Seismic Analysis on Cylindrical Tanks Subjected to Horizontal Acceleration

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Abstract :The dynamic behaviour of inviscid fluid contained in horizontally accelerated cylindrical tanks is considered. Mechanical equations describing the fluid motion are developed and simplified by use of small amplitude wave approximations, enabling expressions for the different modes of vibrations, sloshing frequencies and the free surface displacements to be obtained. The expression for free surface displacement is formulated in such a way that the time histories of the free surface displacement can be calculated for tanks subjected to real earthquake accelerograms. Comparisons of predicted and finite element analysis of different modes of vibration of the cylindrical tank, sloshing frequencies and free surface displacements of a model cylindrical water tank subjected to sinusoidal acceleration was found to be in close agreement.

Keywords: Added mass, sloshing frequency, sloshing time history

I. INTRODUCTION

The seismic performance of liquid retaining structures is a matter of special importance. Without an assured water supply, uncontrolled fires subsequent to a major earthquake may cause substantially more damage than the earthquake itself, as occurred in the great 1906 San Francisco earthquake. If the outbreaks of disease that frequently follow destructive earthquakes are to be avoided, it is essential that safe supplies of drinking water are available. Spillage of toxic can cause damage many times the values of the affected tank and contents.

This paper considers the general problem of horizontally accelerated cylindrical tanks containing in viscid fluid. Small amplitude wave approximations are used to obtain expressions for free surface displacements, pressure distribution. The natural frequencies of vibration of cylinder are calculated by determining the

'added mass' of the fluid. This added mass of the fluid is added up with the structural mass to obtain the different modes of vibration of the cylinder.

II. Mathematical formulation

A. Mathematical formulation for determining the natural frequencies of the cylindrical tank

a. Cylindrical shell of revolution

To calculate the added mass coefficient of the fluid inside the cylinder, a strip model is used. The basic assumption of the strip model is to consider a narrow strip between z and z+dz, located sufficiently far from the ends z=0 and z= H, the axial flow component is negligible; furthermore it is assumed that the end effects extend over a small axial length only.

From the structural standpoint, when writing the shell equations the axial component of motion is discarded and so are the axial variations of displacement field. Therefore radial and tangential Love equations are written as [4]:

$$\frac{E_s e}{1-\nu^2}\left\{\frac{U}{R^2}+\frac{e^2}{12R^2}\left(\frac{\partial^4 U}{R^2\partial\theta^4}-\frac{\partial^3 V}{R^2\partial\theta^3}\right)+\frac{\partial V}{R^2\partial\theta}\right\}+\rho_s e\ddot{U}=p(R,\theta;t) \quad (1)$$

$$\frac{E_s e}{1-\nu^2}\left\{\left(1+\frac{e^2}{12R^2}\right)\left(\frac{\partial^2 V}{R^2\partial\theta^2}\right)+\frac{\partial U}{R^2\partial\theta}-\frac{e^2}{12R^2}\frac{\partial^3 U}{R^2\partial\theta^3}\right\}-\rho_s e\ddot{V}=0 \quad (2)$$

Assuming hoop strain to be negligible, Love eqn. in radial direction is reduced to:

$$\frac{E_s e^3}{12(1-\nu^2)R^4}\left(\frac{\partial^4 U}{\partial\theta^4}+\frac{\partial^2 U}{\partial\theta^2}\right)+\rho_s e\ddot{U}=p(R,\theta;t) \quad (3)$$

The mode shapes are of the following admissible type [1]:

$$\left\{\begin{matrix} u_n(\theta)=\alpha_n\cos n\theta+\beta_n\sin n\theta \\ v_n(\theta)=a_n\cos n\theta+b_n\sin n\theta \end{matrix}\right\}\ \text{n=1,2….} \quad (4)$$

This can be conveniently split into two orthogonal families of mode shapes:

$$\begin{matrix} u_n^{(1)}(\theta)=\cos n\theta;\ v_n^{(1)}=-\frac{1}{n}\sin n\theta \\ u_n^{(2)}(\theta)=\sin n\theta;\ v_n^{(2)}=-\frac{1}{n}\cos n\theta \end{matrix} \quad (5)$$

