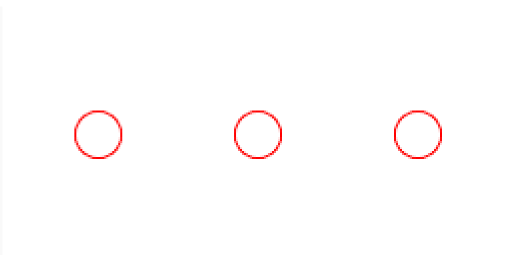
SPECIAL RELATIVITY

The special theory of relativity is, like Newton's laws of motion, a theory of motion: it deals with relations between space, time and matter.

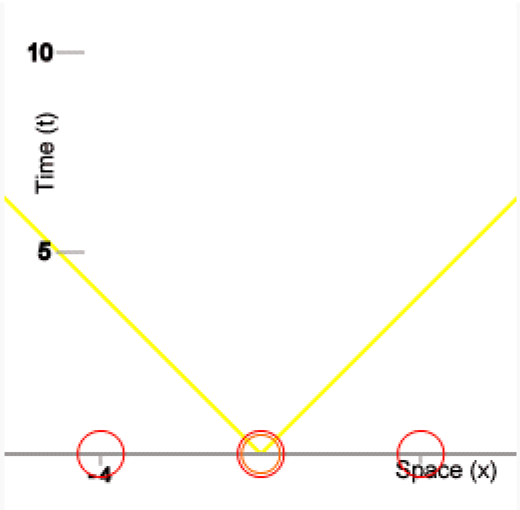
As is the case with Newton's laws of motion, special relativity uses the phenomenon of inertia as the prime organizing principle of dynamics understanding, but on a more profound level. The purpose of this article is to demonstrate that.

synchronization procedure



Picture 1. Animation
Three clocks.

Picture 1 represents three clocks, counting time. To keep those clocks perfectly synchronised you need a way of disseminating time. One way is to use pulses of light, I will get to that later. The next animation shows the three clocks, with smaller clocks shuttling between them.



Picture 2. Animation
Three spaceships and a procedure to maintain synchronised fleet time.

Animation 2 represents a spacetime diagram. The yellow lines represent the worldlines of pulses of light that are emitted at t=0. The consecutive frames of the animation combined represent a single diagram.

An observer can always define his own position as the origin of a coordinate system to map the positions of other objects, such as the ships of a fleet of spaceships. In the animations of this article the fleet consists of three ships, but it can be any number of ships, and those ships can be regarded as forming a grid. That grid provides a coordinate system to assign a coordinate distance and a coordinate time-lapse between two events.

Each ship of the fleet logs the events taking place at its spatial coordinate, recording at what point in fleet time the event took place. The ships of the fleet communicate these logs to each other and on each ship of the fleet the information in the received logs can be put together in a comprehensive spacetime mapping of the events. Animation 2 is an example of such a comprehensive mapping.

The red circles represent clocks onboard the ships, counting time. The two orange circles represent miniclocks shuttling back and forth between the ships of the fleet, the miniclocks are used for a procedure to maintain synchronised fleet time. The synchronization procedure employed here relies on *the isotropy of inertia*. The ships of the fleet take care that every time the miniclocks are sent on their journey they are in both directions propelled with equal force. (More precisely: all the mini-clocks have the same mass, and in propelling each miniclock the ships of the fleet take care to impart the same amount of kinetic energy each time.)

The ships of the red fleet are 4 units of distance apart. Here, 4 *units of distance* means that as measured by the clocks of the fleet pulses of light take 4 units of time to propagate from one ship to another. In this article, distance is measured in terms of time: the amount of transit time of pulses of light. In this animation the miniclocks take 5 units of fleet time to travel from one ship to another, so their velocity relative to the fleet is $^4/_5$th of the speed of light.

Time dilation

The animation depicts special relativity's prediction for this case that during the journey from one ship to another the miniclocks count 3 units of proper time. (Note that the time dilation doesn't prevent the synchronization. The amount of time dilation is perfectly predictable, so it can be accounted for.)

