

# Robust Adaptive Vibration Control with Application to a Robot Beam

J. Fei

**Abstract**—This paper presents the adaptive control scheme with sliding mode compensator for vibration control problem in the presence of disturbance. The dynamic model of the flexible cantilever beam using finite element modeling is derived. The adaptive control with sliding mode compensator using output feedback for output tracking is developed to reject the external disturbance, and to improve the tracking performance. Satisfactory simulation results verify that the effectiveness of adaptive control scheme with sliding mode compensator.

**Keywords**—finite element model, adaptive control, sliding mode control, vibration suppression

## I. INTRODUCTION

PIEZOELECTRIC ceramic patches have received much attention in vibration control of structures in recent years, because piezoelectric ceramic materials have mechanical simplicity, small volume, light weight, large useful bandwidth, efficient conversion between electrical energy and mechanical energy, and easy integration with various metallic and composite structures. Smart structures can exhibit time-variant and non-linear characteristics. It is challenging to control such structures. It is necessary to use adaptation algorithm to compensate the control loop, and to be robust to the changes in the disturbance, and to suppress vibration of the smart structure. Tao and Kokotovic [1], Ioannou and Sun [2] [3] proposed the typical model reference adaptive control structure and developed the adaptive laws. Ma [4] developed novel adaptive filtering algorithm and hybrid control scheme for vibration control of smart structures with bonded PZT Patches. Baumann [5] studied the potential of an adaptive feedback approach to structural vibration suppression. Clark [6] developed an adaptive truss as part of a steel beam flexible structure assembly to control vibrations without the use of a system model. Recently, adaptive control has been applied to a number of flexible systems. Canbolat [7] regulated the vibration of a flexible cable adaptively. New adaptive vibration isolation control strategies are developed based on Lyapunov theory by Ertur [8], and regulation and tracking controllers cancel unknown disturbances while compensating for parametric uncertainty.

Sliding mode control is robust control technique which has many attractive features such as good transient, fast response, easy realization, and insensitivity to the variation of plant parameters and external disturbance. The variable structure sliding mode control is designed and analyzed in [14] by Liu. Song [11], [12] proposed a smooth robust compensator and smooth robust tracking controller for SMA wire actuator. Lee [15], [16] developed a variable structure augmented adaptive controller for a platform and an adaptive variable structure output tracking controller. Wang [13] proposed an adaptive sliding mode controller for a microgravity isolation system. The adaptive sliding mode control has the advantages of combining the robustness of variable structure methods with tracking capability of adaptive control strategies.

This paper investigates the feasibility of adaptive control scheme with sliding mode compensator for vibration suppression of flexible beam in the presence of disturbance. The adaptive control with sliding mode compensator is developed to reject the disturbance, and to improve the tracking performance. A smooth sliding mode compensator is used to reject control chattering. This paper is organized as follows. In section 2, finite element modeling of dynamics response of flexible beam system with PZT patches is derived and analyzed. In section 3 and 4, the adaptive control scheme with sliding mode compensator using output feedback for output tracking is developed. Section 5 describes simulation results. Section 6 summarizes the paper.

## II. FINITE ELEMENT MODEL

The flexible beam shown in Fig. 1 is modeled using the finite element method [9]. The structure is divided into elements that are connected at a finite number of points, called nodes. The motion of the points in the element is defined in terms of nodal displacement and using interpolation functions. Therefore, we first find the stiffness and mass matrices of the elements. The elements are assembled to determine the stiffness and mass matrices of the structure. It was determined that no more than three lowest modes are significant in the response of the appendage and thus would be considered in the simulations. For the analysis, six elements were used to characterize the structure. The model was constructed using Matlab for flexible beams. The flexible arm was divided into six elements and motion was considered to be in-plane bending based on the cantilever action. The system consists of 6 elements and 7 nodes. PZT sensors and actuators are attached to the element 2 of the beam. The PZTs add to the

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beam's stiffness and hence increase the fundamental frequency.