The corresponding mass and stiffness coefficients per unit length are given by [1]:

$$m_s^{(1,2)}(n,n)=\rho_s e\int_0^{2\pi}(1+\frac{1}{n^2})\left\{\begin{matrix} cos^2 n\theta \\ sin^2 n\theta \end{matrix}\right\}Rd\theta=e\pi R(1+\frac{1}{n^2}) \quad (6)$$

$$k_s^{(1,2)}=\frac{E_s e^3 n^2(n^2-1)}{12(1-\nu^2)R^3}\int_0^{2\pi}\left\{\begin{matrix} cos^2 n\theta \\ sin^2 n\theta \end{matrix}\right\}Rd\theta=\frac{E_s e^3\pi n^2(n^2-1)}{12(1-\nu^2)R^3} \quad (7)$$

The natural frequencies of the structure are given by[1]:

$$\omega_n = \sqrt{\frac{k_s(n,n)}{m_s(n,n)}} = \frac{n^2}{R}\left(\frac{e}{R}\right)\sqrt{\frac{E_s}{12(1-\nu^2)}\left(\frac{n^2-1}{n^2+1}\right)} \quad (8)$$

Free and rigid in plane translations corresponds to n=1. The shapes constitute a subspace spanned by the two orthonormal vectors:

$$u_1^{(1)} = \cos\theta; v_1^{(1)} = -\sin\theta; u_1^{(2)} = \sin\theta;\ v_1^{(2)} = \cos\theta \quad (9)$$

The mode shapes n>1 corresponds to axial bending

b. Considering the effect of fluid in the cylinder

We now turn our attention towards motion of the fluid forced by a radial vibration of the shell of the type:

$$U(\theta : t) = \sum_{n=1}^{\infty} q_n^{(1)}\cos n\theta + q_n^{(2)}\sin n\theta \quad (10)$$

Where $q_n^1(t)$ and $q_n^2(t)$ stand for modal displacements of the shell.

Pressure is governed by the boundary value problem [1]:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} = 0 \quad (11)$$

$$\frac{\partial p}{\partial r} = -\rho_f \sum\left(\ddot{q}_n^{(1)}\cos n\theta + \ddot{q}_n^{(2)}\sin n\theta\right);\ r=R \quad (12)$$

Splitting the above eqns. into sine and cosine families:

$$\frac{\partial^2 p_n^{(1)}}{\partial r^2} + \frac{1}{r}\frac{\partial p_n^{(1)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p_n^{(1)}}{\partial \theta^2} = 0 \quad (13)$$

$$\frac{\partial p_n^{(1)}}{\partial r} = -\rho_f \ddot{q}_n^{(1)}\cos n\theta\, ; r = R \quad (14)$$

$$\frac{\partial^2 p_n^{(2)}}{\partial r^2} + \frac{1}{r}\frac{\partial p_n^{(2)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p_n^{(2)}}{\partial \theta^2} = 0 \quad (15)$$

$$\frac{\partial p_n^{(2)}}{\partial r} = -\rho_f \ddot{q}_n^{(2)}\sin n\theta\, ; r = R \quad (16)$$

Solving the above eqns the pressure field can be determined as:

$$p_n^{(1)}(r,\theta;t) = -\rho_f H\frac{\ddot{q}_n^{(1)}}{nR^{n-1}} r^n \cos n\theta \quad (17)$$

$$p_n^{(2)}(r,\theta;t) = -\rho_f H\frac{\ddot{q}_n^{(2)}}{nR^{n-1}} r^n \sin n\theta \quad (18)$$

As the structural and fluid problems have the same axial symmetry it is sufficient to retain only one of the two mode families.

