STRESS ANALYSIS AND BURST PRESSURE DETERMINATION OF TWO LAYER COMPOUND PRESSURE VESSEL

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Abstract:

Multilayer pressure vessel is designed to work under high-pressure condition. This paper introduces the stress analysis and the burst pressure calculation of a two-layer shrink fitted pressure vessel. In the shrink-fitting problems, considering long hollow cylinders, the plane strain hypothesis can be regarded as more natural. Generally hoops stress distribution is non-linear and sharply reduced toward the outer surface. By shrink fitting concentric shells towards the inner shells are placed in residual compression so that the initial compressive hoop stress must be relieved by internal pressure before hoop tensile stress are developed. Therefore the maximum hoop stress will be reduced, resulting more burst pressure. The analytical results of stress distribution and burst pressure is calculated and validated by ANSYS Workbench results.

Keywords*:* Burst pressure, shrink fitting, multilayer cylinder, residual stress, contact pressure.

1. Introduction:

Multilayer compound cylinders are widely used in the field of high pressure technology such as hydraulic presses, forging presses, power plants, gas storages, chemical and nuclear plants, military applications etc. To enhance load bearing capacity and life of multilayer pressure vessels, different processes such as shrink fit and autofrettage are usually employed. Shrink fit increases load capacity. Many researchers have focused on methods to extend lifetimes of vessels. Majzoobi et al. have proposed the optimization of bi-metal compound cylinders and minimized the weight of compound cylinder for a specific pressure [6]. The variables were shrinkage radius and shrinkage tolerance. Patil S. A. has introduced optimum design of two layer compound cylinder and optimized intermediate, outer diameter and shrinkage tolerance to get minimum volume of two layer compound cylinders [7-8]. Niranjan et at.have proposed the optimization of shell thickness in multilayer pressure vessels and also investigated the effect of no. of shell on maximum hoops stress[4]. At the same time Ayub A. Miraje and Sunil A. Patil have proposed the optimization of three layer shrink fitted cylinder for uniform stress distribution[5]..

To increase the pressure capacity of thick walled cylinders, two or more cylinders (multi-layer) are shrunk into each other with different diametric differences to form compound cylinder. When the outer cylinder contracts on cooling the inner cylinder is brought into a state of compression. The outer cylinder will conversely be brought into a state of tension. If this compound cylinder is now subjected to internal pressure the resultant hoop stresses will be the algebraic sum of those resulting from internal pressure and those resulting from shrinkage, thus a much smaller total fluctuation of hoop stress is obtained.

In this paper, the analytical results of stress distribution and burst pressure is calculated and validated by ANSYS WORKBENCH results.

1.1. Introduction to elastic-breakdown pressure:

The pressure necessary for the yield point for the metal fibers in the bore to be reached is known as the elastic-breakdown pressure. At this pressure the maximum fiber stress is the tangential stress at the inner surface. The radial stress has also its maximum value at the bore and this stress is equal to the internal pressure.

1.2. Introduction to Burst Pressure:

Burst pressure is defined as the internal pressure, which required stressing the outer surface to the yield point. As the pressure is raised from elastic-breakdown pressure the region of plastic flow, termed overstrain moves radially outward and causes the tangential stress to decrease in the inner layer and to increase rapidly in the outer layers. Progressive increase in pressure moves the elastic-plastic interface radially outward until the interface reaches the outer radius and no elastic zone remains. In this situation the maximum hoop stress in the outer surface.

According to the Faupel the burst pressure of a hollow monoblock vessel is given by……….

$$p = \frac{2 f_{y.p.}}{\sqrt{3}}\left[\ln\frac{r_o}{r_i}\right]\left[2-\frac{f_{y.p.}}{f_{t.s.}}\right] \text{…………………(1)}$$

Where $f_{y.p.}$ = yield point of the shell material in single tension

$f_{t.s.}$ = Ultimate tensile strength

r_o = Outside radius of the vessel

r_i = Inside radius of the vessel.

2. Mathematical Model:

Let us consider a compound cylinder, consisting of a cylinder and a jacket as shown in fig. The inner diameter of the jacket is slightly smaller than the outer diameter of the cylinder. When the jacket is heated, it expands sufficiently to move over the cylinder. As the jacket cools, it tends to contract onto the inner cylinder, which induces residual compressive stresses. There is a shrinkage pressure P between the cylinder and jacket. The pressure P tends to contract the cylinder and expand the jacket.

