SCROLL WAVE TURBULENCE

Scroll wave turbulence is a self-sustained regime in a three-dimensional autowave (e.g. excitable or oscillatory) medium mediated by an instability ofscroll waves and including persistent multiplication and annihilation of scroll filaments.

Overview

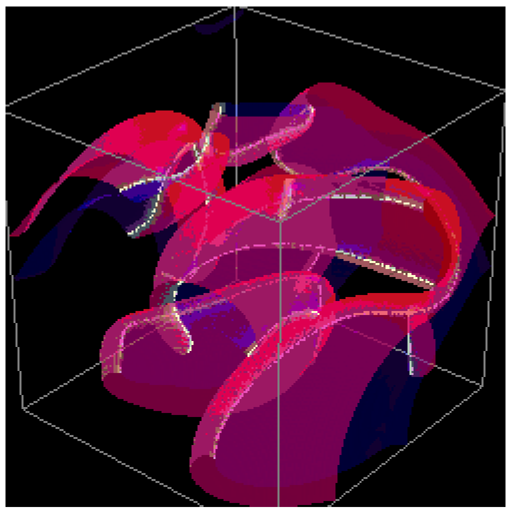

Figure 1: Scroll wave turbulence developed from a scroll with a curved filament. Simulation of FitzHugh-Nagumo model in 3D, parameters as in (Biktashev 1998). Red surfaces are fronts (u=0,v<0) blue surfaces are backs (u=0,v>0) of excitation waves, and yellow lines are singularities (u=0,v=0)rotating around slowly moving scroll filaments, where u,v are respectively the activator and inhibitor fields.

Compared to drift of spiral waves in two spatial dimensions, the scroll waves in three spatial dimensions have more degrees of freedom. While spiral waves rotate around *cores*, scroll waves rotate around *filaments*, which can not only move in space, but also change shape. The phase of rotation may vary along the filament, the feature known as *twist* of the scroll wave. Twist of a scroll wave and curvature of its filament are factors of its dynamics that are specifically three-dimensional. Sometimes interaction of all the factors can make the steady rotation of scroll waves unstable and lead to apparently chaotic regime in which the scroll filaments are bent, increase in length, break up and multiply. Such regime is known as *scroll wave turbulence (SWT)*. This is akin to spiral wave break-up in two spatial dimensions. However, scroll wave turbulence is a 3D phenomenon in that it may be observed in a system which does not show any turbulent behaviour whilst in 2D. This may happen for cases where 2D spiral waves are stationarily rotating or meandering. At the time of writing this article, three specifically three-dimensional mechanisms of SWT in excitable and oscillatory media have been well described. One is mediated by *negative filament tension* which is an intrinsic property of a scroll wave that makes a curved filament to curve even more. Another is the finite-wavelength instability of scroll filaments, which causes them to spontaneously deform into *wrinkled* or *zig-zag* shape. The third one is observed in non-uniformly anisotropic media and is related to development of twist instabilities. Apart from that, any mechanism that could produce turbulence in two dimensions would, of course, reveal itself in three dimensions as well (and these mechanisms are not considered in this article). SWT is thought to be relevant for understanding of fibrillation of the heart. Cardiac tissue is excitable and three-dimensional, and reentrant waves may take the form of scroll waves. Scroll waves can produce tachycardia and unstable 3D waves are associated with fibrillation.

The concept of filament tension

The concept of filament tension can be understood in terms of perturbative dynamics of a scroll wave (Yakushevich, 1984;Keener, 1988;Biktashev et al., 1994). Consider a reaction-diffusion system

$$\partial_t \mathbf{u}=\mathbf{f}(\mathbf{u})+\mathbf{D}\nabla^2\mathbf{u},\mathbf{u}=\mathbf{u}(\vec{r},t),\mathbf{f}=\mathbf{f}(\mathbf{u})\in\mathbb{R}^\ell,\ell\geq 2,\vec{r}\in\mathbb{R}^2 \text{ or } \mathbb{R}^3 \quad (1)$$

and assume existence of stationarily rotating spiral solutions in $\mathbb{R}^2$,