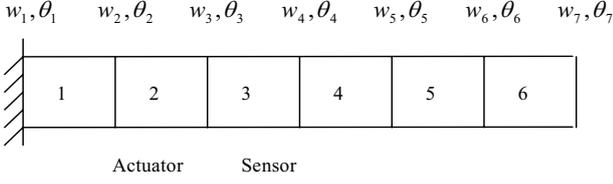


Fig. 1 Elemental model of the flexible beam

The following are equations and procedure in finite element modeling. The general relationship for the electro-mechanical coupling is given by

$$\begin{Bmatrix} D_3 \\ S_1 \end{Bmatrix} = \begin{bmatrix} \varepsilon_3^T & d_{31} \\ d_{31} & s_{11}^E \end{bmatrix} \begin{Bmatrix} E_3 \\ T_1 \end{Bmatrix} \quad (1)$$

where  $D$  is the displacement,  $S$  is the strain,  $E$  is the electric field,  $T$  is the stress,  $s$  is the compliance and  $d$  is the piezoelectric constant. The subscripts are tensor notation where the 1- and 2-axes are arbitrary in the plane perpendicular to the 3-axis poling direction of the piezoelectric material. Using the fact that the elastic constant for piezoceramic material,  $s$ , is the inverse of its Young's modulus,  $E_p$ , this equation can be written as

$$\begin{Bmatrix} D_3 \\ T_1 \end{Bmatrix} = \begin{bmatrix} \varepsilon_3^T - d_{31}^2 E_p & d_{31} E_p \\ -d_{31} E_p & E_p \end{bmatrix} \begin{Bmatrix} E_3 \\ S_1 \end{Bmatrix} \quad (2)$$

The equation for the elemental potential energy is given by

$$-U = \frac{1}{2} \int (-T_1 S_1 + D_3 E_3) dV \quad (3)$$

where the two terms in the integral represent mechanical energy and electrical energy respectively. Using  $W_p$  as width of the piezoceramic wafer,  $t_p$  is thickness of piezoceramic,  $\zeta$  is half of the thickness of beam,  $\varepsilon_3$  is the permittivity of piezoelectric material,  $E_p$  is the elastic modulus,  $d_{31}$  is the piezoelectric charge coefficient and  $E_3$  is the applied field intensity. The general form of the energy equation is

$$-U = \frac{1}{2} \gamma e^2 - q^T B e - \frac{1}{2} q^T K q \quad (4)$$

where

$$\gamma = \frac{W_p h}{t_p} (\varepsilon_3^T - d_{31}^2 E_p), e = t_p E_3 \quad (5)$$

$$b_1 = b_3 = 0 \quad (6)$$

$$b_2 = -b_4 = -d_{31} E_p W_p \left( \zeta + \frac{t_p}{2} \right)$$

where  $K = k_b + k_p$ ,  $k_b$  is stiffness matrix for the structure,  $k_p$  is stiffness matrix for the piezoceramic,  $q$  is the generalized coordinate and  $B$  is the electro-mechanical coupling term

which represents the conversion of electrical voltage to mechanical displacement.  $B^T = [b_1 \ b_2 \ b_3 \ b_4]$ .

The detailed physical parameter meaning and specifications of the flexible beam and piezoceramic properties on the flexible beam are given in Table 1 and 2 in section 5. The total kinetic energy is given by

$$T = \frac{1}{2} \dot{q}^T M \dot{q} \quad (7)$$

where  $M = M_b + M_p$ , and  $M_b$  is mass matrix for beam,  $M_p$  is mass matrix for PZT.

The Lagrangian function  $L$  is

$$L = T - U = \frac{1}{2} \dot{q}^T M_p \dot{q} + \frac{1}{2} \gamma e^2 - q^T B e - \frac{1}{2} q^T K q \quad (8)$$

The Lagrangian equation is

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = 0 \quad (9)$$

The equation for the actuator is

$$M \dot{q} + K q = -B e_a \quad (10)$$

where  $M = M_b + M_p$  and  $e_a$  is applied voltage.

