Quantum Mechanics_quantum chromodynamics (QCD)

In theoretical physics, **quantum chromodynamics** (**QCD**) is a theory ofstrong interactions, a fundamental forcedescribing the interactions between quarksand gluons which make up hadrons such as the proton, neutron and pion. QCD is a type of Quantum field theory called a non-abelian gauge theory with symmetry group SU(3). The QCD analog of electric charge is a property called 'color'. Gluons are the force carrier of the theory, like photons are for the electromagnetic force in quantum electrodynamics. The theory is an important part of the Standard Model of Particle physics. A huge body of experimental evidence for QCD has been gathered over the years.

QCD enjoys two peculiar properties:

- **Confinement**, which means that the force between quarks does not diminish as they are separated. Because of this, when you do split the quark the energy is enough to create another quark thus creating another quark pair; they are forever bound into hadrons such as theproton and the neutron or the pion and kaon. Although analytically unproven, confinement is widely believed to be true because it explains the consistent failure of free quark searches, and it is easy to demonstrate in lattice QCD.
- **Asymptotic freedom**, which means that in very high-energy reactions, quarks and gluons interact very weakly creating a quark-gluon plasma. This prediction of QCD was first discovered in the early 1970s by David Politzer and by Frank Wilczek and David Gross. For this work they were awarded the 2004 Nobel Prize in Physics.

There is no known phase-transition line separating these two properties; confinement is dominant in low-energy scales but, as energy increases, asymptotic freedom becomes dominant.

Terminology

The word *quark* was coined by American physicist Murray Gell-Mann (b. 1929) in its present sense. It originally comes from the phrase "Three quarks for Muster Mark" in *Finnegans Wake* by James Joyce. On June 27, 1978, Gell-Mann wrote a private letter to the editor of the *Oxford English Dictionary*, in which he related that he had been influenced by Joyce's words: "The allusion to three quarks seemed perfect." (Originally, only three quarks had been discovered.) Gell-Mann, however, wanted to pronounce the word with (ô) not (ä), as Joyce seemed to indicate by rhyming words in the vicinity such as *Mark*. Gell-Mann got around that "by supposing that one ingredient of the line 'Three quarks for Muster Mark' was a cry of 'Three quarts for Mister . . . ' heard in H.C. Earwicker's pub", a plausible suggestion given the complex punning in Joyce's novel.[1] The three kinds of charge in QCD (as opposed to one in quantum electrodynamicsor QED) are usually referred to as "color charge" by loose analogy to the three kinds of color (red, green and blue) perceived by humans. Other than this nomenclature, the quantum parameter "color" is completely unrelated to the everyday, familiar phenomenon of color.

Since the theory of electric charge is dubbed "electrodynamics", the Greek word "chroma" Χρώμα (meaning color) is applied to the theory of color charge, "chromodynamics".

History

With the invention of bubble chambers and spark chambers in the 1950s, experimental Particle physics discovered a large and ever-growing number of particles called hadrons. It seemed that such a large number of particles could not all be fundamental. First, the particles were classified by charge and isospin byEugene Wigner and Werner Heisenberg; then, in 1953, according to strangenessby Murray Gell-Mann and Kazuhiko Nishijima. To gain greater insight, the hadrons were sorted into groups having similar properties and masses using the*eightfold way*, invented in 1961 by Gell-Mann and Yuval Ne'eman. Gell-Mann andGeorge Zweig, correcting an earlier approach of Shoichi Sakata, went on to propose in 1963 that the structure of the groups could be explained by the existence of three flavors of smaller particles inside the hadrons: the quarks.

Perhaps the first remark that quarks should possess an additional quantum number was made[2] as a short footnote in the preprint of Boris Struminsky[3] in connection with Ω^- hyperon composed of three strange quarks with parallel spins (this situation was peculiar, because since quarks are fermions, such combination is forbidden by the Pauli exclusion principle):

Three identical quarks cannot form an antisymmetric S-state. In order to realize an antisymmetric orbital S-state, it is necessary for the quark to have an additional quantum number.