$$\mathbf{u}(\vec{r},t)=\mathbf{U}(\rho,\theta+\omega t), \quad (2)$$

where ρ,θ are polar coordinates in $\mathbb{R}^2$, and the rotation frequency ω is a fixed constant. A simple extension of spiral wave solution to the third spatial dimension is called a straight scroll wave. More generically, a scroll wave solution in $\mathbb{R}^3$ may be viewed as a solution of the form

$$\mathbf{u}(\vec{R}+\vec{N}\rho\cos\theta+\vec{B}\rho\sin\theta,t)=\mathbf{U}(\rho,\theta+\omega t-\Phi)+O(\varepsilon),$$

where ε is a formal small parameter measuring deformation of a scroll wave compared to the straight scroll, $\vec{R}=\vec{R}(\sigma,t)$ is the parametric equation of filament position at time t ,$\Phi=\Phi(\sigma,t)$ is the distribution of the rotation phase along the filament, $\vec{N}=\vec{N}(\sigma,t)$ and $\vec{B}=\vec{B}(\sigma,t)$ are the unit principal normal and binormal vectors to the filament at point $\vec{R}(\sigma,t)$ .Vectors $\vec{N}$ and $\vec{B}$ together with tangent vector $\vec{T}$ make a Frenet-Serret triple which can be defined using arc length differentiation operator $D_s u(\sigma)=|\partial_\sigma \vec{R}(\sigma)|^{-1}\partial_\sigma u(\sigma)$. Then the tangent vector is $\vec{T}=D_s\vec{R}$, the curvature κ and the normal unit vector $\vec{N}$ are defined by $\kappa\vec{N}=D_s\vec{T}$, and the binormal vector $\vec{B}=\vec{T}\times\vec{N}$ completes the triad. The Frenet-Serret description is easy to understand but it has a significant technical disadvantage: it becomes degenerate at the points of zero filament curvature, $\kappa=0$. An alternative description, suggested by Verschelde et al. (2007), (see also Dierckx and Verschelde, 2013) free from this disadvantage, is in terms Fermi-Walker coordinates, corresponding to a Levi-Civita (torsion-free metric) connectionalong the filament (hereafter called torsion-free coordinates for brevity). Either way, the equation of motion of the filament, in the assumption of small filament curvature, $\kappa=O(\varepsilon)$, and small twist, $D_s\Phi=O(\varepsilon)$, can be written as

$$\partial_t\vec{R}=\alpha D_s^2\vec{R}+\beta[D_s\vec{R}\times D_s^2\vec{R}]+O(\varepsilon^2) \quad (3)$$

Equation (3) is written in the assumption that parameterization of the filament is chosen in such a way that a point with a fixed σ moves orthogonally to the filament. The coefficients in the equation of motion are calculated using the response functions $\mathbf{W}_1$ (see Drift of spiral waves) as

$$\alpha+i\beta=-\frac{1}{2}\int_0^\infty\oint[\mathbf{W}_1(\rho,\theta)]^{+}\mathbf{D}e^{-i\theta}\left(\partial_\rho-\frac{i}{\rho}\partial_\theta\right)\mathbf{U}(\rho,\theta)\,\mathrm{d}\theta\,\rho\,\mathrm{d}\rho.$$

Following Biktashev et al. (1994), let us now consider the total length of the filament, defined at each t:

$$S(t)=\int\mathrm{d}s=\int||\partial_\sigma\vec{R}||\,\mathrm{d}\sigma,\quad(4)$$

where the integral is taken over the whole filament. Differentiation of (4), with account of (3) and using integration by parts, reveals that neglecting boundary effects (which are absent for closed filaments and vanish for smooth impermeable boundaries), the rate of change of the total length is described by

$$\frac{\mathrm{d}S}{\mathrm{d}t}\approx-\alpha\int\kappa^2\,\mathrm{d}s.\quad(5)$$

This implies that unless the filament is straight, and within the applicability of the perturbation theory, the total length of the filament will decrease if $\alpha>0$ and increase if $\alpha<0$. Hence the coefficient α is often called *filament tension*.