The piezoceramic sensor voltage output is  $\gamma e_s = B^T q$ . We have considered only an element. The equation for the global form is determined by combining the equations.

### III. ADAPTIVE CONTROL DESIGN

For control of the flexible beam, the model reference adaptive control scheme is well suited for vibration suppression problem in the presence of disturbance to minimize the displacement at the tip of flexible beam.

Consider the  $n$ th order linear time-invariant plant described by

$$y_p = G_p(s)(u_p + d) = k_p \frac{Z_p(s)}{R_p(s)} (u_p + d) \quad (11)$$

where  $r$  is the reference input which is assumed to be a uniformly bounded and piecewise continuous function of time,  $d$  is a bounded disturbance. The disturbance  $d$  satisfies

$$|d(t)| \leq \bar{d}, \quad \forall t \geq 0 \quad (12)$$

where  $\bar{d}$  is positive constant.

The transfer function of the reference model is given by

$$y_m = W_m(s) r = k_m \frac{Z_m(s)}{R_m(s)} r \quad (13)$$

The model reference control objective is to determine the plant input  $u_p$  so that all signals are bounded and the plant output  $y_p$  tracks the reference model output  $y_m$  as close as possible for any given reference input  $r(t)$ . Without loss of generality, assume the relative degree  $n^*$  of the plant is  $n^* = 2$ . The plant with relative degree  $n^* > 2$  can be discussed in the similar way. The plant and reference model satisfy the assumptions as [1] and [2]. Following the procedure

as Tao and Kokotovic [1], Ioannou and Sun [2], [3], the standard adaptive control scheme

$$u_p = \theta_1^T \frac{\alpha(s)}{\Lambda(s)} u_p + \theta_2^T \frac{\alpha(s)}{\Lambda(s)} y_p + \theta_3 y_p + \theta_4 r \quad (14)$$

where  $\alpha(s) = [s^{n-2}, s^{n-3}, \dots, s, 1]^T$  for  $n \geq 2$ ,  $\theta_3$  and  $\theta_4 \in R^1$ ,  $\theta_1, \theta_2 \in R^{n-1}$ ,  $\Lambda(s)$  is an arbitrary monic Hurwitz polynomial of degree  $n-1$  that contains  $Z_m(s)$  as a factor.

In the adaptive control problem, the parameters of  $G_p(s)$  are unknown so that  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$  and  $\theta_4$  cannot be determined a priori and have to be updated from an adaptive law. The simplified control law is

$$\dot{w}_1 = Fw_1 + gu_p, \quad w_1(0) = 0 \quad (15)$$

$$\dot{w}_2 = Fw_2 + gy_p, \quad w_2(0) = 0 \quad (16)$$

$$u_p = \theta^T w \quad (17)$$

where  $\theta$  is the estimate of  $\theta^*$ ,  $w = [w_1^T, w_2^T, y_p, r]^T$ ,  $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$  and  $\theta^* = [\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*]^T$ ,  $(F, g)$  is the state space realization of  $\frac{\alpha(s)}{\Lambda(s)}$ .

The error equations can be derived as

$$\dot{e} = A_c e + B_c(u_p - \theta^{*T} w) \quad (18)$$

$$e_1 = y_p - y_m = C_c^T e = W_m(s) \rho^*(u_p - \theta^{*T} w) \quad (19)$$

where  $\rho^* = k_p / k_m$ ,  $e$  is the state error. The state space representation of the plant and controller can be described as

$$\begin{aligned} \dot{Y}_C &= A_C Y_C + B_C u_p \\ y_p &= C_C^T Y_C \\ u_p &= \theta^T \omega \end{aligned} \quad (20)$$

where

$$A_C = \begin{bmatrix} A_p + B_p \theta_3^* C_p^{*T} & B_p \theta_1^{*T} & B_p \theta_2^{*T} \\ g \theta_3^* C_p^{*T} & F + g \theta_1^{*T} & g \theta_2^{*T} \\ g C_p^{*T} & 0 & F \end{bmatrix}, \quad B_C = \begin{bmatrix} B_p \\ 0 \\ 0 \end{bmatrix} \quad (21)$$

and  $Y_C = [x_p^T \quad \omega_1^T \quad \omega_2^T]^T$ ,  $C_C^T = [C_p^T \quad 0 \quad 0]$ .

