the bow has the same period as the vibration of the string*. This might be less than a thousandth of a second for a violin or several hundredths of a second on a double bass. Do not confuse this cycle with the forward and backward motion of the bow: while the player is moving the bow in one direction hundreds of stick-slip cycles may occur. (In the animation, only three cycles of stick-slip motion are shown, to keep the file size acceptable. Further, it is only an up-bow. When the bow reverses direction, there is inevitably a small but usually noticeable discontinuity in the wave motion, which is a key element of articulation for string instruments. Animation © Heidi Hereth)

The bow allows the player to put energy into the vibrating string and to play long sustained notes. It also allows a range of different transient effects and articulations.

The bow string interaction is important in less obvious ways, too. Over a limited range of force applied by the player (players call this "pressure"), the cycle of stick and slip is governed by the **standing wave** in the string. (See Waves in strings.) When this happens the motion of the string is nearly exactly periodic, and it therefore makes a sound with an almost exactly harmonic spectrum. (See also What is a sound spectrum.) This means that any inharmonic effects of the string are reduced by bowing, which is not the case when the string is plucked. The periodic motion of the string includes rather sudden changes in direction, and these imply substantial power in the high harmonics. The regular action of the stick-slip action thus puts power into the high harmonics and contributes to the richness, brightness and loudness of the violin's sound.



Helmholtz motion is the name given to the idealised motion shown in the animation above. (Strictly, it occurs only for an idealised, one dimensional string. Initially, we shan't worry about complications, but we'll mention a couple below).

At all times, the shape of the string is two straight lines, joined by a kink that travels around the envelope shown, which is made of two parabolic segments. For a bow moving upwards, the kink travels anti-clockwise, as shown in the animation. In the graphs of velocity *vs* time t, let's say that at the bowing point (A), an upwards moving bow starts the stick phase at time t = 0. So at t = 0, v is positive, as shown in figure b. While the string and bow move together, the kink travels to the right hand end of the string, reflects and returns (check this on the animation). When the kink returns to A, there is a large transient force on the string (the tension on both sides of the string now has downwards components). This starts the slip phase ($v < 0$ in figure b) during which the kink travels to the left hand end, is reflected and returns to the bow, ready to being the stick phase again. At the midpoint of the string (B), the up and down speeds are equal (figure c).

Now we can work out the displacement of the string at the bowing point. The kink travels at a constant speed V. Let the string's length be L. If the frequency is f so one cycle takes time 1/f. So the kink travels 2L in time 1/f so $V = 2Lf$. Suppose we are bowing it at a position L/n from the closer end, usually the bridge. During the stick phase, the kink travels to the far end and back, a distance $D = 2L(n-1)/n$. At speed V, this takes a time

$$D/V = 2L(n-1)/nV = (n-1)/nf.$$

Now the bow speed is v, so during the stick phase the bow and the string together travel a distance

$$A = v(n-1)/nf,$$

where A is the amplitude of the motion of the string at the bowing point. So, at the same bowing position, the amplitude of the motion is proportional to the bowing speed and inversely proportional to the frequency. (In the animation above, v and f are both very small.) So, if you bow with greater speed, the stick point will travel further and the amplitude will be greater. This of course makes it louder, which is what conductors want when the say "use more bow".

If you vary the bowing position but keep the bow speed constant, you are changing only n and A in the equation above. Measuring the bowing position from the closer end of the string means that n can vary between 2 and a large number. So the amplitude *at the bowing position* increases from v/2f if you bow at the middle, towards v/f as you get close the bridge. However, the maximum amplitude of the string's motion is greatest at the middle. As you move the bowing point towards the bridge, the maximum amplitude of the string increases for this reason as well.

The diagram and the explanation above are simplified. See real measurements on this link, which also explains one of the important complications.

Bow force comes into this because, for a given bowing speed and bowing position, there is only a certain range of bow force that will maintain Helmholtz motion. For a given bowing speed, the required force is a little higher as you bow closer to the bridge. Further, the permitted range becomes narrower, so you must judge the force more accurately. The pay-off is that, with more applied force, you excite more higher harmonics so the tone is brighter and richer.

For bowing **harmonics**, the story is more complicated. For the second harmonic, there are two cycles of stick slip in the time that a kink takes to make one complete return trip along the string. However, there are two kinks travelling at any time, L/2 apart. Each time one of them arrives at the bow after a reflection from the distant end, it initiates a slip. When it arrives from a reflection at the closer end, it initiates a stick phase. The differences between bowing the fundamental and bowing the second harmonic are shown in a graphic at the bottom of the page.

Now let's return to address some of the **complications** neglected above. A real bowed string never passes through a position where it is completely straight. Instead, it retains a small displacement at the bow in the bowing direction. The sharp corner of the kink produces all of the harmonics, except for those that have a node at the bowing point. However, because a string has a certain stiffness, the kink is never perfectly sharp and the finite curvature limits the number of upper harmonics. Bowing with greater force (usually closer to the bridge) gives a sharper kink and therefore more high harmonics and a brighter tone.

Source: http://www.phys.unsw.edu.au/jw/Bows.html