

Module

6

Actuators

Lesson 30

Pneumatic Control Systems

Instructional Objectives

At the end of this lesson, the student should be able to

- Sketch the schematic diagram of a pneumatic proportional controller.
- Apply linearisation technique to develop the transfer function of a pneumatic proportional controller.
- Identify the major difference in construction among pneumatic P, P-D and P-I controllers.
- Identify the varying element by which the proportional gain of a P-controller can be adjusted.
- Identify the varying elements for adjusting the derivative and integral times in P-D and P-I controllers.
- Develop the transfer function of a pneumatic P-D controller.

Introduction

In the last lesson we have discussed on the construction and principles of operation of various pneumatic components. In this lesson, we shall try to understand how these components can be combined to make a complete pneumatic control system. We shall particularly concentrate on the working of various PID controllers. These controllers together with the final control elements (diaphragm type valve positioners) provide the essentials of a pneumatic control system. Pneumatic controllers are the earliest type of controllers used in industry and still find regular use in many applications. But probably the more interesting part of pneumatic control is its principle of operation and how the derivative and integral parts can be generated by simply throttling the valves in the air line. The implementation issues of PID controllers have been discussed in Lesson 14. The details of the implementation of a pneumatic PID controller have been elaborated in this lesson.

Pneumatic Proportional Controller

Consider the pneumatic system shown in Fig.1. It consists of several pneumatic components discussed in Lesson 29. The components, which can be easily identified, are: flapper nozzle amplifier, air relay, bellows and springs, feedback arrangement etc. The overall arrangement is known as a pneumatic proportional controller. It acts as a controller in a pneumatic system generating output pressure proportional to the displacement e at one end of the link. The principle of operation is similar to that of Fig. 4 in lesson 29. The input to the system is a small linear displacement e and the output is pressure P_o . The input displacement may be caused by a small differential pressure to a pair of bellows, or by a small current driving an electromagnetic unit. There are two springs K_2 and K_f those exert forces against the movements of the bellows A_2 and A_f . For a positive displacement of e (towards right) will cause decrease of pressure in the flapper nozzle. This will cause an upward movement of the bellows A_2 (decrease in y). Consequently the output pressure of the air relay will increase. The increase in output pressure will move the free end of the feedback bellows towards left, bringing in the gap between the flapper and nozzle to almost its original value.

We will first develop the closed loop representation of the scheme and from there the input-output relationship will be worked out. The air is assumed to be incompressible here.