— B. V. Struminsky, *Magnetic moments of barions in the quark model*, JINR-Preprint P-1939, Dubna, Submitted on January 7, 1965

Boris Struminsky was a PhD student of Nikolay Bogolyubov. The problem considered in this preprint was suggested by Nikolay Bogolyubov, who advised Boris Struminsky in this research.[3] In the beginning of 1965, Nikolay Bogolyubov, Boris Struminsky and Albert Tavkhelidze wrote a preprint with a more detailed discussion of the additional quark quantum degree of freedom.[4]This work was also presented by Albert Tavchelidze without obtaining consent of his collaborators for doing so at an international conference in Trieste (Italy), in May 1965.[5][6]

A similar mysterious situation was with the Δ^{++} baryon; in the quark model, it is composed of three up quarks with parallel spins. In 1965, Moo-Young Han withYoichiro Nambu and Oscar W. Greenberg independently resolved the problem by proposing that quarks possess an additional **SU**(3) gauge degree of freedom, later called color charge. Han and Nambu noted that quarks might interact via an octet of vector gauge bosons: the gluons.

Since free quark searches consistently failed to turn up any evidence for the new particles, and because an elementary particle back then was *defined* as a particle which could be separated and isolated, Gell-Mann often said that quarks were merely convenient mathematical constructs, not real particles. The meaning of this statement was usually clear in context: He meant quarks are confined, but he also was implying that the strong interactions could probably not be fully described by quantum field theory.

Richard Feynman argued that high energy experiments showed quarks are real particles: he called them *partons* (since they were parts of hadrons). By particles, Feynman meant objects which travel along paths, elementary particles in a field theory. The difference between Feynman's and Gell-Mann's approaches reflected a deep split in the theoretical physics community. Feynman thought the quarks have a distribution of position or momentum, like any other particle, and he (correctly) believed that the diffusion of parton momentum explained diffractive scattering. Although Gell-Mann believed that certain quark charges could be localized, he was open to the possibility that the quarks themselves could not be localized because space and time break down. This was the more radical approach of S-matrix theory.

James Bjorken proposed that pointlike partons would imply certain relations should hold in deep inelastic scattering of electrons and protons, which were spectacularly verified in experiments at SLAC in 1969. This led physicists to abandon the S-matrix approach for the strong interactions.

The discovery of Asymptotic freedom in the strong interactions by David Gross,David Politzer and Frank Wilczek allowed physicists to make precise predictions of the results of many high energy experiments using the quantum field theory technique of perturbation theory. Evidence of gluons was discovered in three-jet events at PETRA in 1979. These experiments became more and more precise, culminating in the verification of perturbative QCD at the level of a few percent at the LEP in CERN.

The other side of asymptotic freedom is Confinement. Since the force between color charges does not decrease with distance, it is believed that quarks and gluons can never be liberated from hadrons. This aspect of the theory is verified within lattice QCD computations, but is not mathematically proven. One of theMillennium Prize Problems announced by the Clay Mathematics Instit
requires a claimant to produce such a proof. Other aspects of non-perturbativeQCD are the exploration of phases of quark matter, including the quark-gluon plasma.

The relation between the short-distance particle limit and the confining long-distance limit is one of the topics recently explored using string theory, the modern form of S-matrix theory.[7][8]

Theory

QCD in the non-perturbativeregime:

- **Confinement**: the equations of QCD remain unsolved atenergy scalesrelevant for describingatomic nuclei. How does QCD give rise to the physics of nuclei and nuclear constituents?
- **quark matter**: the equations of QCD predict that a plasma (or soup) of quarks and gluons should be formed at high temperature and density. What are the properties of this phase of matter?

List of unsolved problems in physics

Some definitions

Every field theory of Particle physics is based on certain symmetries of nature whose existence is deduced from observations. These can be

- local symmetries, that is the symmetry acts independently at each point in space-time. Each such symmetry is the basis of a Gauge theory and requires the introduction of its own gauge bosons.
- global symmetries, which are symmetries whose operations must be simultaneously applied to all points of space-time.

QCD is a gauge theory of the SU(3) gauge group obtained by taking the color charge to define a local symmetry.

Since the strong interaction does not discriminate between different flavors of quark, QCD has approximate **flavor symmetry**, which is broken by the differing masses of the quarks.

There are additional global symmetries whose definitions require the notion ofchirality, discrimination between left and right-handed. If the spin of a particle has a positive projection on its direction of motion then it is called left-handed; otherwise, it is right-handed. Chirality and handedness are not the same, but become approximately equivalent at high energies.