Over the course of a complete synchronization cycle you see the red clocks go round 10 times. That is, for the red clocks 10 units of time elapse. Conversely, you see the orange clocks go round 6 times; for the shuttles a complete cycle takes 6 units of shuttle time.

Transmission delays not fundamental

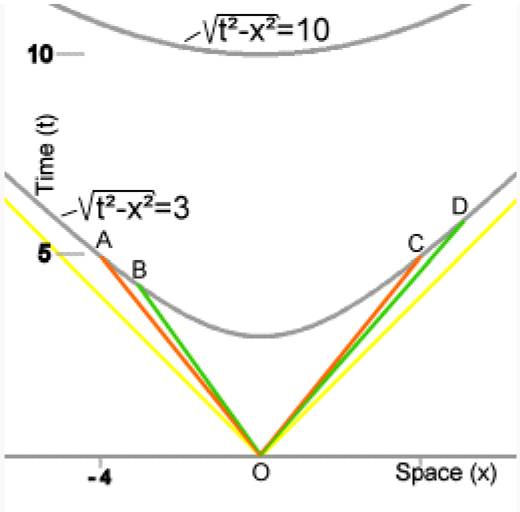
It's important to note that transmission delays play no part in special relativity. They must be accounted for, of course, in order to assemble a comprehensive spacetime mapping, but the substance of special relativity begins only *after* transmission delays have been taken into account. In order to get the relativistic effect in focus the animation is designed to avoid observational differences that arise from transmission delay.

No explanation

Relativistic physics does not provide an explanation as to how and why time dilation occurs. The starting point of relativistic physics is to assume that this is how things are, and the content of the theory consists of working out the ramifications of this assumption. The justification of the assumption is in the success of relativistic theory in applied physics.

Minkowski spacetime geometry

The conceptual shift in the transition from classical dynamics to the spacetime of special relativity is a shift from euclidean space and time to Minkowski spacetime.



Picture 3. Image
The chrono-geometry of spacetime intervals.

The line on which the points A, B, C, and D are grouped connects all the points in spacetime that have in common that for an object moving with uniform velocity from point O in spacetime to any point on that line, 3 units of proper time elapse. For instance, the lines OA and OC in image 3 correspond to the wordlines of the miniclocks in animation 2.

Spacetime interval

The concept of spacetime interval in Minkowski spacetime is somewhat analogous to the concept of radial distance in Euclidean space. In Euclidean space with 2 dimensions of space there is the relation:

$r^2 = x^2 + y^2$

Which is of course Pythagoras' theorem.

The radial distance between two points is an *invariant*, in the sense that it is independent of the particular choice of mapping a space with a cartesian coordinate system. More precisely, radial distance between two points is invariant under a coordinate transformation that corresponds to a spatial rotation.

In this article the word 'space' is used in a very abstract sense, in a meaning that is quite different from the everyday meaning of the word. In this article, everything is described in terms of **time**. Spatial distance is measured in terms of the amount of time that it takes light to cover the distance. **Both time and spatial distance are counted in units of time.**

The invariant spacetime interval of Minkowski spacetime geometry embodies a relation between space and time. The size of the spacetime interval is counted in units of time: the proper time as measured by co-moving clocks. The standard symbol for proper time is τ (the Greek letter 'tau'.)

$\tau^2 = t^2 - x^2$

The radical difference is the presence of the *minus sign*.

When represented geometrically, the spacetime interval is associated with a hyperbola curve, as depicted in image 3, whereas radial distance in Euclidean space is of course associated with a circle. Note that image 3 represents a *mapping* of a Minkowski space onto Euclidean space, rather than representing a Minkowski space directly.

Physical consequences

The shuttles are going back and forth between the ships of the red fleet. Each time they return to the ship that they came from and dock there they are still the same size as when they left. Space would be very strange indeed if returning from a journey you would find yourself to be smaller than when you left.

Now to the physical effect that does occur: the effect on elapsed time
The shuttles are taking a path that is not the spatially shortest path. Whenever that happens then on rejoining an object that did move along the spatially shortest path less proper time has elapsed for the traveller.

Metric of Minkowski spacetime

The measure of distance between two points in Euclidean space that is invariant under a rotation of the mapping coordinate system is called 'the metric of Euclidean space'.