Therefore, in the present problem, the added mass matrix, as expressed in the structural mode basis is diagonal and the mode shapes of the shell are the same as in vacuum. The generalized force exerted by the fluid on a shell strip of unit length is [1]:

$$Q_n = -\rho_f \frac{\ddot{q}_n^{(1)}}{n} HR^2 \int_0^{2\pi} (\vec{u}.\vec{u}) cos^2 n\theta d\theta = \frac{-\rho_f \pi H R^2}{n}\ddot{q}_n^{(1)} \quad (19)$$

Where $\vec{u}$ is the unit normal vector in the radial direction, the added mass coefficients per unit length of the shell is given by:

$$M_a = \frac{\rho_f \pi R^2 H}{n} \quad (20)$$

Where the mode shape is normalized by the condition max $u_n^{(1,2)}(\theta)=1$ and $m_f = \rho\pi R^2 H$, stands for the physical mass of the fluid contained in the shell of unit length

B. Free surface sloshing time history

The sloshing of the fluid inside the cylindrical tank takes place due to the x-component of the acceleration imparted due to the seismic activity. The complete set of the stated problem is given by the following system of equations[2]:

$$-\nabla(p - z) = \vec{U}.\nabla\vec{U} + \frac{\partial \vec{U}}{\partial t} + a(t)\hat{\imath} \quad (21)$$

$$\nabla.\vec{U} = 0 \quad (22)$$

$$\vec{U}(x,y,z,0) = 0 \quad (23)$$

$$\vec{U}.\widehat{en} = 0\ , (x,y,z\ on\ tank\ surface) \quad (24)$$

$$p = 0, [x,y,z\ on\ z = \eta(x,y,t)] \quad (25)$$

$$\vec{U}.\nabla(\eta - z) + \frac{\partial \eta}{\partial t} = 0, [x,y,z\ on\ z = \eta(x,y,t) \quad (26)$$

$$\eta(x,y,0) = 0 \quad (27)$$

The relative velocity, $\vec{U}$; possesses a velocity potential, φ. Introduction of this into above set of eqns. and introducing the linearizing approximation of first order wave mechanics, which assumes that free surface waves have a relatively small amplitude, consists of neglecting all non linear terms and evaluating the resulting free surface boundary conditions on the plane z=0. This leads to the following boundary value problem for φ.

$$\nabla^2\varphi = 0 \quad (29)$$

$$\varphi(x,y,z,0) = 0 \quad (30)$$

$$\frac{\partial \varphi}{\partial n} = 0, (x,y,z\ on\ tank\ surface) \quad (31)$$

$$\frac{\partial \varphi(x,y,0,t)}{\partial z} = \frac{\partial^2 \varphi(x,y,0,t)}{\partial t^2} + x\dot{a(t)} \quad (32)$$

$$\frac{\partial\varphi(x,y,z,0)}{\partial t} = 0 \quad (33)$$

Once φ has been found from the above set of eqns. the free surface coordinates can be determined from the equation [2]:

$$\eta(x,y,t) = \frac{\partial\varphi(x,y,0,t)}{\partial t} + xa(t) \quad (34)$$

For cylindrical tank, the solution can be found by writing the eqns. in cylindrical coordinates. Then separation of variables can be used to obtain the solution for φ [2]

$$\varphi = \cos\theta \sum_{n=1}^{\infty} F_n(t) \frac{J_1(\lambda_n r)}{J_1(\lambda)} \frac{\cosh\lambda_n(D-z)}{\cosh\lambda_n D} \quad (35)$$

The numbers λ_n are roots of the first derivative of the first order Bessel function of the first kind $[J_1'(\lambda_n) = 0 \; for \; n = 1,2,3 \ldots]$. The right side of the eqn has been chosen so that it satisfies the governing differential eqns[2].

$$\cos\theta \sum_{n=1}^{\infty} \frac{J_1(\lambda_n r)}{J_1(\lambda_n)} \left[\ddot{F}_n(t) + \beta_n^2 F_n(t)\right] + r\dot{a}(t)\cos\theta = 0\,, (\beta_n \equiv \sqrt{\lambda_n \tanh\lambda_n D}) \quad (36)$$