The error equations can be transformed as

$$\dot{e} = A_c e + \bar{B}_c (s + p_0) \rho^* \tilde{\theta}^T \varphi \quad (22)$$

$$e_1 = C_c^T e = W_m(s) (s + p_0) \rho^* \tilde{\theta}^T \varphi \quad (23)$$

where  $\bar{B}_c = B_c \theta_4^*$  and  $u_f = \frac{1}{s + p_0} u_p$ ,  $\varphi = \frac{1}{s + p_0} w$ ,  $\tilde{\theta} = \theta - \theta^*$ , by using the transform  $\bar{e} = e - \bar{B}_c \rho^* \tilde{\theta}^T \varphi$ , then the equation (38) can be transformed into the desired form

$$\dot{\bar{e}} = A_c \bar{e} + B_1 \rho^* \tilde{\theta}^T \varphi, \quad e_1 = C_c^T \bar{e} \quad (24)$$

where  $B_1 = A_c \bar{B}_c + \bar{B}_c p_0$  and  $C_c^T \bar{B}_c = C_p^T B_p \theta_4^* = 0$  due to  $n^* = 2$ . Consider the Lyapunov-like function

$$V(\tilde{\theta}, \bar{e}) = \frac{\bar{e}^T P_c \bar{e}}{2} + \frac{\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}}{2} |\rho^*| \quad (25)$$

where  $\Gamma = \Gamma^T > 0$  and  $P_c = P_c^T > 0$  satisfy the algebraic equation

$$P_c A_c + A_c^T P_c = -Q \quad P_c \bar{B}_c = C_c \quad (26)$$

The time derivative  $\dot{V}$  of  $V$  is given by

$$\dot{V} = -\frac{\bar{e}^T Q \bar{e}}{2} - \bar{e}^T P_c \bar{B}_c \rho^* \tilde{\theta}^T \varphi + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} |\rho^*| \quad (27)$$

Because  $\bar{e}^T P_c \bar{B}_c = e_1$  and  $\rho^* = |\rho^*| \text{sgn}(\rho^*)$ , by choosing

$$\dot{\tilde{\theta}} = \dot{\theta} = -\Gamma e_1 \varphi \text{sgn}(\rho^*) \quad (28)$$

In fact this is the part of desired adaptive law, which leads to

$$\dot{V} = -\frac{\bar{e}^T Q \bar{e}}{2} \leq 0 \quad (29)$$

This proves that  $V$ ,  $\bar{e}$  and  $\tilde{\theta} \in L^\infty$ ,  $\bar{e}$  and  $e_1 \in L^2$ . Since  $e_1 = y_p - y_m$ , we also have  $y_p \in L^\infty$ . since  $w$ ,  $\varphi$  and  $e_1 \in L^\infty$ , we have  $\dot{\theta} \in L^\infty$  and  $u_p \in L^\infty$ , and therefore all signal in the closed-loop plant are bounded. From equation (24) we also have  $\bar{e} \in L^\infty$ ,  $\dot{e}_1 \in L^\infty$ , together with  $e_1 \in L^\infty \cap L^2$ , implies that  $e_1(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

The equation  $u_p = (s + p_0) u_f = (s + p_0) \theta^T \varphi$  implies that  $u_p = \theta^T w + \dot{\theta}^T \varphi$ . Because  $\dot{\theta}$  is made available by the adaptive law, the control laws can be implemented without the use of differentiators, so the control law and adaptive law are

$$u_p = \theta^T w + \dot{\theta}^T \varphi = \theta^T w - \varphi^T \Gamma \varphi e_1 \text{sgn}(\rho^*) \quad (30)$$

$$\dot{\theta} = -\Gamma e_1 \varphi \text{sgn}(k_p / k_m) \quad (31)$$

The MRAC scheme guarantees that all signals in the closed-loop plant are bounded and tracking error  $e_1$  converges to zero asymptotically under enough rich excitation signals.