In the case of Minkowski spacetime it is common to refer to its properties as 'geometry of Minkowski spacetime'. (A more accurate expression would be 'chrono-geometry of Minkowski spacetime', but that expression is rarely used.) By analogy with the concept of a metric in Euclidean context the formula for the invariant spacetime interval is referred to as 'the metric of Minkowski spacetime'.

The expression 'metric of Minkowski spacetime' is common usage, but because of the difference with the general concept of a metric it is also referred to as a 'pseudo-metric'. This signals that while in mathematical expressions the pseudo-metric performs exactly the same function as a metric, it is fundamentally different from a metric.

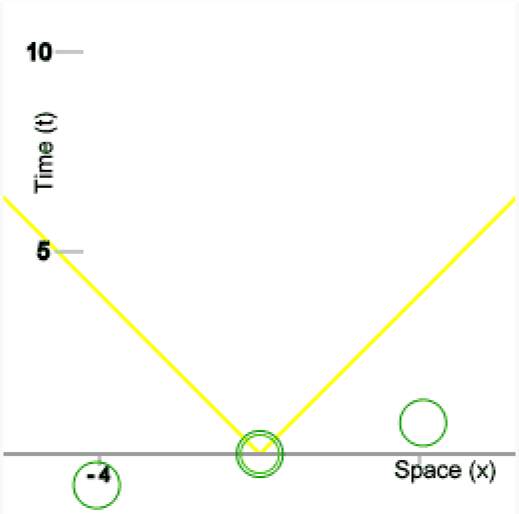
The concept of a metric can be applied in many different geometric contexts; A simple example of a metric is the metric of the way that in the game of chess the King moves around. To go from one corner to another along a column or a row takes 7 steps, and to go diagonally also takes 7 steps. That metric is an example of a non-euclidean metric, for Pythagoras' theorem does not apply.

The metric of Minkowski spacetime, with the square of one dimension being *subtracted* from the square of another dimension, is unexplained. For the question of how the structure of space and time can be like the way it is there is no established theory. At present, the Minkowski spacetime geometry must be *assumed* in order to be able to formulate a theory at all.

Spacetime interval

Special relativity implies that the *spacetime interval* is more fundamental than spatial distance. Special relativity implies that space cannot be thought of as an entity with an independent existence. Rather, physicists feel compelled to think of space as a sort of 3D *silhouette* of some more fundamental entity: the spacetime continuum, involving three spatial dimensions and one time dimension. Depending on how the spacetime is mapped spatial distances come out differently, in a way that is reminiscent of projective geometry.

Equivalence of different coordinate mappings



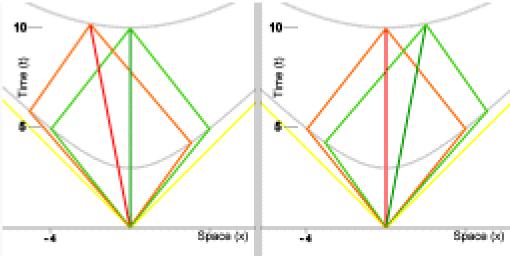
Picture 4. Animation
Three spaceships and a procedure to maintain synchronised fleet time.

The three dark green circles represent a fleet of spaceships. As in the first animations miniclocks are shuttling back and forth between the ships of the fleet, as part of a procedure to maintain synchronised fleet time. The trajectories of the green miniclocks correspond to the worldlines OB and OD in image 3.

The green fleet has a velocity with respect to the red fleet of $^2/_5$th of the speed of light. The spacetime diagram in animation 4 represents how the motion of the green fleet is mapped in a coordinate system that is co-moving with the red fleet.

The metric of Minkowsk spacetime describes how everything will proceed for the green fleet. The central ship sends the miniclocks in opposite directions and each miniclock has a relative velocity of $^4/_5$th of the speed of light with respect to the fleet. For each leg of the procedure, the miniclocks count 3 units of proper time, and the fleet clocks count 5 units of proper time for each leg of the procedure.