$F_n(t)$ must satisfy the initial conditions $\dot{F}_n(0) = F_n(0) = 0$. Thus the solution for $F_n(t)$ gives

$$F_n(t) = \frac{2}{\beta_n(1-\lambda_n^2)} \int_0^t \dot{a}(\tau)\sin\beta_n(t-\tau)d\tau \quad (37)$$

Once the function for a(t) has been specified $\dot{F}_n(t)$ can be calculated. Suppose a(t) is sinusoidal(a_o $\sin\omega t$), then

$$\dot{F}_n(t) = \frac{2a_o\omega}{(1-\lambda_n^2)} \frac{\omega\sin\omega t - \beta_n \sin\beta_n t}{\omega^2 - \beta_n^2} \quad (38)$$

From this an expression for the free surface coordinates can be calculated as [2]:

$$\eta(r,\theta,t) = \left[r\,a(t) + \sum_{n=1}^{\infty} \dot{F}_n(t) \frac{J_1(\lambda_n r)}{J_1(\lambda_n)}\right]\cos\theta \quad (39)$$

C. Sloshing Frequency

Theoretically the sloshing frequency of the liquid contained in a cylindrical tank can be determined from the expression [3]

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{R}\tanh(\lambda_n \frac{H}{R})} \quad (40)$$

III.FEA Modeling of the Cylindrical Tank

The 3D cylindrical tank and the fluid subjected to seismic excitation can be converted into a 2D axisymmetric problem [6] and solved using the commercial FEA packages available like Ansys 11.0.

A detailed explanation for the FEA modeling has been avoided in this section. For the modeling the sloshing problem in Ansys, the cylinder wall is modeled with quadrilateral 4 node element PLANE 42 with each node having 2 degrees of freedom i.e u_x and u_y

The fluid domain has been modeled with 4 node quadrilateral elements FLUID 79, having 2 degrees of freedom at each node i.e the translation DOF u_x and u_y.

The base of the tank is arrested for all DOF. The seismic excitation data (here a sinusoidal wave) can be input to the structure at the base in a tabular column. At the fluid structure interface the fluid elements should be coupled in such a way, so that it transmits only normal forces to the cylinder wall and is always in contact with the surface of the cylinder i.e it can move only in transverse direction. On the line of symmetry the fluid elements are constrained in the radial direction.

Modal analysis is carried out using Block Lanczos method for the first 5 modes and mode shapes.

IV. Results and Discussion

From the mathematical expression derived earlier, the natural frequency of the tank containing fluid, the sloshing frequency and the time history of the free surface subjected to seismic forces are calculated and compared to the results obtained analytically from Ansys.

A. Modes of vibration of cylinder

The theoretical and the analytical results obtained are given below:

Table 1- Modes of vibration of the cylinder

Mode no.	Theoretical(Hz)	Numerical(Hz)
1	54.10	53.23
2	144.16	147.87
3	273.43	298.34
4	446.78	453.56
5	678.76	684.78

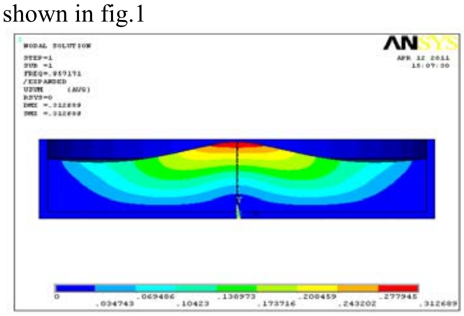
B. Sloshing Frequencies and Mode Shapes

The sloshing frequencies calculated theoretically and analytically along with the mode shapes obtained from Ansys are given below:

Table 2- Sloshing frequencies of the fluid

Mode No.	Theoretical(HZ)	Numerical(Hz)
1	.7434	.9571
2	1.599	1.3526
3	2.0565	1.8653
4	2.4116	2.3030
5	2.717	2.3314

