Quantum Mechanics_Friction

Friction is the Force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. There are several types of friction:

- **Dry friction** resists relative lateral motion of two solid surfaces in contact. Dry friction is subdivided into *static friction* ("stiction") between non-moving surfaces, and *kinetic friction* between moving surfaces.
- **Fluid friction** describes the friction between layers of a viscous fluid that are moving relative to each other.[1][2]
- **Lubricated friction** is a case of fluid friction where a fluid separates two solid surfaces.[3][4][5]
- **Skin friction** is a component of drag, the force resisting the motion of a fluid across the surface of a body.
- **Internal friction** is the force resisting motion between the elements making up a solid material while it undergoesdeformation.[2]

When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into heat. This property can have dramatic consequences, as illustrated by the use of friction created by rubbing pieces of wood together to start a fire. Kinetic energy is converted to heat whenever motion with friction occurs, for example when a viscous fluid is stirred. Another important consequence of many types of friction can bewear, which may lead to performance degradation and/or damage to components. Friction is a component of the science oftribology.

Friction is not itself a fundamental force but arises from interatomic and intermolecular forces between the two contacting surfaces. The complexity of these interactions makes the calculation of friction from first principlesimpractical and necessitates the use ofempirical methods for analysis and the development of theory.

History

This section requires expansion. *(July 2010)*

The classic rules of sliding friction were discovered by Leonardo da Vinci (1452–1519), but remained unpublished in his notebooks.[6][7][8] They were rediscovered by Guillaume Amontons (1699). Amontons presented the nature of friction in terms of surface irregularities and the force required to raise the weight pressing the surfaces together. This view was further elaborated by Belidor (representation of rough surfaces with spherical asperities, 1737)[6] and Leonhard Euler (1750), who derived the angle of repose of a weight on an inclined plane and first distinguished between static and kinetic friction.[9] A different explanation was provided by Desaguliers (1725), who demonstrated the strong cohesion forces between lead spheres of which a small cap is cut off and which were then brought into contact with each other.

The understanding of friction was further developed by Charles-Augustin de Coulomb (1785). Coulomb investigated the influence of four main factors on friction: the nature of the materials in contact and their surface coatings; the extent of the surface area; the normal pressure (or load); and the length of time that the surfaces remained in contact (time of repose).[6] Coulomb further considered the influence of sliding velocity, temperature and humidity, in order to decide between the different explanations on the nature of friction that had been proposed. The distinction between static and dynamic friction is made in Coulomb's friction law (see below), although this distinction was already drawn by Johann Andreas von Segner in 1758.[6] The effect of the time of repose was explained by Musschenbroek (1762) by considering the surfaces of fibrous materials, with fibers meshing together, which takes a finite time in which the friction increases.

John Leslie (1766–1832) noted a weakness in the views of Amontons and Coulomb. If friction arises from a weight being drawn up the inclined plane of successive asperities, why isn't it balanced then through descending the opposite slope? Leslie was equally skeptical about the role of adhesion proposed by Desaguliers, which should on the whole have the same tendency to accelerate as to retard the motion.[6] In his view friction should be seen as a time-dependent process of flattening, pressing down asperities, which creates new obstacles in what were cavities before.

Arthur Morrin (1833) developed the concept of sliding versus rolling friction.Osborne Reynolds (1866) derived the equation of viscous flow. This completed the classic empirical model of friction (static, kinetic, and fluid) commonly used today in engineering.[7]

The focus of research during the last century has been to understand the physical mechanisms behind friction. F. Phillip Bowden and David Tabor (1950) showed that at a microscopic level, the actual area of contact between surfaces is a very small fraction of the apparent area.[8] This actual area of contact, caused by "asperities" (roughness) increases with pressure, explaining the proportionality between normal force and frictional force. The development of the atomic force microscope (1986) has recently enabled scientists to study friction at the atomic scale.[7]

Laws of dry friction

The elementary property of sliding (kinetic) friction were discovered by experiment in the 15th to 18th centuries and were expressed as three empirical laws:

- **Amontons' First Law**: The force of friction is directly proportional to the applied load.
- **Amontons' Second Law**: The force of friction is independent of the apparent area of contact.
- **Coulomb's Law of Friction**: Kinetic friction is independent of the sliding velocity.

Dry friction

Dry friction resists relative lateral motion of two solid surfaces in contact. The two regimes of dry friction are 'static friction' ("stiction") between non-moving surfaces, and *kinetic friction* (sometimes called sliding friction or dynamic friction) between moving surfaces.