- **Chiral** symmetries involve independent transformations of these two types of particle.
- **Vector** symmetries (also called diagonal symmetries) mean the same transformation is applied on the two chiralities.
- **Axial** symmetries are those in which one transformation is applied on left-handed particles and the inverse on the right-handed particles.

Additional remarks: duality

As mentioned, *asymptotic freedom* means that at large energy – this corresponds also to *short distances* – there is practically no interaction between the particles. This is in contrast – more precisely one would say *dual* – to what one is used to, since usually one connects the absence of interactions with *large* distances. However, as already mentioned in the original paper of Franz Wegner,[9] a solid state theorist who introduced 1971 simple gauge invariant lattice models, the high-temperature behaviour of the *original model*, e.g. the strong decay of correlations at large distances, corresponds to the low-temperature behaviour of the (usually ordered!) *dual model*, namely the asymptotic decay of non-trivial correlations, e.g. short-range deviations from almost perfect arrangements, for short distances. Here, in contrast to Wegner, we have only the dual model, which is that one described in this article.[10]

Symmetry groups

The color group **SU**(3) corresponds to the local symmetry whose gauging gives rise to QCD. The electric charge labels a representation of the local symmetry group **U**(1) which is gauged to give QED: this is an abelian group. If one considers a version of QCD with N_f flavors of massless quarks, then there is a global (chiral) flavor symmetry group $\mathbf{SU}_L(N_f) \times \mathbf{SU}_R(N_f) \times \mathbf{U}_B(1) \times \mathbf{U}_A(1)$. The chiral symmetry is spontaneously broken by the QCD vacuum to the vector (L+R) $\mathbf{SU}_V(N_f)$ with the formation of a chiral condensate. The vector symmetry, $\mathbf{U}_B(1)$ corresponds to the baryon number of quarks and is an exact symmetry. The axial symmetry $\mathbf{U}_A(1)$ is exact in the classical theory, but broken in the quantum theory, an occurrence called an anomaly. Gluon field configurations called instantons are closely related to this anomaly.

There are two different types of **SU**(3) symmetry: there is the symmetry that acts on the different colors of quarks, and this is an exact gauge symmetry mediated by the gluons, and there is also a flavor symmetry which rotates different flavors of quarks to each other, or *flavor SU(3)*. Flavor **SU**(3) is an approximate symmetry of the vacuum of QCD, and is not a fundamental symmetry at all. It is an accidental consequence of the small mass of the three lightest quarks.

In the QCD vacuum there are vacuum condensates of all the quarks whose mass is less than the QCD scale. This includes the up and down quarks, and to a lesser extent the strange quark, but not any of the others. The vacuum is symmetric under **SU**(2) isospin rotations of up and down, and to a lesser extent under rotations of up, down and strange, or full flavor group **SU**(3), and the observed particles make isospin and **SU**(3) multiplets.

The approximate flavor symmetries do have associated gauge bosons, observed particles like the rho and the omega, but these particles are nothing like the gluons and they are not massless. They are emergent gauge bosons in an approximate string description of QCD.

Lagrangian

The dynamics of the quarks and gluons are controlled by the quantum chromodynamics Lagrangian. The gauge invariant QCD Lagrangian is

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\, \delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

where $\psi_i(x)$ is the quark field, a dynamical function of space-time, in thefundamental representation of the SU(3) gauge group, indexed by $i, j, \ldots$; $\mathcal{A}^a_\mu(x)$ are the gluon fields, also dynamical functions of space-time, in the adjoint representation of the **SU**(3) gauge group, indexed by *a*, *b*,... The γ^μ are Dirac matrices connecting the spinor representation to the vector representation of theLorentz group.