#### IV. ADAPTIVE SLIDING MODE CONTROL

The sliding mode control is a robust control technique which has many attractive features such as robustness to parameter variation and insensitivity to disturbance, fast response. But it also has some limitation such as chattering or high frequency oscillation in application. It is necessary to integrate adaptive control and sliding mode control. The adaptive sliding mode control has the advantages of combining the robustness of variable structure methods with

the tracking capability of adaptive control strategies. The adaptive control with sliding mode compensator is developed to reject the disturbance, and to improve the tracking performance. Furthermore, a smooth sliding mode controller is used to compensate for the nonlinearity and external disturbance of system and to increase the control accuracy and stability. The sliding line  $s(t) = 0$  is defined in the state space of errors, define sliding surface

$$s(t) = \dot{e}_1 + \lambda e_1 \quad (32)$$

and  $\lambda$  is a positive constant,  $e_1$  is tracking error. A reaching law is a differential equation describing the evolution of the distance  $s(t)$  under sliding mode control. The control law is designed to satisfy a prescribed reaching law. Gao [10] proposed simple reaching law such as

$$\dot{s} = -\eta \operatorname{sgn}(s) \quad (33)$$

where  $\eta > 0$ , corresponding to a reaching law with constant reaching speed  $\eta$ . In order to eliminate the control discontinuities, a smooth sliding mode control  $-\eta \operatorname{Tanh}(as)$  that can reduce chattering problem is introduced. The term with the sign of  $s(t)$  in the control law results in the control discontinuities which causes chattering problem and may excite the high frequency unmodeled dynamics, the sliding mode control  $-\eta \operatorname{Tanh}(as)$  is to guarantee the system reach the sliding line and remain on it. The surface defined by  $s = 0$  represents the "sliding surface," so that when the dynamics are restricted to this surface,  $e_1 = 0$  and  $\dot{e}_1 = 0$  is an asymptotically stable equilibrium point. Therefore, when the system is restricted to the sliding surface, the control errors vanish as  $t \rightarrow \infty$ .

The sliding mode controller  $u_s$  with the adaptive controller  $u_p$  is proposed as

$$u = u_p + u_s \quad (34)$$

Since  $u_p = \theta^T w - \varphi^T \Gamma \varphi e_1 \operatorname{sgn}(\rho^*)$  and  $u_s = -\eta \operatorname{Tanh}(as)$ , therefore the adaptive controller with sliding mode controller is derived as

$$u = \theta^T w - \varphi^T \Gamma \varphi e_1 \operatorname{sgn}(\rho^*) - \eta \operatorname{Tanh}(as) \quad (35)$$

where  $\eta > \bar{d}$ . The term  $-\eta \operatorname{Tanh}(as)$  is a robust compensator and is used to compensate for the nonlinearity of the system and to increase the control accuracy and stability. In this control approach,  $s = 0$  functions as the sliding surface on which the system is asymptotic stable, i.e., the control error is zero. The robust compensator  $-\eta \operatorname{Tanh}(as)$  is continuously differentiable with respect to the control variable  $s$ , and it generates a smooth control action. Compared with the commonly used bang-bang or saturation robust controllers, the smooth robust controller has advantages in ensuring both smooth control input and the ultimately globally uniform stability of the closed-loop system.