Coulomb friction, named after Charles-Augustin de Coulomb, is an approximate model used to calculate the force of dry friction. It is governed by the model:

$$F_\mathrm{f} \leq \mu F_\mathrm{n}$$

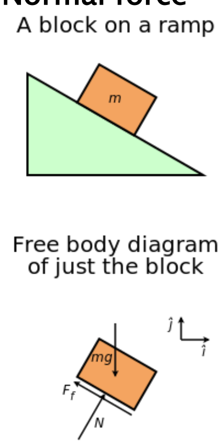
where

- F_f is the force of friction exerted by each surface on the other. It is parallel to the surface, in a direction opposite to the net applied force.
- μ is the coefficient of friction, which is an empirical property of the contacting materials,
- F_n is the normal force exerted by each surface on the other, directed perpendicular (normal) to the surface.

The Coulomb friction F_f may take any value from zero up to μF_n, and the direction of the frictional force against a surface is opposite to the motion that surface would experience in the absence of friction. Thus, in the static case, the frictional force is exactly what it must be in order to prevent motion between the surfaces; it balances the net force tending to cause such motion. In this case, rather than providing an estimate of the actual frictional force, the Coulomb approximation provides a threshold value for this force, above which motion would commence. This maximum force is known as traction.

The force of friction is always exerted in a direction that opposes movement (for kinetic friction) or potential movement (for static friction) between the two surfaces. For example, a curling stone sliding along the ice experiences a kinetic force slowing it down. For an example of potential movement, the drive wheels of an accelerating car experience a frictional force pointing forward; if they did not, the wheels would spin, and the rubber would slide backwards along the pavement. Note that it is not the direction of movement of the vehicle they oppose, it is the direction of (potential) sliding between tire and road.

Normal force



Free-body diagram for a block on a ramp. Arrows are vectors indicating directions and magnitudes of forces. *N* is the normal force, *mg* is the force of gravity, and F_f is the force of friction.

Main article: normal force

The normal force is defined as the net force compressing two parallel surfaces together; and its direction is perpendicular to the surfaces. In the simple case of a mass resting on a horizontal surface, the only component of the normal force is the force due to gravity, where $N = mg$. In this case, the magnitude of the friction force is the product of the mass of the object, the acceleration due to gravity, and the coefficient of friction. However, the coefficient of friction is not a function of mass or volume; it depends only on the material. For instance, a large aluminum block has the same coefficient of friction as a small aluminum block. However, the magnitude of the friction force itself depends on the normal force, and hence on the mass of the block.

If an **object is on a level surface** and the force tending to cause it to slide is horizontal, the normal force N between the object and the surface is just its weight, which is equal to its Mass multiplied by the Acceleration due to earth's gravity, *g*. If the **object is on a tilted surface** such as an inclined plane, the normal force is less, because less of the force of gravity is perpendicular to the face of the plane. Therefore, the normal force, and ultimately the frictional force, is determined using vector analysis, usually via a free body diagram. Depending on the situation, the calculation of the normal force may include forces other than gravity.

Coefficient of friction

The **coefficient of friction** (COF), often symbolized by the Greek letter µ, is adimensionless scalar value which describes the ratio of the force of friction between two bodies and the force pressing them together. The coefficient of friction depends on the materials used; for example, ice on steel has a low coefficient of friction, while rubber on pavement has a high coefficient of friction. Coefficients of friction range from near zero to greater than one.

For surfaces at rest relative to each other $\mu = \mu_\mathrm{s}$, where μ_s is the *coefficient of static friction*. This is usually larger than its kinetic counterpart.

For surfaces in relative motion $\mu = \mu_\mathrm{k}$, where μ_k is the *coefficient of kinetic friction*. The Coulomb friction is equal to F_f, and the frictional force on each surface is exerted in the direction opposite to its motion relative to the other surface.

Arthur Morin introduced the term and demonstrated the utility of the coefficient of friction.[6] The coefficient of friction is an empirical measurement – it has to be measured experimentally, and cannot be found through calculations.[*citation needed*] Rougher surfaces tend to have higher effective values. Both static and kinetic coefficients of friction depend on the pair of surfaces in contact; for a given pair of surfaces, the coefficient of static friction is *usually* larger than that of kinetic friction; in some sets the two coefficients are equal, such as teflon-on-teflon.