The symbol $G^a_{\mu\nu}$ represents the gauge invariant gluon field strength tensor, analogous to the electromagnetic field strength tensor, $F^{\mu\nu}$, in quantum electrodynamics. It is given by:[11]

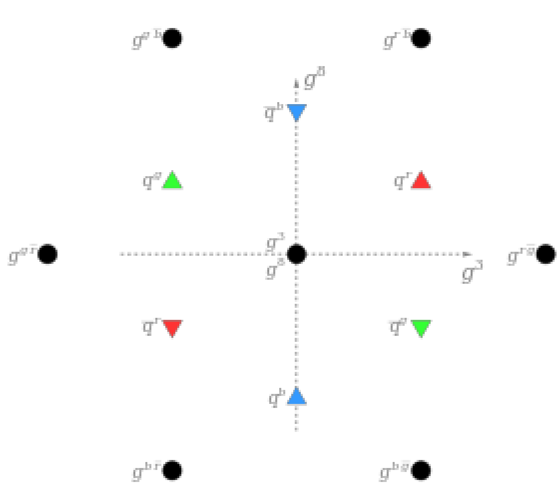
$$G^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu \,,$$

where f_{abc} are the structure constants of **SU**(3). Note that the rules to move-up or pull-down the *a*, *b*, or *c* indexes are *trivial*, (+,... +), so that $f^{abc} = f_{abc} = f^a{}_{bc}$ whereas for the μ or ν indexes one has the non-trivial *relativistic* rules, corresponding e.g. to the metric signature (+ − − −).

The constants *m* and *g* control the quark mass and coupling constants of the theory, subject to renormalization in the full quantum theory.

An important theoretical notion concerning the final term of the above Lagrangian is the *Wilson loop* variable. This loop variable plays a most-important role in discretized forms of the QCD (see lattice QCD), and more generally, it distinguishes confined and deconfined states of a gauge theory. It was introduced by the Nobel prize winner Kenneth G. Wilson and is treated in a separate article.

Fields



The pattern of strong charges for the three colors of quark, three antiquarks, and eight gluons (with two of zero charge overlapping).

Quarks are massive spin-1/2 fermions which carry a color charge whose gauging is the content of QCD. Quarks are represented by Dirac fields in the fundamental representation **3** of the gauge group **SU**(3). They also carry electric charge (either −1/3 or 2/3) and participate in weak interactions as part of weak isospin doublets. They carry global quantum numbers including the baryon number, which is 1/3 for each quark, hypercharge and one of the flavor quantum numbers.

Gluons are spin-1 bosons which also carry color charges, since they lie in theadjoint representation **8** of **SU**(3). They have no electric charge, do not participate in the weak interactions, and have no flavor. They lie in the singlet representation**1** of all these symmetry groups.

Every quark has its own antiquark. The charge of each antiquark is exactly the opposite of the corresponding quark.

Dynamics

According to the rules of Quantum field theory, and the associated Feynman diagrams, the above theory gives rise to three basic interactions: a quark may emit (or absorb) a gluon, a gluon may emit (or absorb) a gluon, and two gluons may directly interact. This contrasts with QED, in which only the first kind of interaction occurs, since photons have no charge. Diagrams involving Faddeev-Popov ghosts must be considered too (except in the unitarity gauge).

Area law and confinement

Detailed computations with the above-mentioned Lagrangian[12] show that the effective potential between a quark and its anti-quark in a meson contains a term $\propto r$, which represents some kind of "stiffness" of the interaction between the particle and its anti-particle at large distances, similar to the entropic elasticity of a rubber band (see below). This leads to *confinement* [13] of the quarks to the interior of hadrons, i.e. mesons and nucleons, with typical radii R_c, corresponding to former "Bag models" of the hadrons[14] . The order of magnitude of the "bag radius" is 1 fm (=10^{-15} m). Moreover, the above-mentioned stiffness is quantitatively related to the so-called "area law" behaviour of the expectation value of the Wilson loop product P_W of the ordered coupling constants around a closed loop W; i.e. $\langle P_W \rangle$ is proportional to the *area* enclosed by the loop. For this behaviour the non-abelian behaviour of the gauge group is essential.

Methods

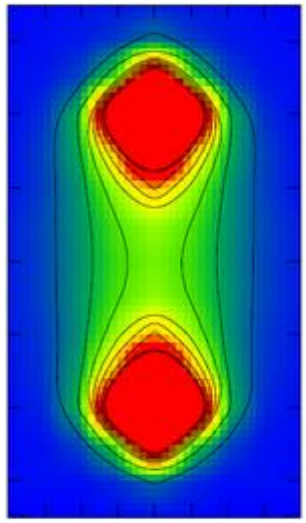
Further analysis of the content of the theory is complicated. Various techniques have been developed to work with QCD. Some of them are discussed briefly below.