Filament tension can be defined via the asymptotic rate of collapse or expansion of scroll rings of large radius and exact axial symmetry, in which case the problem becomes mathematically equivalent to that of a drift of a spiral wave in an electric field (Hakim, Karma 1999;Henry, Hakim 2002). This definition can be formally extended to the case when unperturbed spiral waves are meandering, even though at the moment of writing this article, an asymptotic theory of evolution of meandering scroll does not yet exist.

Filament tension depends on parameters of the medium, typically becoming negative in media with lower excitability, where spiral waves have larger radii (Panfilov and Rudenko, 1987;Brazhnik et al., 1987; Krinsky et al., 1996; Hakim and Karma (1999)), although there are exceptions (Alonso and Panfilov, 2008) (this is important as cardiac tissue is typically highly excitable). Henry (2004) has noted a correspondence between the sign of filament tension and the [[Meander of spiral waves|spiral wave meandering]]: namely, that the manifold $\alpha=0$ in the parametric space of (1) in 3D correlates with the locus of spontaneously drifting spirals in (1) in 2D, separating "outward petal" and "inward petal" meandering patterns, which is related to the resonance between the Hopf and the Euclidean frequencies.

SWT mediated by negative filament tension

As was noted already by Brazhnik et al (1987), negative filament tension should make a straight scroll unstable, as any sufficiently smooth small deviation from the straight shape will grow. The constant increase in filament length due to curvature-induced drift as predicted by (4) is stopped when other factors, not accounted for by the perturbation theory quoted above, become significant. These include high curvatures, high twists, interaction of different scroll waves or different parts of the same scroll wave with each other and with medium boundaries. One possible outcome is that mutual interaction of scrolls may stabilise them so that double or multiple scrolls are observed. Alternatively, if stabilization does not occur, then the SWT can develop (Biktashev (1998), Alonso et al. (2004), Alonso et al. (2006b), Alonso and Panfilov (2007) see also Alonso et al. (2013) for a review). A third possibility is extinction of all scroll waves. Empirical studies of SWT suggest that in smaller media, extinction or mutual stabilization prevail, whereas for sufficiently large media, SWT becomes more prevalent. However, it is not known if stabilization and extinction may still be possible, just less probable, in larger media. The chaotic character of SWT is currently only a conjecture. Note that for $\beta=0$ and planar filament, equation (3) is similar to Kuramoto-Sivashinsky equation without its regularization term, and chaotic nature of solutions of Kuramoto-Sivashinsky equation is known. Comparative statistical analysis of numerical solutions of SWT due to negative tension and due to the two-dimensional spiral break-up instabilities, see e.g. Zaritsky et al. 2005,Clayton 2008,Davidsen et al. 2008, may be considered as indirect confirmation of their chaotic nature. At the moment of writing this article, such studies have been mostly empirical, although attempts have been made to summarise the findings in terms of Markov models (Davidsen et al. 2008).

The above quoted perturbation theory is based on the assumption that 2D spirals are stationarily rotating, formalized by (2). At the time of writing this article, an extension to the meanderingcase has not been done yet; however, filament tension still can be defined via the rate of change of average radius of axisymmetric scroll rings. For cases where meandering scroll rings expanded, the full 3D simulations also show SWT.

The search for possible low-voltage defibrillation methods motivates control of SWT with the view of elimination of all scroll waves. Alonso et al. (2003, 2006a) explored periodic time-dependent variation of parameters of the mathematical model, representing repetitive low-voltage electrical stimulation. They focused on *super-resonant* case with stimulation frequency higher than that of the scrolls. Such stimulation can sometimes effectively reverse the sign of filament tension from negative to positive, thus stopping the breeding of scroll waves and "taming" the SWT. This, however, does not guarantee elimination of already existing scroll waves if they are not scroll rings topologically. Such elimination can be achieved by *resonant*stimulation, i.e. with the frequency equal to that of the scroll waves, exploiting resonant drift phenomenon (Morgan et al. 2008). Super-resonant *localized* stimulation can cause induced driftof scroll waves, which can also be used for suppressing the SWT (Zhang et al. 2005).