## V. SIMULATION RESULTS

In this section, in order to demonstrate the feasibility of the proposed MRAC with sliding mode controller numerical

simulation for the adaptive control with sliding mode compensator is performed using MATLAB and SIMULINK. For a flexible cantilevered with a pair of PZT-patch actuator, we consider the voltage of the signal sent to the power amplifier for the PZT as the input and the sensor output as the output. According to section II, finite element model is calculated. Specifications of the flexible beam and piezoceramic properties on the flexible beam are given in Table I and Table II. The individual stiffness and mass matrices for each element is individually computed and finally the global stiffness and mass matrices were constructed. The fundamental frequency of the flexible beam was found to be about 1.6 Hz by experiment. Through finite element analysis for the flexible beam, the first three modes table calculated from Matlab program is presented in Table III.

From the FEM modal analysis, there are variations of natural frequencies of beams with the bonded actuators and sensor. Numerical results show that the bonded actuators and sensors lead to increase in natural frequencies. The dynamic effects of mass and stiffness of the piezoelectric patch are considered in the model procedure.

In the MRAC controller used in the vibration suppression of flexible beam, we make use of the control law and adaptive law, choose filter item  $p_0 = 1$  and  $\Lambda(s) = s + 2$ , then get the controller and adaptive law as follows:

$$\begin{aligned} \dot{w}_1 &= -2w_1 + u_p, & \dot{w}_2 &= -2w_2 + y_p \\ \dot{\varphi} &= -\varphi + w, & u_p &= \theta^T w - \varphi^T \Gamma \varphi e_1 \end{aligned} \quad (36)$$

The adaptive law is  $\dot{\theta} = -\Gamma e_1 \varphi$ ,  $\Gamma = \Gamma^T > 0$  is adaptive gain,

$\theta = [\theta_1, \theta_2, \theta_3, \theta_4]$  and  $w = [w_1, w_2, y_p, r]^T$ . Starting

with  $\theta(0) = [-4, 2, -5, -2]^T$ , adaptive law gain  $\Gamma = \operatorname{diag}\{18, 10, 10, 10\}$ . The reference model is chosen as  $y_m = \frac{1}{s^2 + s + 101}$ . The parameters of the sliding mode

term  $-\eta \operatorname{Tanh}(as)$  are  $\lambda = 4$ ,  $\eta = 0.5$ ,  $a = 0.5$ . The disturbance noise is band-limited white noise with noise power 0.05,  $\eta = 2$  in the presence of transient natural frequency excitation,  $\eta = 4$  in the presence of persistent natural frequency excitation, and chip signal excitation.

Fig 2 shows the control results when the natural frequency excitation is applied during the first 5s and the control action begin at 5s, and it compares the non-adaptive and adaptive control with and without sliding mode compensator under transient frequency excitation. Fig 3 compares non-adaptive and adaptive control with and without sliding mode compensator under transient frequency excitation in the presence of disturbance noise. Fig. 4 shows the control results with persistent natural frequency excitation when the natural frequency excitation is applied during the 30s and the control action begins at 5s, it compared the non-adaptive and adaptive control with and without sliding mode compensator under persistent frequency excitation. Fig 5

compares non-adaptive and adaptive control with and without sliding mode compensator under persistent frequency excitation in the presence of disturbance noise. It is observed that adaptive control with sliding mode compensator has better results than adaptive control without sliding mode compensator no matter whether excitations are applied transiently and continuously.

The important criterion to evaluate the robustness of a controller is how the controller deals with unexpected changes in the system. A simulation is conducted that uses a sinusoidal disturbance whose frequency changes in time, for example, a linear chip signal. Fig 6 shows the control results when the chip signal disturbance is applied continuously and the control action begin at 0s, and it compares the non-adaptive and adaptive control with and without sliding mode compensator. The linear chip signal is the sine wave whose frequency varies linearly with time, in this simulation, the chip signal is linearly changing from 0.1 Hz to 4 Hz during 40s time period. The simulation results show that the adaptive control with sliding mode compensator can deal with the unexpected changes in the system better than adaptive control without sliding mode compensator.