Most dry materials in combination have friction coefficient values between 0.3 and 0.6. Values outside this range are rarer, but teflon, for example, can have a coefficient as low as 0.04. A value of zero would mean no friction at all, an elusive property –

even magnetic levitation vehicles have drag. Rubber in contact with other surfaces can yield friction coefficients from 1 to 2. Occasionally it is maintained that μ is always < 1, but this is not true. While in most relevant applications μ < 1, a value above 1 merely implies that the force required to slide an object along the surface is greater than the normal force of the surface on the object. For example, silicone rubber or acrylic rubber–coated surfaces have a coefficient of friction that can be substantially larger than 1.

While it is often stated that the COF is a "material property," it is better categorized as a "system property." Unlike true material properties (such as conductivity, dielectric constant, yield strength), the COF for any two materials depends on system variables like temperature, Velocity, atmosphere and also what are now popularly described as aging and deaging times; as well as on geometric properties of the interface between the materials. For example, acopper pin sliding against a thick copper plate can have a COF that varies from 0.6 at low speeds (metal sliding against metal) to below 0.2 at high speeds when the copper surface begins to melt due to frictional heating. The latter speed, of course, does not determine the COF uniquely; if the pin diameter is increased so that the frictional heating is removed rapidly, the temperature drops, the pin remains solid and the COF rises to that of a 'low speed' test.[*citation needed*]

Negative coefficient of friction

As of 2012, a single study has demonstrated the potential for a negative coefficient of friction, meaning that a decrease in normal force leads to an increase in friction. This contradicts everyday experience in which an increase in normal force leads to an increase in friction.[10] This was reported in the journal *Nature* in October 2012 and involved the friction encountered by an atomic force microscope stylus when dragged across a graphene sheet in the presence of graphene-adsorbed oxygen.[10]

Approximate coefficients of friction

		Static friction, μ_s	
Materials		Dry and clean	Lubricated
Aluminium	Steel	0.61	
Copper	Steel	0.53	
Brass	Steel	0.51	
Cast iron	Copper	1.05	
Cast iron	Zinc	0.85	

Concrete	Rubber	1.0	0.30 (wet)
Concrete	Wood	0.62[11]	
Copper	Glass	0.68	
Glass	Glass	0.94	
Metal	Wood	0.2–0.6[11]	0.2 (wet)[11]
Polyethene	Steel	0.2[12]	0.2[12]
Steel	Steel	0.80[12]	0.16[12]
Steel	PTFE (Teflon)	0.04[12]	0.04[12]
PTFE (Teflon)	PTFE (Teflon)	0.04[12]	0.04[12]
Wood	Wood	0.25–0.5[11]	0.2 (wet)[11]

An $AlMgB_{14}$–TiB_2 composite has an approximate coefficient of friction of 0.02 in water-glycol-based lubricants,[13][14] and 0.04–0.05 when dry.[15] Under certain conditions, some materials have even lower friction coefficients. An example is (highly ordered pyrolytic) graphite, which can have a friction coefficient below 0.01.[16] This ultralow-friction regime is called superlubricity.

Static friction

Static friction is friction between two or more solid objects that are not moving relative to each other. For example, static friction can prevent an object from sliding down a sloped surface. The coefficient of static friction, typically denoted as μ_s, is usually higher than the coefficient of kinetic friction.

The static friction force must be overcome by an applied force before an object can move. The maximum possible friction force between two surfaces before sliding begins is the product of the coefficient of static friction and the normal force: $F_{max} = \mu_s F_n$. When there is no sliding occurring, the friction force can have any value from zero up to F_{max}. Any force smaller than F_{max} attempting to slide one surface over the other is opposed by a frictional force of equal magnitude and opposite direction. Any force larger than F_{max} overcomes the force of static friction and causes sliding to occur. The instant sliding occurs, static friction is no longer applicable—the friction between the two surfaces is then called kinetic friction.

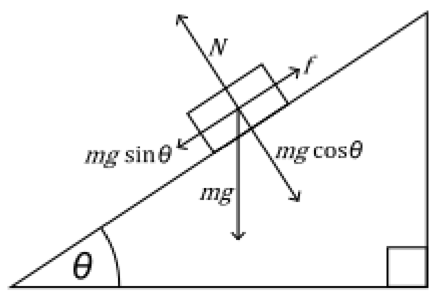
An example of static friction is the force that prevents a car wheel from slipping as it rolls on the ground. Even though the wheel is in motion, the patch of the tire in contact with the ground is stationary relative to the ground, so it is static rather than kinetic friction.