The corresponding mode shapes obtained are shown in fig.1



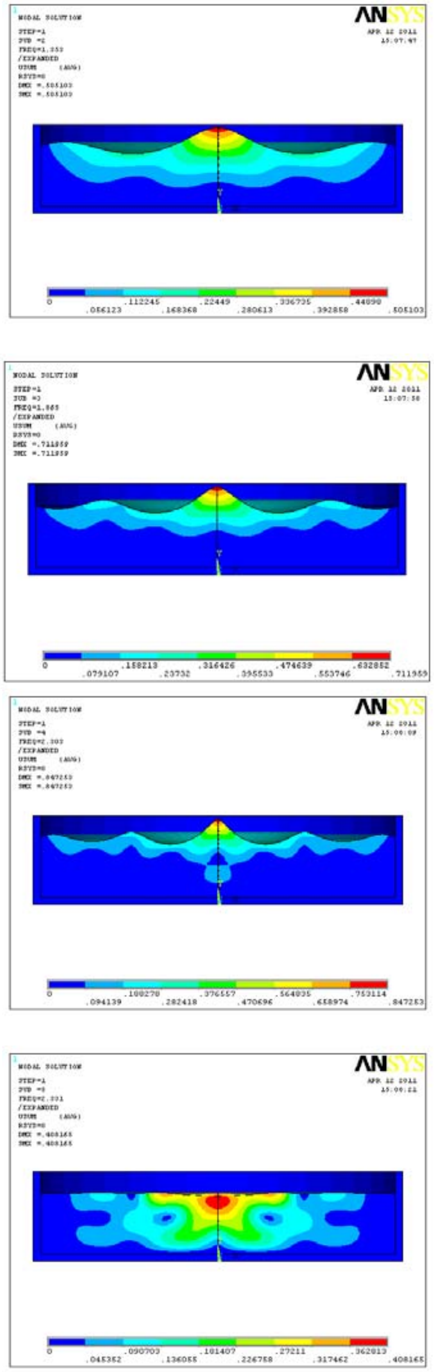


Fig. 1- Sloshing mode shapes of the fluid.

The theoretical and the analytically obtained sloshing frequencies are found to be agreeing.

C. **Free surface sloshing time history**

The expression for free surface sloshing time history has been determined from. A MATLAB program was written for plotting the free surface sloshing time history of the tank when it was subjected to a sinusoidal acceleration. The obtained graph is shown in the fig2.

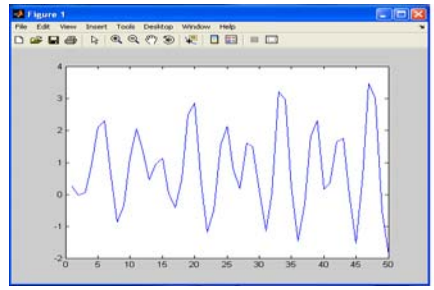


Fig. 2- Time history of free surface sloshing(analytical)

The corresponding free surface sloshing time history obtained from Ansys is shown in fig.3:

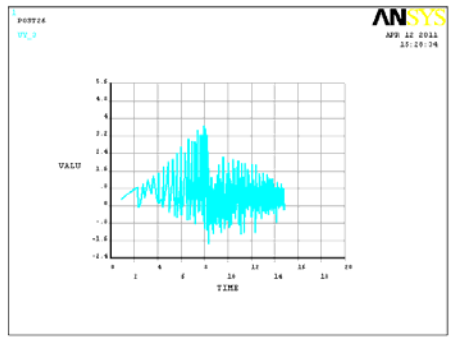


Fig.3- Time history of free surface sloshing (Numerical)

From the obtained graphs, it can be seen that the theoretically obtained time history and the analytically obtained time history vary, this can be attributed to ignoring of non linear terms from the governing differential eqns.

V. Conclusion

The natural frequencies of the cylinder, sloshing frequency and the time history of the free surface sloshing was determined theoretically and validated using Ansys 11.0.

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