TABLE I SPECIFICATION OF THE FLEXIBLE BEAM

| PROPERTY                            | SYMBOL | VALUE               |
|-------------------------------------|--------|---------------------|
| Young's Modulus (N/m <sup>2</sup> ) | $E$    | 7.03E10             |
| Beam width (m)                      | $W$    | 0.0531              |
| Beam density (kg/m <sup>3</sup> )   | $\rho$ | 2690                |
| Beam thickness (m)                  | $T$    | 1x10 <sup>-03</sup> |
| Beam length (m)                     | $L$    | 0.826               |

TABLE II PIEZOCERAMIC PROPERTIES OF THE FLEXIBLE BEAM

| PROPERTY                               | SYMBOL       | VALUE                    |
|--|--------------|--------------------------|
| Piezo Modulus (N/m <sup>2</sup> )      | $E_p$        | 6.3E10                   |
| Piezo density (kg/m)                   | $\rho_p$     | 7.5x10 <sup>3</sup>      |
| Piezo lateral strain coefficient (m/V) | $d_{31}$     | 1.8x10 <sup>-10</sup>    |
| Piezo permittivity (N/V <sup>2</sup> ) | $\epsilon_3$ | 1.5x10 <sup>-08</sup>    |
| Piezo thickness (m)                    | $t_p$        | 2.53x10 <sup>-03</sup>   |
| Piezo actuator width (m)               | $W_{pa}$     | 33.274x10 <sup>-03</sup> |
| Piezo sensor width (m)                 | $W_{ps}$     | 7x10 <sup>-03</sup>      |
| Piezo actuator length (m)              | $L_{pa}$     | 46x10 <sup>-03</sup>     |
| Piezo sensor length (m)                | $L_{ps}$     | 14x10 <sup>-03</sup>     |

TABLE II.COMPARISON OF FIRST THREE MODES OF THE FLEXIBLE BEAM

| Mode | Without actuators and sensors |        | With actuators and sensors |        |
|------|-------------------------------|--------|----------------------------|--------|
|      | Rad/sec                       | Hz     | Rad/sec                    | Hz     |
| 1    | 9.5663                        | 1.5225 | 10.266                     | 1.6339 |
| 2    | 60.085                        | 9.5286 | 63.415                     | 10.093 |
| 3    | 170.17                        | 27.083 | 176.96                     | 28.164 |

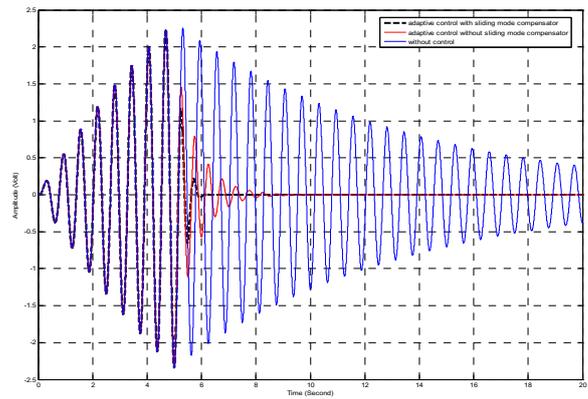


Fig. 2 Comparison of non-adaptive and adaptive control with and without sliding mode compensator under transient frequency excitation

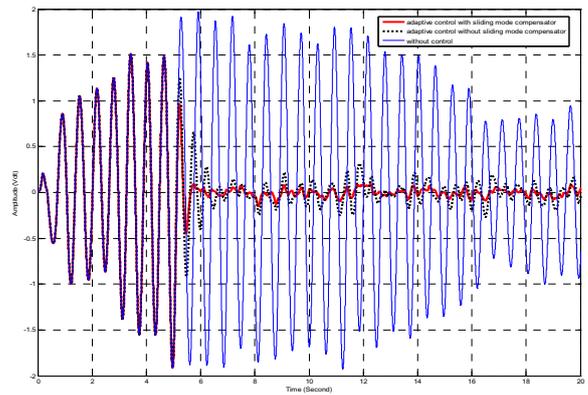


Fig. 3 Comparison of non-adaptive and adaptive control with and without sliding mode compensator under transient frequency excitation in the presence of disturbance noise