Symmetry

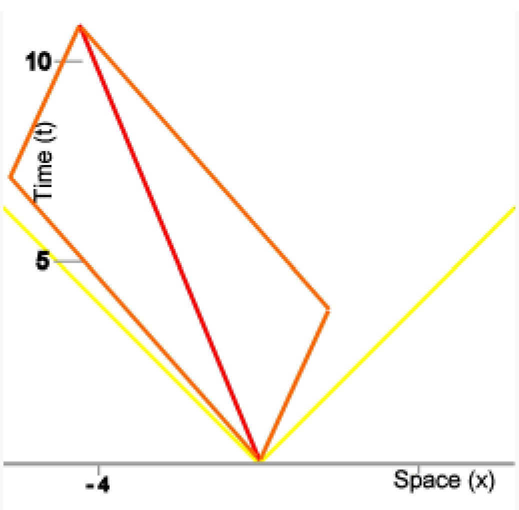


Picture 5. Image

Image 5 shows spacetime diagrams that map both the procedure of the red fleet and the procedure of the green fleet. The diagram on the left shows a mapping of events in spacetime in a coordinate system that is co-moving with the green fleet, the diagram on the right shows a mapping of events in spacetime in a coordinate system that is co-moving with the red fleet.

In this particular case the synchronization procedure and its mapping in a spacetime diagram was introduced with the red fleet first, mapping the physics in a coordinate system that is co-moving with the red fleet. It could also have been introduced with the green fleet first, mapping the physics in a coordinate system that is co-moving with the green fleet. According to special relativity there is *complete symmetry* between the two coordinate representations.

Equivalence class of coordinate systems



Picture 6. Animation
Equivalence class of coordinate systems.

In animation 6 the complete symmetry illustrated with image 5 is represented as an animation. Here, the sequence of frames is a sequence of coordinate systems with a velocity relative to each other. Each frame represents the same physics series of events: the synchronization procedure as outlined above. All individual frames of the animation represent the physics taking place equally well. Together the set of all frames in which the physics taking place is represented equally well constitutes an equivalence class of coordinate systems.

In particular the spacetime interval is the same in all spacetime mappings. On the other hand, in each frame simultaneity comes out differently relative to other frames.

Relativity of simultaneity

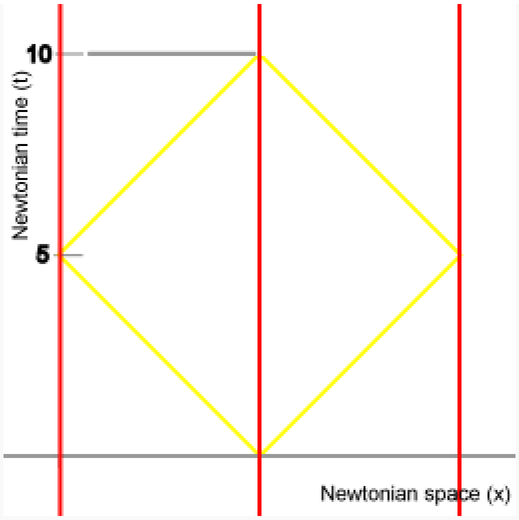
A tacit assumption in classical mechanics is that motion and simultaneity are distinct entities. That assumption does not carry over to special relativity. There is no inherent criterium to regard two events as simultaneous or not simultaneous. This is called the relativity of simultaneity.

While there is no inherent criterium for assigning simultaneity, there is an economy criterium. If you take as definition of simultaneity the synchronization procedure as above (synchronization that uses the symmetries of inertia as reference), then the physical laws, such as the equations for electromagnetism, are in their simplest form.

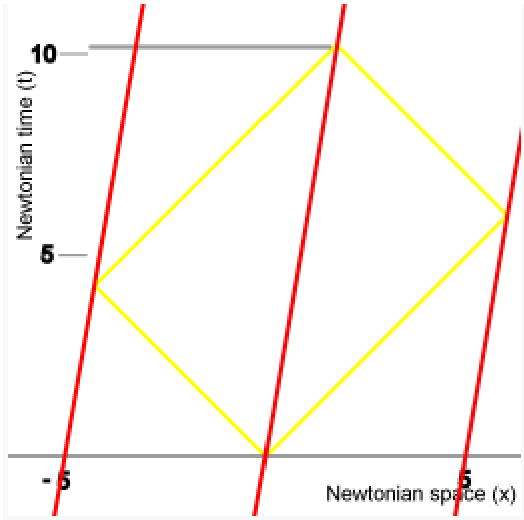
Comparison: luminiferous ether and Minkowski spacetime

Diagrams 7 to 10 represent cases of a *synchronization procedure* being applied. The yellow lines represent the worldlines of pulses of light.