The maximum value of static friction, when motion is impending, is sometimes referred to as **limiting friction**,[17] although this term is not used universally.[1]It is also known as traction.[*citation needed*]

Kinetic friction

Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together (like a sled on the ground). The coefficient of kinetic friction is typically denoted as μ_k, and is usually less than the coefficient of static friction for the same materials.[18][19] However, Richard Feynman comments that "with dry metals it is very hard to show any difference."[20]

New models are beginning to show how kinetic friction can be greater than static friction.[21] Kinetic friction is now understood, in many cases, to be primarily caused by chemical bonding between the surfaces, rather than interlocking asperities;[22] however, in many other cases roughness effects are dominant, for example in rubber to road friction.[21] Surface roughness and contact area, however, do affect kinetic friction for micro- and nano-scale objects where surface area forces dominate inertial forces.[23]



Angle of friction, θ, when block just starts to slide.

Angle of friction

For the maximum angle of static friction between granular materials, see angle of repose.

For certain applications it is more useful to define static friction in terms of the maximum angle before which one of the items will begin sliding. This is called the*angle of friction* or *friction angle*. It is defined as:

$$\tan\theta = \mu_s$$

where θ is the angle from horizontal and μ_s is the static coefficient of friction between the objects.[24] This formula can also be used to calculate μ_s from empirical measurements of the friction angle.

Friction at the atomic level

Determining the forces required to move atoms past each other is a challenge in designing nanomachines. In 2008 scientists for the first time were able to move a single atom across a surface, and measure the forces required. Using ultrahigh vacuum and nearly zero temperature (5 K), a modified atomic force microscope was used to drag a cobalt atom, and a carbon monoxide molecule, across surfaces of copper and platinum.[25]

Limitations of the Coulomb model

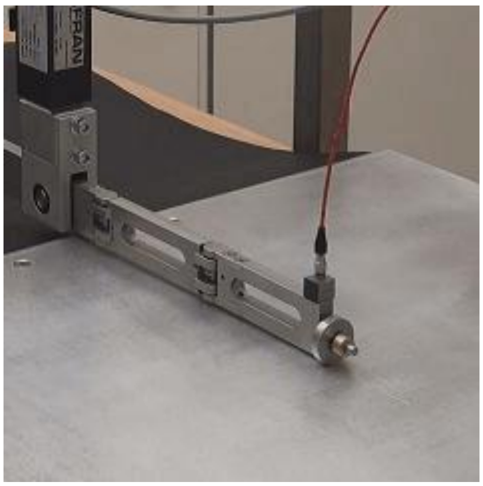
The Coulomb approximation mathematically follows from the assumptions that surfaces are in atomically close contact only over a small fraction of their overall area, that this contact area is proportional to the normal force (until saturation, which takes place when all area is in atomic contact), and that the frictional force is proportional to the applied normal force, independently of the contact area (you can see the experiments on friction from Leonardo da Vinci). Such reasoning aside, however, the approximation is fundamentally an empirical construct. It is a rule of thumb describing the approximate outcome of an extremely complicated physical interaction. The strength of the approximation is its simplicity and versatility – though in general the relationship between normal force and frictional force is not exactly linear (and so the frictional force is not entirely independent of the contact area of the surfaces), the Coulomb approximation is an adequate representation of friction for the analysis of many physical systems.

When the surfaces are conjoined, Coulomb friction becomes a very poor approximation (for example, adhesive tape resists sliding even when there is no normal force, or a negative normal force). In this case, the frictional force may depend strongly on the area of contact. Some drag racing tires are adhesive for this reason. However, despite the complexity of the fundamental physics behind friction, the relationships are accurate enough to be useful in many applications.

Numerical simulation of the Coulomb model

Despite being a simplified model of friction, the Coulomb model is useful in manynumerical simulation applications such as multibody systems and granular material. Even its most simple expression encapsulates the fundamental effects of sticking and sliding which are required in many applied cases, although specific algorithms have to be designed in order to efficiently numerically integratemechanical systems with Coulomb friction and bilateral and/or unilateral contact.[26][27][28][29][30] Some quite nonlinear effects, such as the so-calledPainlevé paradoxes, may be encountered with Coulomb friction.[31]

Dry friction and instabilities



A physical model of the so-called 'Ziegler column', a two-degree-of-freedom system, exhibiting flutter instability as induced by dry friction. Watch the complete movie for more details.

Dry friction can induce several types of instabilities in mechanical systems which display a stable behaviour in the absence of friction.[32] For instance, friction-related dynamical instabilities are thought to be responsible for brake squeal and the 'song' of a glass harp,[33][34] phenomena which involve stick and slip, modelled as a drop of friction coefficient with velocity.[35]

A practically important case is the self-oscillation of the strings of bowed instruments such as the violin, cello, hurdy-gurdy, erhu etc.

