Quantum Mechanics _ fractional quantum mechanics

In physics, fractional quantum mechanics is a generalization of standard Quantum mechanics, which naturally comes out when the Brownian-like quantum paths substitute with the Lévy-like ones in the Feynman path integral. It has been discovered by Nick Laskin who coined the term fractional quantum mechanics.[1]

Fundamentals

Standard quantum mechanics can be approached in three different ways: the matrix mechanics, the Schrödinger equation and the Feynman path integral.

The Feynman path integral[2] is the path integral over Brownian-like quantum-mechanical paths. Fractional quantum mechanics has been discovered by Nick Laskin (1999) as a result of expanding the Feynman path integral, from the Brownian-like to the Lévy-like quantum mechanical paths. A path integral over the Lévy-like quantum-mechanical paths results in a generalization of Quantum mechanics.[3] If the Feynman path integral leads to the well known Schrödinger equation, then the path integral over Lévy trajectories leads to the fractional Schrödinger equation.[4] The Lévy process is characterized by the Lévy index α, 0 < α ≤ 2. At the special case when α = 2 the Lévy process becomes the process of Brownian motion. The fractional Schrödinger equation includes a space derivative of fractional order α instead of the second order (α = 2) space derivative in the standard Schrödinger equation. Thus, the fractional Schrödinger equation is a fractional differential equation in accordance with modern terminology.[5] This is the main point of the term fractional Schrödinger equation or a more general term fractional quantum mechanics. As mentioned above, at α = 2 the Lévy motion becomes Brownian motion. Thus, fractional quantum mechanics includes standard quantum mechanics as a particular case at α = 2. The quantum-mechanical path integral over the Lévy paths at α = 2 becomes the well-known Feynman path integral and the fractional Schrödinger equation becomes the well-known Schrödinger equation.

Fractional Schrödinger equation

The fractional Schrödinger equation discovered by Nick Laskin has the following form (see, Refs.[1,3,4])

\[ i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = D_\alpha(-\hbar^2 \Delta)^{\alpha/2} \psi(\mathbf{r}, t) + V(\mathbf{r}, t) \psi(\mathbf{r}, t), \]

using the standard definitions:

- \( \mathbf{r} \) is the 3-dimensional position vector.
• $\hbar$ is the reduced Planck constant.
• $\psi(r, t)$ is the wavefunction, which is the quantum mechanical function that determines the probability amplitude for the particle to have a given position $r$ at any given time $t$.
• $V(r, t)$ is a potential energy.
• $\Delta = \frac{\partial^2}{\partial r^2}$ is the Laplace operator.

Further,
• $D_\alpha$ is a scale constant with physical dimension $[D_\alpha] = [\text{energy}]^{-\alpha}[\text{length}]^\alpha[\text{time}]^{-\alpha}$, at $\alpha = 2$, $D_2 = 1/2m$, where $m$ is a particle mass,
• the operator $(-\hbar^2 \Delta)^{\alpha/2}$ is the 3-dimensional fractional quantum Riesz derivative defined by (see, Ref.[4]);
  \[
  (-\hbar^2 \Delta)^{\alpha/2} \psi(r, t) = \frac{1}{(2\pi \hbar)^3} \int d^3 p e^{i p \cdot r / \hbar} |p|^{\alpha} \varphi(p, t),
  \]
Here, the wave functions in the position and momentum spaces; $\psi(r, t)$ and $\varphi(p, t)$ are related each other by the 3-dimensional Fourier transforms:
\[
\psi(r, t) = \frac{1}{(2\pi \hbar)^3} \int d^3 p e^{i p \cdot r / \hbar} \varphi(p, t), \quad \varphi(p, t) = \int d^3 r e^{-i p \cdot r / \hbar} \psi(r, t).
\]
The index $\alpha$ in the fractional Schrödinger equation is the Lévy index, $1 < \alpha \leq 2$.

See also
• Quantum mechanics
• matrix mechanics
• Fractional calculus
• Fractional dynamics
• fractional Schrödinger equation
• Non-linear Schrödinger equation
• Path integral formulation
• Relation between Schrödinger's equation and the path integral formulation of quantum mechanics
• Lévy process

References


Further reading


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