Total shrinkage interference (δ) is given by,

$$\delta = \frac{PD_2}{E}\left[\frac{2D_2^{\,2}\left(D_3^{\,2}-D_1^{\,2}\right)}{\left(D_3^{\,2}-D_2^{\,2}\right)\left(D_2^{\,2}-D_1^{\,2}\right)}\right] \text{……………..(2)}$$

3. Analytical method:

Let consider, a tube, with 50mm and 70 mm as inner diameters respectively, is reinforced by shrinking a jacket with an outer diameter of 100 mm. Here, for both layer material is structural steel, E = 210 kN/mm^2.

4. Stress analysis:

Let us assume that an internal pressure of 35 Mpa is applied at the inner face of the compound cylinder.

Here, D_1= 50 mm D_2= 75 mm D_3= 100 mm E= 210 kN/mm^2 and P_i= 35 Mpa

The resultant stresses in the tubes are obtained by superimposing the stresses due to internal pressure and those due to shrinkage pressure.

4.1. Stress due to internal pressure (P_i):

Table 1. Stresses due to internal pressure:

r	25	37.5	50
σ_t	58.35	32.42	23.34

4.2. Stress due to shrinkage pressure (P):

Table 2. Stresses due to shrinkage pressure:

For jacket

r	37.5	50
σ_t	3.57 P	2.57 P

For inner tube

r	25	37.5
σ_t	-3.6 P	-2.6 P

4.3. Calculation of Shrinkage pressure (P):

Equating stresses at the inner surfaces of tube and jacket,

$$58.35 - 3.6\,P = 32.42 + 3.57\,P$$

or $$P = 3.62 \text{ Mpa}$$

Table 3. Distribution of the stresses:

	Inner tube		Jacket	
	r = 25	r =37.5	r = 37.5	r = 50
Stresses due to P_i	58.35	32.42	32.42	23.34
Stresses due to P (P= 3.62 Mpa)	-13.03	-9.41	12.92	9.30
Resultant stresses	45.32	23	45.34	32.64

4.4. Calculation of Shrinkage interference (δ):

Substituting the proper value in Eq (6), we got the shrinkage interference (δ) = 0.008 mm

The inner diameter of the jacket should be (75 – 0.008) or 74.992 mm.

4.5. Validation using FEM and ANSYS results:

Table 4. Data for modeling in ANSYS

D_1	D_2	D_{2i}	D3	δ
50	75	74.992	100	0.008

Where,

D_1 ,D_2 = inner and outer diameter of the tube

D_{2i}, D_3 = inner and outer diameter of the jacket

Shrink fit is applied between tube and jacket in ANSYS Workbench. Contact between tube and jacket is applied using contact tool in ANSYS Workbench.

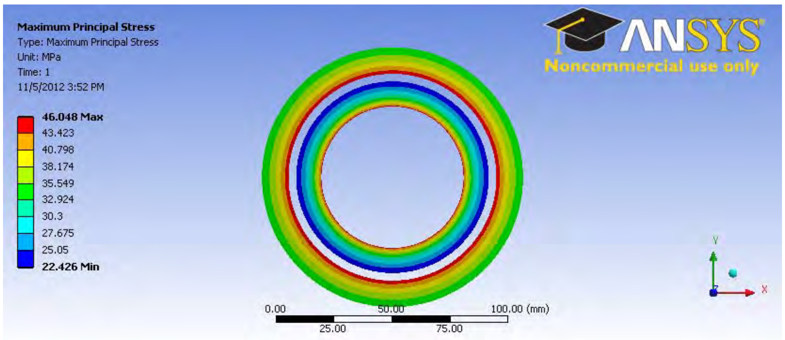

Fig. 1. Maximum principle stress at inner tube and jacket due to internal pressure 35Mpa

4.6 Discussion:

Table 5. Resultant stresses:

	Inner tube		Jacket	
	r =25	r =37.5	r =37.5	r =50
Analytical	45.32	23.00	45.34	32.64
ANSYS Workbench	44.729	22.581	46.048	33.038
%Error	1.304	1.82	-1.56	-1.22

5. Elastic-breakdown pressure Analysis:

Let assume that elastic-breakdown pressure of the compound cylinder= $P_{eb,}$ shrinkage pressure between inner tube and jacket = P.