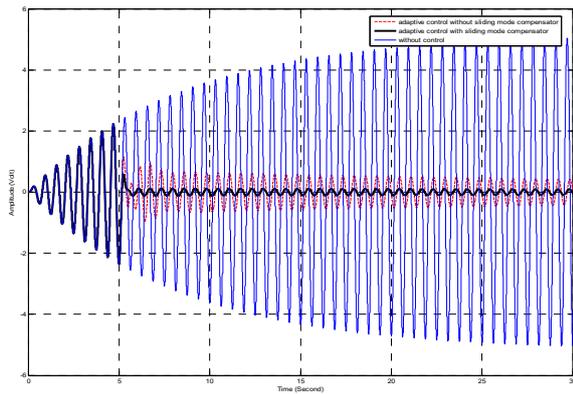


Fig. 4 Comparison of non-adaptive and adaptive control with and without sliding mode compensator under persistent natural frequency excitation

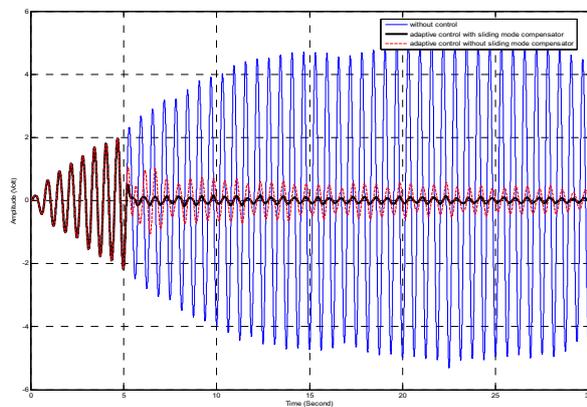


Fig. 5 Comparison of non-adaptive and adaptive control with and without sliding mode compensator under persistent natural frequency excitation in the presence of noise

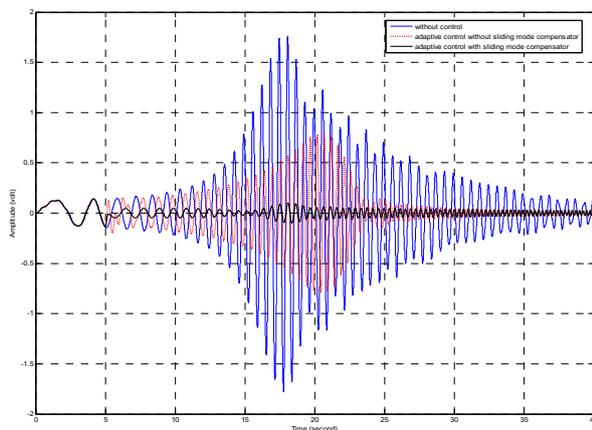


Fig. 6 Comparison of non-adaptive and adaptive control with and without sliding mode compensator under the linear chip signal

## VI. CONCLUSION

This paper investigates the feasibility of adaptive output feedback control with sliding mode control for the vibration suppression of a flexible cantilever beam model. Model reference adaptive controller whose parameters are updated directly from the Lyapunov-based adaptive laws is described in detail and the adaptive law is developed. A smooth sliding mode controller is used to compensate for the nonlinearity of system and to increase the control accuracy and stability. Numerical simulations show that the adaptive control with sliding mode control has satisfactory performance and robustness in vibration suppression of flexible cantilever beam compared with adaptive control without sliding mode control. The simulation results clearly demonstrate the adaptive control with sliding mode compensator successfully suppresses the vibration and therefore is superior to standard adaptive control without sliding mode compensator.

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