A connection between dry friction and flutter instability in a simple mechanical system has been discovered.[36]

Frictional instabilities can lead to the formation of new self-organized patterns (or "secondary structures") at the sliding interface, such as in-situ formed tribofilms which are utilized for the reduction of friction and wear in so-called self-lubricating materials.[37]

Fluid friction

Main article: Viscosity

Fluid friction occurs between layers within a fluid that are moving relative to each other. This internal resistance to flow is described as *Viscosity*. In everyday terms viscosity of a fluid is said to have "thickness". Thus, water is "thin", having a lower viscosity, while honey is "thick", having a higher viscosity. The less viscous the fluid, the greater its ease of movement.

All real fluids (except superfluids) have some resistance to stress and therefore are viscous, but a fluid which has no resistance to shear stress is known as an ideal fluid or inviscid fluid.

Lubricated friction

Main article: Lubrication

Lubricated friction is a case of fluid friction where a fluid separates two solid surfaces. Lubrication is a technique employed to reduce wear of one or both surfaces in close proximity moving relative to each another by interposing a substance called a lubricant between the surfaces.

In most cases the applied load is carried by pressure generated within the fluid due to the frictional viscous resistance to motion of the lubricating fluid between the surfaces. Adequate lubrication allows smooth continuous operation of equipment, with only mild wear, and without excessive stresses or seizures at bearings. When lubrication breaks down, metal or other components can rub destructively over each other, causing heat and possibly damage or failure.

Skin friction

Main article: Parasitic drag

Skin friction arises from the friction of the fluid against the "skin" of the object that is moving through it. Skin friction arises from the interaction between the fluid and the skin of the body, and is directly related to the area of the surface of the body that is in contact with the fluid. Skin friction follows the drag equationand rises with the square of the velocity.

Skin friction is caused by viscous drag in the boundary layer around the object. There are two ways to decrease skin friction: the first is to shape the moving body so that smooth flow is possible, like an airfoil. The second method is to decrease the length and cross-section of the moving object as much as is practicable.

Internal friction

Main article: Plastic deformation of solids

See also: Deformation (engineering)

Internal friction is the force resisting motion between the elements making up a solid material while it undergoes plastic deformation.

Plastic deformation in solids is an irreversible change in the internal molecular structure of an object. This change may be due to either (or both) an applied force or a change in temperature. The change of an object's shape is called strain. The force causing it is called stress. Stress does not necessarily cause permanent change. As deformation occurs, internal forces oppose the applied force. If the applied stress is not too large these opposing forces may completely resist the applied force, allowing the object to assume a new equilibrium state and to return to its original shape when the force is removed. This is what is known in the literature as elastic deformation (or elasticity). Larger forces in excess of the elastic limit may cause a permanent (irreversible) deformation of the object. This is what is known as plastic deformation.

Other types of friction

Rolling resistance

Main article: Rolling resistance

Rolling resistance is the force that resists the rolling of a wheel or other circular object along a surface caused by deformations in the object and/or surface. Generally the force of rolling resistance is less than that associated with kinetic friction.[38] Typical values for the coefficient of rolling resistance are 0.001.[39]One of the most common examples of rolling resistance is the movement of motor vehicle tires on a road, a process which generates heat and sound as by-products.[40]

Triboelectric effect

Main article: Triboelectric effect

Rubbing dissimilar materials against one another can cause a build-up ofelectrostatic charge, which can be hazardous if flammable gases or vapours are present. When the static build-up discharges, explosions can be caused by ignition of the flammable mixture.

Belt friction

Main article: Belt friction

Belt friction is a physical property observed from the forces acting on a belt wrapped around a pulley, when one end is being pulled. The resulting tension, which acts on both ends of the belt, can be modeled by the belt friction equation.

In practice, the theoretical tension acting on the belt or rope calculated by the belt friction equation can be compared to the maximum tension the belt can support. This helps a designer of such a rig to know how many times the belt or rope must be wrapped around the pulley to prevent it from slipping. Mountain climbers and sailing crews demonstrate a standard knowledge of belt friction when accomplishing basic tasks.

Reducing friction

Devices

Devices such as wheels, ball bearings, roller bearings, and air cushion or other types of fluid bearings can change sliding friction into a much smaller type of rolling friction.

Many thermoplastic materials such as nylon, HDPE and PTFE are commonly used in low friction bearings. They are especially useful because the coefficient of friction falls with increasing imposed load.[*citation needed*] For improved wear resistance, very high molecular weight grades are usually specified for heavy duty or critical bearings.