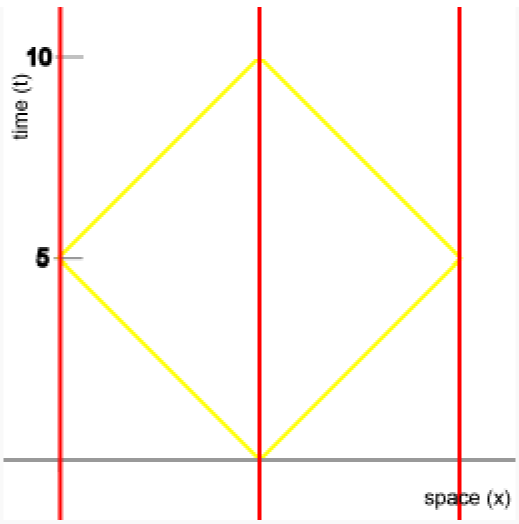
In the previous examples the synchronization procedure used miniclocks. The advantage of that is that at each encounter you can see how much proper time has elapsed for the miniclocks during their journey, enabling you to check the fidelity of the synchronization. Every encounter it is checked that for each miniclock the same amount of proper time has elapsed during its journey. When synchronizing with pulses of light that information isn't there: no comparison of transit time is possible.



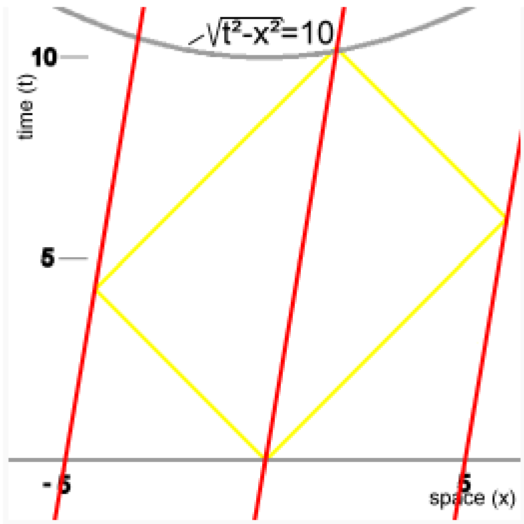
Picture 7. Diagram
The synchronization procedure in classical spacetime, using pulses of light. The procedure takes 10 units of time to complete.



Picture 8. Diagram
The case of a fleet that has a velocity relative to the luminiferous ether: the procedure takes more than 10 units of time to complete.



Picture 9. Diagram
The synchronization procedure in Minkowski spacetime. The coordinate system is co-moving with the fleet of spaceships. The procedure takes 10 units of time to complete.



Picture 10. Diagram
The synchronization procedure mapped in a coordinate system that is moving relative to the fleet. After the procedure is completed 10 units of proper time have elapsed.

Classical physics

Diagrams 7 and 8 represent what you expect to happen when the light signals are supposed to propagate in a medium, usually referred to as the luminiferous ether. If a fleet of spaceships has a velocity with respect to the luminiferous ether the overall pathlength of the light signals is longer, and the procedure will take more time than when the spaceships are stationary with respect to the luminiferous ether.

Minkowski spacetime

Diagrams 9 and 10 represent what you expect to happen when the environment is Minkowski spacetime. Every mapping of the procedure will indicate that 10 units of proper time will elapse. In other words, the synchronization procedure will not reveal anything about a velocity with respect to some background structure.

Einstein synchronization procedure

When pulses of light are used for synchronization of clocks the procedure is referred to as Einstein synchronization procedure. In the article, 'On the electrodynamics of moving bodies' in which Einstein introduced special relativity Einstein had introduced that procedure as a definition of simultaneity.