Table 6. Stresses due to elastic-breakdown pressure (P_{eb}):

r	25	37.5	50
σ_t	$1.67P_{eb}$	$0.93\,P_{eb}$	$0.67\,P_{eb}$

Shrinkage pressure (P):

Equating the stresses at the inner surface of tube and jacket,

$$1.67\,P_{eb} - 3.6\,P = 0.93\,P_{eb} + 3.57\,P$$

or, $P = 0.1032\,P_{eb}$

so, stress at the inner surface of the tube

$1.67\,P_{eb} - (3.6*0.1032\,P_{eb}) = 250$ (yield strength of structural steel is 250 Mpa)

P_{eb}= 192 Mpa

So elastic-breakdown pressure of the compound cylinder is 192 Mpa

The elastic-breakdown pressure of single layer with same outer and inner dia. wii be 150 Mpa.

6. Burst pressure Analysis:

Let assume pressure required in the inner surface to burst the compound cylinder = P_b

Here shrinkage pressure P = 0.1032 P_b

Interference pressure due to P_b:

$$0.93P_b = \frac{P_{inter} * 75^2}{100^2 - 75^2}\left[\left(\frac{50}{37.5}\right)^2 + 1\right]$$

Or, $P_{inter} = 0.2604P_b$

So, total interference pressure

$\left(P_{inter}\right)_{total}$ = (Shrinkage pressure (P) + interference pressure due to P_b)

Or, $\left(P_{inter}\right)_{total}$ = 0.3636 P_b

Let assume, pressure required at the inner surface of the jacket to burst the jacket = P_{bj}

From equ. (2), we get,

P_{bj} = 121 Mpa.

Here, for structural steel $f_{y.p.}$ =250Mpa and $f_{t.s.}$ =460Mpa

At the time of bursting, shrinkage pressure (P) will be equal to P_{bj}.

Hence, $P_{bj} = \left(P_{inter}\right)_{total}$

Or 121= 0.3636 P_b

Or, P_b = 332.78 Mpa

Therefore, burst pressure of the compound cylinder is 332.78 Mpa

Table 7. Resultant stresses:

	Inner tube		Jacket	
	r = 25	r =37.5	r = 37.5	r = 50
Stresses due to P_b (P_b=332.78Mpa)	555.74	309.49	309.49	222.96
Stresses due to P (P= 34.34 Mpa)	-123.4	-89.28	122.59	88.25
Resultant stresses	432.34	220.21	432.08	311.21

6.1. ANSYS Workbench Results:

Table 8. Data for modeling in ANSYS

D_1	D_2	D_{2i}	D3	δ
50	75	74.924	100	0.076

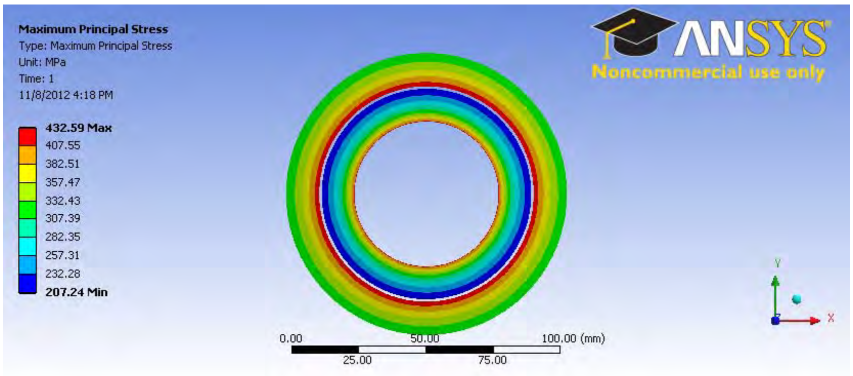

Fig.2. Maximum principle stress due to burst pressure

Using Design of Experiment in ANSYS Workbench, the pressure required to generate maximum principle stress 332.34Mpa is 327Mpa.

6.2. Discussion:

Table 9. Comparison of burst pressure

	Burst pressure(P_b)
Analytical	332.78
ANSYS Workbench	327
%Error	1.74

7. Conclusion:

From the discussion it is clear that the difference in analytical and ANSYS Software results is within acceptable limits. This difference is due to numerical techniques of Finite Element Method in ANSYS. Since analytical results are validated by FEM calculations, the design methodology proposed in this paper can be successfully applied into the real-world mechanical applications for the stress analysis and to determine the burst pressure of multi-layered compound cylinders to assure best utilization of material. Without shrink-fit the burst pressure of the cylinder with same outer and inner diameter would be 291Mpa (Faupel Formula) which is lesser than 332.78Mpa. So, we can conclude that shrink-fitted compound pressure vessels have higher-pressure capacity.

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