Lubricants

A common way to reduce friction is by using a lubricant, such as oil, water, or grease, which is placed between the two surfaces, often dramatically lessening the coefficient of friction. The science of friction and lubrication is called tribology. Lubricant technology is when lubricants are mixed with the application of science, especially to industrial or commercial objectives.

Superlubricity, a recently discovered effect, has been observed in graphite: it is the substantial decrease of friction between two sliding objects, approaching zero levels. A very small amount of frictional energy would still be dissipated.

Lubricants to overcome friction need not always be thin, turbulent fluids or powdery solids such as graphite and talc; acoustic lubrication actually uses sound as a lubricant. Another way to reduce friction between two parts is to superimpose micro-scale vibration to one of the parts. This can be sinusoidal vibration as used in ultrasound-assisted cutting or vibration noise, known as dither.

Energy of friction

According to the law of conservation of energy, no energy is destroyed due to friction, though it may be lost to the system of concern. Energy is transformed from other forms into heat. A sliding hockey puck comes to rest because friction converts its kinetic energy into heat. Since heat quickly dissipates, many early philosophers, including Aristotle, wrongly concluded that moving objects lose energy without a driving force.

When an object is pushed along a surface along a path C, the energy converted to heat is given by a line integral, in accordance with the definition of work.

$$E_{th} = \int_C \mathbf{F}_{\mathrm{fric}}(\mathbf{x}) \cdot d\mathbf{x} \ = \int_C \mu_{\mathrm{k}} \ \mathbf{F}_{\mathrm{n}}(\mathbf{x}) \cdot d\mathbf{x}$$

where

- $\mathbf{F}_{fric}$ is the friction force,
- $\mathbf{F}_{n}$ is the vector obtained by multiplying the magnitude of the normal force by a unit vector pointing *against* the object's motion,
- μ_{k} is the coefficient of kinetic friction, which is inside the integral because it may vary from location to location (e.g. if the material changes along the path)
- $\mathbf{x}$ is the position of the object

Energy lost to a system as a result of friction is a classic example of thermodynamic irreversibility.

Work of friction

In the reference frame of the interface between two surfaces, static friction does *no* work, because there is never displacement between the surfaces. In the same reference frame, kinetic friction is always in the direction opposite the motion, and does *negative* work.[41] However, friction can do *positive* work in certain frames of reference. One can see this by placing a heavy box on a rug, then pulling on the rug quickly. In this case, the box slides backwards relative to the rug, but moves forward relative to the frame of reference in which the floor is stationary. Thus, the kinetic friction between the box and rug accelerates the box in the same direction that the box moves, doing *positive* work.[42]

The work done by friction can translate into deformation, wear, and heat that can affect the contact surface properties (even the coefficient of friction between the surfaces). This can be beneficial as in polishing. The work of friction is used to mix and join materials such as in the process of friction welding. Excessive erosion or wear of mating sliding surfaces occurs when work due frictional forces rise to unacceptable levels. Harder corrosion particles caught between mating surfaces in relative motion (fretting) exacerbates wear of frictional forces. Bearing seizure or failure may result from excessive wear due to work of friction. As surfaces are worn by work due to friction, fit and surface finish of an object may degrade until it no longer functions properly.[43]

Applications

Friction is an important factor in many engineering disciplines.

Transportation

- Automobile brakes inherently rely on friction, slowing a vehicle by converting its kinetic energy into heat. Incidentally, dispersing this large amount of heat safely is one technical challenge in designing brake systems.
- Rail adhesion refers to the grip wheels of a train have on the rails, see Frictional contact mechanics.
- Road slipperiness is an important design and safety factor for automobiles
  - Split friction is a particularly dangerous condition arising due to varying friction on either side of a car.
  - Road texture affects the interaction of tires and the driving surface.

Measurement

- A tribometer is an instrument that measures friction on a surface.
- A profilograph is a device used to measure pavement surface roughness.

Household usage

- Friction is used to heat and ignite matchsticks (friction between the head of a matchstick and the rubbing surface of the match box).

References

1. ^ a b Beer, Ferdinand P.; E. Russel Johnston, Jr. (1996). *Vector Mechanics for Engineers* (Sixth ed.). McGraw-Hill. p. 397. ISBN 0-07-297688-8.