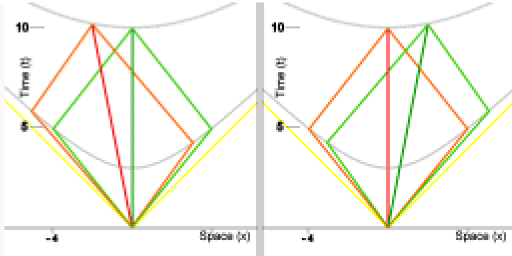
The diagrams illustrate in which environment the Einstein synchronization procedure is applicable. In space and time as envisioned prior to relativistic physics you expect the synchronisation procedure to take more time when the senders/receivers have a velocity relative to the luminiferous ether. That difference will give rise to inconsistencies, making the procedure unfit. In Minkowski spacetime, however, the Einstein synchronization procedure is the appropriate setup.

consequences for measurements of the speed of light

The only way to measure the speed of light is to set up a back and forth trip. If the environment is Newtonian space and time then you expect to find a different value for the speed of light, depending on the velocity of the measurement rig to the luminiferous ether.

Diagrams 8 and 9 illustrate how it works out in Minkowski spacetime. Different measurement setups, with a velocity relative to each other, will each find the same value for the speed of light. That means that the speed of light is an invariant.

Symmetric velocity time dilation



Picture 11. Image

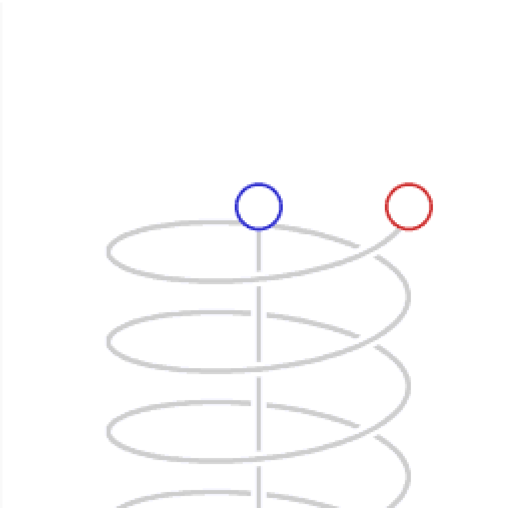
The situation is symmetrical. The red fleet and the green fleet have a velocity relative to each other, so for each unit of red time less than one unit of green time elapses, and for each unit of green time, less than one unit of red time elapses.

At time t=0 the two central ships of both fleets pass each other. At t=0, let the red ship emit a signal with a particular frequency, as measured in red fleet time. The green ship receives that signal, and that signal will be shifted to a lower frequency, as measured by green fleet time.

Conversely: at t=0 let the green ship emit a signal with a particular frequency, as measured in green fleet time. The red ship receives that signal, and that signal will be shifted to a lower frequency, as measured by red fleet time.

This type of time dilation is called symmetric velocity time dilation.
An example of that is the trajectories of the time-disseminating shuttles in the animations. At all times the shuttles have a velocity relative to each other, so there is a corresponding symmetric velocity time dilation. When the shuttles rejoin it is seen that among the shuttles there no difference in the amount of elapsed proper time.

Nonsymmetric velocity time dilation



Picture 12. Animation
A straight worldline and a helical worldline.

Schematic representation of asymmetric velocity time dilation. The animation represents motion as mapped in a Minkowski spacetime diagram, with two dimensions of space, (the horizontal plane) and position in time vertically. The circles represent clocks, counting lapse of proper time. The Minkowski coordinate system is co-moving with the non-accelerating clock.

The clock in circular motion counts a smaller *amount* of proper time than the non-accelerating clock. Here, the difference in the amount of proper time that elapses is in a ratio of 1:2, which corresponds to a transversal velocity of 0.866 times the speed of light.

Any light emitted by the non-accelerating clock and received by the circling clock is received as a blue-shifted signal, in a ratio of 1:2. Any light emitted by the circling clock and received by the non-accelerating clock is received as a red-shifted signal, in a ratio of 2:1 .

In this situation, symmetry is broken, and there is a difference in*amount* of proper time that elapses.

Source:
http://www.cleonis.nl/physics/phys256/special.php#section_1