Perturbative QCD

Main article: perturbative QCD

This approach is based on asymptotic freedom, which allows perturbation theoryto be used accurately in experiments performed at very high energies. Although limited in scope, this approach has resulted in the most precise tests of QCD to date.

Lattice QCD



A quark and an antiquark (red color) are glued together (green color) to form a meson (result of a lattice QCD simulation by M. Cardoso et al.[15])

Among non-perturbative approaches to QCD, the most well established one islattice QCD. This approach uses a discrete set of space-time points (called the lattice) to reduce the analytically intractable path integrals of the continuum theory to a very difficult numerical computation which is then carried out onsupercomputers like the QCDOC which was constructed for precisely this purpose. While it is a slow and resource-intensive approach, it has wide applicability, giving insight into parts of the theory inaccessible by other means, in particular into the explicit forces acting between quarks and antiquarks in a meson. However, the numerical sign problem makes it difficult to use lattice methods to study QCD at high density and low temperature (e.g. nuclear matter or the interior of neutron stars).

1/N expansion

Main article: 1/N expansion

A well-known approximation scheme, the 1/N expansion, starts from the premise that the number of colors is infinite, and makes a series of corrections to account for the fact that it is not. Until now, it has been the source of qualitative insight rather than a method for quantitative predictions. Modern variants include theAdS/CFT approach.

Effective theories

For specific problems effective theories may be written down which give qualitatively correct results in certain limits. In the best of cases, these may then be obtained as systematic expansions in some parameter of the QCD Lagrangian. One such effective field theory is chiral perturbation theory or ChiPT, which is the QCD effective theory at low energies. More precisely, it is a low energy expansion based on the spontaneous chiral symmetry breaking of QCD, which is an exact symmetry when quark masses are equal to zero, but for the u,d and s quark, which have small mass, it is still a good approximate symmetry. Depending on the number of quarks which are treated as light, one uses either **SU**(2) ChiPT or**SU**(3) ChiPT . Other effective theories are heavy quark effective theory (which expands around heavy quark mass near infinity), and soft-collinear effective theory (which expands around large ratios of energy scales). In addition to effective theories, models like the Nambu-Jona-Lasinio model and the chiral model are often used when discussing general features.

QCD sum rules

Main article: QCD sum rules

Based on an Operator product expansion one can derive sets of relations that connect different observables with each other.

Nambu-Jona-Lasinio model

In one of his recent works, Kei-Ichi Kondo derived as a low-energy limit of QCD, a theory linked to the Nambu-Jona-Lasinio model since it is basically a particular non-local version of the Polyakov-Nambu-Jona-Lasinio model.[16] The later being in its local version, nothing but the Nambu-Jona-Lasinio model in which one has included the Polyakov loop effect, in order to describe a 'certain confinement'.

The Nambu-Jona-Lasinio model in itself is, among many other things, used because it is a 'relatively simple' model of chiral symmetry breaking, phenomenon present up to certain conditions (Chiral limit i.e. massless fermions) in QCD itself. In this model, however, there is no confinement. In particular, the energy of an isolated quark in the physical vacuum turns out well defined and finite.

Experimental tests

The notion of quark flavors was prompted by the necessity of explaining the properties of hadrons during the development of the quark model. The notion of color was necessitated by the puzzle of the Δ++. This has been dealt with in the section on the history of QCD.

The first evidence for quarks as real constituent elements of hadrons was obtained in deep inelastic scattering experiments at SLAC. The first evidence for gluons came in three jet events at PETRA.

Several good quantitative tests of perturbative QCD exist:

- The running of the QCD coupling as deduced from many observations
- Scaling violation in polarized and unpolarized deep inelastic scattering
- Vector boson production at colliders (this includes the Drell-Yan process)
- Jet cross sections in colliders
- Event shape observables at the LEP
- Heavy-quark production in colliders

Quantitative tests of non-perturbative QCD are fewer, because the predictions are harder to make. The best is probably the running of the QCD coupling as probed through lattice computations of heavy-quarkonium spectra. There is a recent claim about the mass of the heavy meson B_c [4]. Other non-perturbative tests are currently at the level of 5% at best. Continuing work on masses and form factorsof hadrons and their weak matrix elements are promising candidates for future quantitative tests. The whole subject of quark matter and the quark-gluon plasmais a non-perturbative test bed for QCD which still remains to be properly exploited.