2. ^ a b Meriam, J. L.; L. G. Kraige (2002). *Engineering Mechanics* (fifth ed.). John Wiley & Sons. p. 328. ISBN 0-471-60293-0.
3. ^ Ruina, Andy; Rudra Pratap (2002). *Introduction to Statics and Dynamics*. Oxford University Press. p. 713.
4. ^ Hibbeler, R. C. (2007). *Engineering Mechanics* (Eleventh ed.). Pearson, Prentice Hall. p. 393. ISBN 0-13-127146-6.
5. ^ Soutas-Little, Robert W.; Inman, Balint (2008). *Engineering Mechanics*. Thomson. p. 329. ISBN 0-495-29610-4.
6. ^ a b c d e f Dowson, Duncan (1997). *History of Tribology, 2nd Edition*. Professional Engineering Publishing. ISBN 1-86058-070-X.
7. ^ a b c Armstrong-Hélouvry, Brian (1991). *Control of machines with friction*. USA: Springer. p. 10. ISBN 0-7923-9133-0.
8. ^ a b van Beek, Anton. "History of Science Friction". tribology-abc.com. Retrieved 2011-03-24.
9. ^ "Leonhard Euler". *Friction Module*. Nano World website. 2002. Retrieved 2011-03-25.
10. ^ a b Deng, Zhao; et al. (October 14, 2012), "Adhesion-dependent negative friction coefficient on chemically modified graphite at the nanoscale", *Nature*,Bibcode:2012NatMa..11.1032D, doi:10.1038/nmat3452, retrieved November 18, 2012, lay summary – *R&D Magazine* (October 17, 2012)
11. ^ a b c d e Coefficient of Friction. EngineersHandbook.com
12. ^ a b c d e f g h "The Engineering Toolbox: Friction and Coefficients of Friction". Retrieved 2008-11-23.
13. ^ Kleiner, Kurt (2008-11-21). "Material slicker than Teflon discovered by accident". Retrieved 2008-12-25.
14. ^ Higdon, C.; Cook, B.; Harringa, J.; Russell, A.; Goldsmith, J.; Qu, J.; Blau, P. (2011). "Friction and wear mechanisms in AlMgB14-TiB2 nanocoatings". *Wear* **271** (9-10): 2111. doi:10.1016/j.wear.2010.11.044.
15. ^ Tian, Y.; Bastawros, A. F.; Lo, C. C. H.; Constant, A. P.; Russell, A. M.; Cook, B. A. (2003). "Superhard self-lubricating AlMgB[sub 14] films for microelectromechanical devices". *Applied Physics Letters* **83** (14): 2781.Bibcode:2003ApPhL..83.2781T. doi:10.1063/1.1615677.

16. ^ Dienwiebel, Martin et al. (2004). "Superlubricity of Graphite". *Phys. Rev. Lett.* **92** (12): 126101. Bibcode:2004PhRvL..92l6101D. doi:10.1103/PhysRevLett.92.126101.
17. ^ Bhavikatti, S. S.; K. G. Rajashekarappa (1994). *Engineering Mechanics*. New Age International. p. 112. ISBN 978-81-224-0617-7. Retrieved 2007-10-21.
18. ^ Sheppard, Sheri; Tongue, Benson H. and Anagnos, Thalia (2005). *Statics: Analysis and Design of Systems in Equilibrium*. Wiley and Sons. p. 618. ISBN 0-471-37299-4. "In general, for given contacting surfaces, $\mu_k < \mu_s$"
19. ^ Meriam, James L.; Kraige, L. Glenn and Palm, William John (2002). *Engineering Mechanics: Statics*. Wiley and Sons. p. 330. ISBN 0-471-40646-5. "Kinetic friction force is usually somewhat less than the maximum static friction force."
20. ^ Feynman, Richard P.; Leighton, Robert B. and Sands, Matthew (1964). "The Feynman Lectures on Physics, Vol. I, p. 12-5". Addison-Wesley. Retrieved 2009-10-16.
21. ^ *a b* Persson, B. N.; Volokitin, A. I (2002). "Theory of rubber friction: Nonstationary sliding". *Physical Review B* **65** (13): 134106. Bibcode:2002PhRvB..65m4106P. doi:10.1103/PhysRevB.65.134106.
22. ^ Beatty, William J. "Recurring science misconceptions in K-6 textbooks". Retrieved 2007-06-08.
23. ^ Persson, B. N. J. (2000). *Sliding friction: physical principles and applications*. Springer. ISBN 978-3-540-67192-3. Retrieved 12 December 2011.
24. ^ Nichols, Edward Leamington; Franklin, William Suddards (1898). *The Elements of Physics* **1**. Macmillan. p. 101.
25. ^ Ternes, Markus; Lutz, Christopher P.; Hirjibehedin, Cyrus F.; Giessibl, Franz J.; Heinrich, Andreas J. (2008-02-22). "The Force Needed to Move an Atom on a Surface". *Science* **319** (5866): 1066-1069. Bibcode:2008Sci...319.1066T. doi:10.1126/science.1150288. PMID 18292336.
26. ^ Haslinger, J.; Nedlec, J.C. (1983). "Approximation of the Signorini problem with friction, obeying the Coulomb law". *Mathematical Methods in the Applied Sciences* **5**: 422-437. Bibcode:1983MMAS....5..422H. doi:10.1002/mma.1670050127.