SWT due to twisted anisotropy

Three-dimensional regimes phenomenologically very similar to negative tension SWT have been observed in simulations of scroll waves in media with spatially non-uniform anisotropy where the diffusion matrix components are not scalars but tensors, and vary in space:

$$\frac{\partial u_j}{\partial t}=f_j(u_k)+\sum_{k=1}^{\ell}\sum_{\mu,\nu=1}^{3}\frac{\partial}{\partial x_\mu}\left(D_{j,k}^{\mu,\nu}(\vec{r})\frac{\partial}{\partial x_\mu}u_k\right), j,k\in\{1,\ldots,\ell\}, \mu,\nu\in\{1,2,3\}. \quad (6)$$

In cardiac excitation models, $D_{j,k}^{\mu,\nu}(\vec{r})=P_{i,j}Q^{\mu,\nu}$ where matrix component $P_{1,1}=1$ (assuming 1 is the index of the transmembrane voltage field) and all other $P_{i,j}=0$, and tensor $Q_{\mu,\nu}$is related to local direction of fibres (see Models of heart). A number of numerical studies have been done with "rotational anisotropy": in a rectangular volume, the filament direction is within (x,y) plane, and this direction linearly changes depending on the z-coordinate (Panfilov and Keener, 1995; Fenton and Karma 1998a and 1998b). This roughly corresponds to the structure of ventricular walls. In such numerical studies, fiber rotation could cause scroll wave breakup. A more detailed analysis revealed a typical motif: fiber rotation creates regions of highly localized filament twist, called "twistons". These twistons migrate along filaments, and their collision with boundaries causes a scroll wave filament to break, producing daughter scroll waves. At the time of writing this article, there is no published theory of this mechanism of SWT, although the perturbation theory described above can be extended to cover non-uniformly anisotropic models (6) (Verschelde et al. 2007, Dierckx et al. 2009).

SWT due to finite-wavenumber instability

A peculiar mechanism of three-dimensional instability of scroll waves that can also lead to SWT has been described in the [[complex Ginzburg-Landau equation]] (see e.g. Aranson and Bishop (1997), Aranson and Kramer (2002), Rousseau et al. (2008), Reid et al. (2011) and references therein).

$$\partial_t A=A+(1+ib)\nabla^2 A-(1+ic)|A|^2 A, A\in\mathbb{C}. (7)$$

Although the filament tension in this oscillatory autowave model is always positive, $\alpha=1+b^2$, so a straight scroll is stable against perturbations that are sufficiently smooth in space, at some combinations of parameters b,c it still can develop a three-dimensional instability with respect to finite wavenumber (related to two-dimensional "accelerating" instability, which is in turn related to near-Galilean invariance of (7) in the limit $b\gg1$). This instability can lead to spontaneously twisted vortices with helical-shaped filaments, or to SWT (called "spatiotemporal intermittency").

Notably, a finite-wavenumber three-dimensional instability of scroll waves, leading to helical filaments, was also described for Barkley model by Aranson and Mitkov (1998) and Henry and Hakim (2002); however they did not report that this instability could lead to SWT in that model.

"Winfree turbulence"

In literature, SWT is sometimes referred to as "Winfree turbulence", due to high-impact publications by A.T. Winfree (1994a and 1994b). In the Science paper, Winfree discussed plausible mechanisms of cardiac fibrillation, including its possible links with three-dimensional instabilities of scroll waves. In that context, Winfee mentioned the topological opportunities allowed by the 3D space for filaments to "lie on their sides" and to "snake about, fragment and close in rings". As to possible causes of "turbulent" behaviour, Winfree mentioned inhomogeneity and anisotropy, but also referred to the cases of "turbulent" behaviour in a perfectly homogeneous and isotropic medium, presented in his earlier Nature paper. These were in the form of numerical solutions in the FitzHugh-Nagumo model with knotted or entangled filaments, which were topologically persistent but did not remain periodic in time hence called "turbulent". The parameters of those "turbulent" examples corresponded to meandering rather than rigidly rotating spirals, which was the apparent cause of the non-periodicity.

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