One qualitative prediction of QCD is that there exist composite particles made solely of gluons called glueballs that have not yet been definitively observed experimentally. A definitive observation of a glueball with the properties predicted by QCD would strongly confirm the theory. In principle, if glueballs could be definitively ruled out, this would be a serious experimental blow to QCD. But, as of 2013, scientists are unable to confirm or deny the existence of glueballs definitively, despite the fact that particle accelerators have sufficient energy to generate them.

Cross-relations to solid state physics

There are unexpected cross-relations to solid state physics. For example, the notion of gauge invariance forms the basis of the well-known Mattis spin glasses,[17] which are systems with the usual spin degrees of freedom $s_i = \pm 1$ for $i = 1,\ldots,N$, with the special fixed "random" couplings $J_{i,k} = \epsilon_i \, J_0 \, \epsilon_k$. Here the ϵ_i and ϵ_k quantities can independently and "randomly" take the values ± 1, which corresponds to a most-simple gauge transformation $(s_i \to s_i \cdot \epsilon_i \quad J_{i,k} \to \epsilon_i J_{i,k} \epsilon_k \quad s_k \to s_k \cdot \epsilon_k)$. This means that thermodynamic expectation values of measurable quantities, e.g. of the energy $\mathcal{H} := -\sum s_i \, J_{i,k} \, s_k$, are invariant.

However, here the *coupling degrees of freedom* $J_{i,k}$, which in the QCD correspond to the *gluons*, are "frozen" to fixed values (quenching). In contrast, in the QCD they "fluctuate" (annealing), and through the large number of gauge degrees of freedom the entropy plays an important role (see below).

For positive J_0 the thermodynamics of the Mattis spin glass corresponds in fact simply to a "ferromagnet in disguise", just because these systems have no "frustration" at all. This term is a basic measure in spin glass theory.[18]Quantitatively it is identical with the loop-product $P_W := \; J_{i,k} J_{k,l} \ldots J_{n,m} J_{m,i}$ along a closed loop *W*. However, for a Mattis spin glass – in contrast to "genuine" spin glasses – the quantity P_W never becomes negative.

The basic notion "frustration" of the spin-glass is actually similar to the Wilson loop quantity of the QCD. The only difference is again that in the QCD one is dealing with SU(3) matrices, and that one is dealing with a "fluctuating" quantity. Energetically, perfect absence of frustration should be non-favorable and atypical for a spin glass, which means that one should add the loop-product to the Hamiltonian, by some kind of term representing a "punishment". – In the QCD the Wilson loop is essential for the Lagrangian rightaway.

The relation between the QCD and "disordered magnetic systems" (the spin glasses belong to them) were additionally stressed in a paper by Fradkin, Huberman und Shenker,[19] which also stresses the notion of duality.

A further analogy consists in the already mentioned similarity to polymer physics, where, analogously to Wilson Loops, so-called "entangled nets" appear, which are important for the formation of the entropy-elasticity (force proportional to the length) of a rubber band. The non-abelian character of the SU(3) corresponds thereby to the non-trivial "chemical links", which glue different loop segments together, and "Asymptotic freedom" means in the polymer analogy simply the fact that in the short-wave limit, i.e. for $0 \leftarrow \lambda_w \ll R_c$ (where R_c is a characteristic correlation-length for the glued loops, corresponding to the above-mentioned "bag radius", while λ_w is the wavelength of an excitation) any non-trivial correlation vanishes totally, as if the system had crystallized.[20]

There is also a correspondence between confinement in QCD - the fact that the color-field is only different from zero in the interior of hadrons - and the behaviour of the usual magnetic field in the theory of type-II superconductors: there the magnetism is confined to the interiour of the Abrikosov flux-line lattice,[21] i.e., the London penetration depth λ of that theory is analogous to the confinement radius R_c of quantum chromodynamics. Mathematically, this correspondendence is supported by the second term, $\propto gG^a_\mu \bar{\psi}_i \gamma^\mu T^a_{ij} \psi_j$, on the r.h.s. of the Lagrangian.

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