27.^ Alart, P.; Curnier, A. (1991). "A mixed formulation for frictional contact problems prone to Newton like solution method". *Computer Methods in Applied Mechanics and Engineering* **92** (3): 353–375. Bibcode:1991CMAME..92..353A.doi:10.1016/0045-7825(91)90022-X.
28.^ Acary, V.; Cadoux, F.; Lemaréchal, C.; Malick, J. (2011). "A formulation of the linear discrete Coulomb friction problem via convex optimization". *Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik* **91** (2): 155–175. Bibcode:2011ZaMM...91..155A.doi:10.1002/zamm.201000073.
29.^ De Saxcé, G.; Feng, Z.-Q. (1998). "The bipotential method: A constructive approach to design the complete contact law with friction and improved numerical algorithms". *Mathematical and Computer Modelling* **28** (4): 225–245.doi:10.1016/S0895-7177(98)00119-8.
30.^ Simo, J.C.; Laursen, T.A. (1992). "An augmented lagrangian treatment of contact problems involving friction". *Computers and Structures* **42** (2): 97–116.doi:10.1016/0045-7949(92)90540-G.
31.^ Acary, V.; Brogliato, B. (2008). *Numerical Methods for Nonsmooth Dynamical Systems. Applications in Mechanics and Electronics* **35**. Springer Verlag Heidelberg.
32.^ Bigoni, D. *Nonlinear Solid Mechanics: Bifurcation Theory and Material Instability*. Cambridge University Press, 2012. ISBN 9781107025417.
33.^ Flint, J. and Hultén, J. (2002). "Lining-deformation-induced modal coupling as squeal generator in a distributed parameter disk brake model". *J. Sound and Vibration* **254**: 1–21. Bibcode:2002JSV...254....1F. doi:10.1006/jsvi.2001.4052.
34.^ Kröger, M.; Neubauer, M. and Popp, K. (2008). "Experimental investigation on the avoidance of self-excited vibrations". *Phil. Trans. R. Soc. A* **366** (1866): 785–810. Bibcode:2008RSPTA.366..785K. doi:10.1098/rsta.2007.2127. PMID 17947204.
35.^ Rice, J. R. and Ruina, A. L. (1983). "Stability of Steady Frictional Slipping".*Journal of Applied Mechanics* **50** (2): 343–349. Bibcode:1983JAM....50..343R.doi:10.1115/1.3167042.
36.^ Bigoni, D. and Noselli, G. (2011). "Experimental evidence of flutter and divergence instabilities induced by dry friction". *Journal of the Mechanics and*

Physics of Solids **59** (10): 2208–2226. Bibcode:2011JMPSo..59.2208B.doi:10.1016/j.jmps.2011.05.007.

37. ^ Nosonovsky, Michael (2013). *Friction-Induced Vibrations and Self-Organization: Mechanics and Non-Equilibrium Thermodynamics of Sliding Contact*. CRC Press. p. 333. ISBN 978-1466504011.
38. ^ Silliman, Benjamin (1871) *Principles of Physics, Or Natural Philosophy*, Ivison, Blakeman, Taylor & company publishers
39. ^ Butt, Hans-Jürgen; Graf, Karlheinz and Kappl, Michael (2006) *Physics and Chemistry of Interfaces*, Wiley, ISBN 3-527-40413-9
40. ^ Hogan, C. Michael (1973). "Analysis of highway noise". *Water, Air, and Soil Pollution* **2** (3): 387. doi:10.1007/BF00159677.
41. ^ Den Hartog, J. P. (1961). *Mechanics*. Courier Dover Publications. p. 142. ISBN 0-486-60754-2.
42. ^ Leonard, William J (2000). *Minds-on Physics*. Kendall/Hunt. p. 603. ISBN 0-7872-3932-1.
43. ^ Bayer, Raymond George (2004). *Mechanical wear*. CRC Press. pp. 1, 2. ISBN 0-8247-4620-1. Retrieved 2008-07-07.