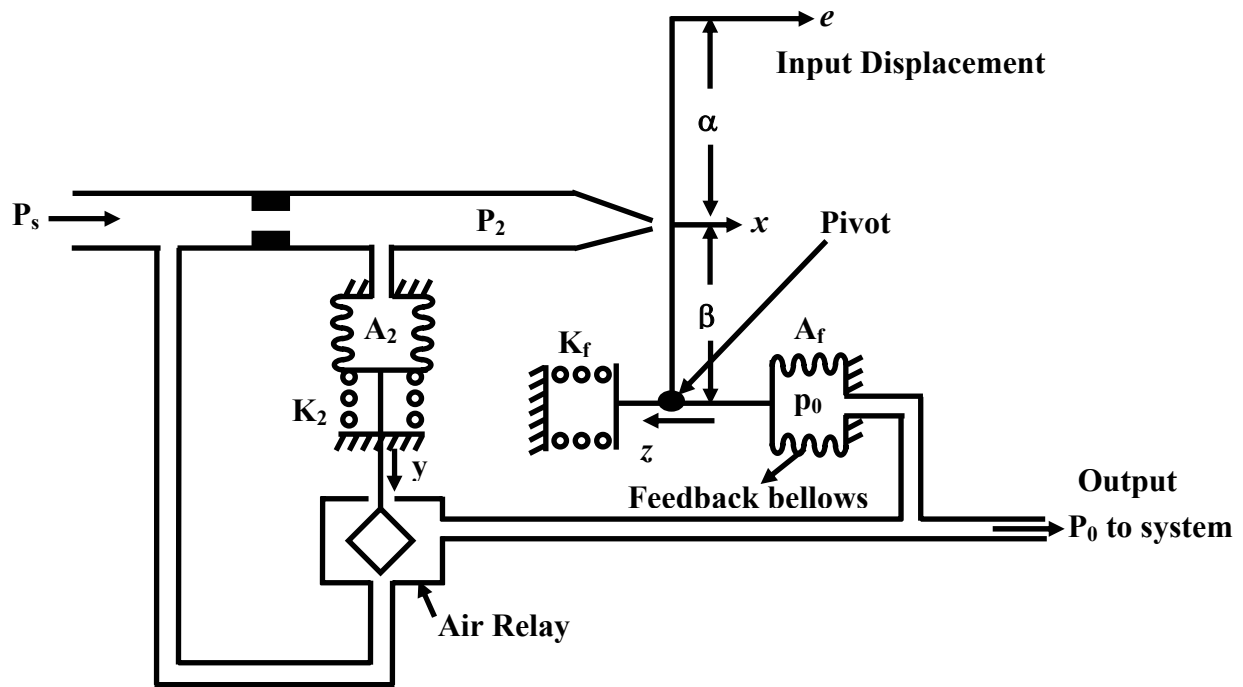


Fig. 1 A pneumatic proportional controller

The first problem we encounter in order to obtain the linearised relationship is the nonlinear characteristics of the flapper nozzle amplifier as shown in Fig.2. The problem can be circumvented by linearising the characteristics over an operating point. Suppose $X_{i,o}$ is the nominal gap between the flapper and the nozzle and the pressure at this operating point is $P_{2,o}$. Any incremental change in displacement X_i (say x) will cause an incremental change in pressure of p_2 . The linearised relationship can be obtained by taking into account of the first order term in Taylor series expansion as:

$$p_2 = \frac{\partial P_{2,o}}{\partial X_{i,o}} x = -K_1 x \quad (1)$$

The term $\frac{\partial P_{2,o}}{\partial X_{i,o}}$ indicates the slope of the flapper nozzle characteristics at the operating point.

This slope can be taken as constant and is denoted here by $-K_1$, indicating a negative slope.

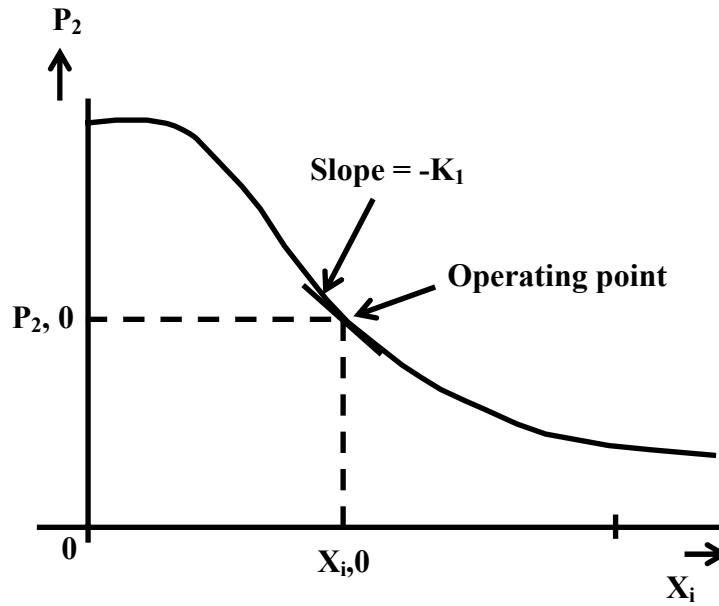


Fig. 2 Characteristics of a flapper nozzle amplifier.

Similarly, for the air relay we can obtain linearised characteristics as given by:

$$p_o = -K_o y \quad (2)$$

indicating decrease in output pressure for positive (downward) y .

Now the two bellows move against two springs. Let the constants of the springs be K_2 and K_f .

We assume the areas of the bellows be A_2 and A_f .

Then,

$$K_2 y = p_2 A_2; \text{ or, } y = \frac{A_2}{K_2} p_2 \quad (3)$$

Similarly,

$$K_f z = p_o A_f; \text{ or, } z = \frac{A_f}{K_f} p_o. \quad (4)$$

In all the cases the variables represented by small letters denote the deviations from the values at the operating points.

Now consider the movement of the link. It has two independent inputs (e and z). Applying superposition, the net displacement signal at the flapper nozzle is given by:

$$x = \frac{\beta}{\alpha + \beta} e - \frac{\alpha}{\alpha + \beta} z \quad (5)$$

Now we can draw the block diagram of the closed loop feedback using the above expressions and as shown in Fig. 3 (a). This block diagram can be further simplified as shown in Fig. 3(b)-(d), where

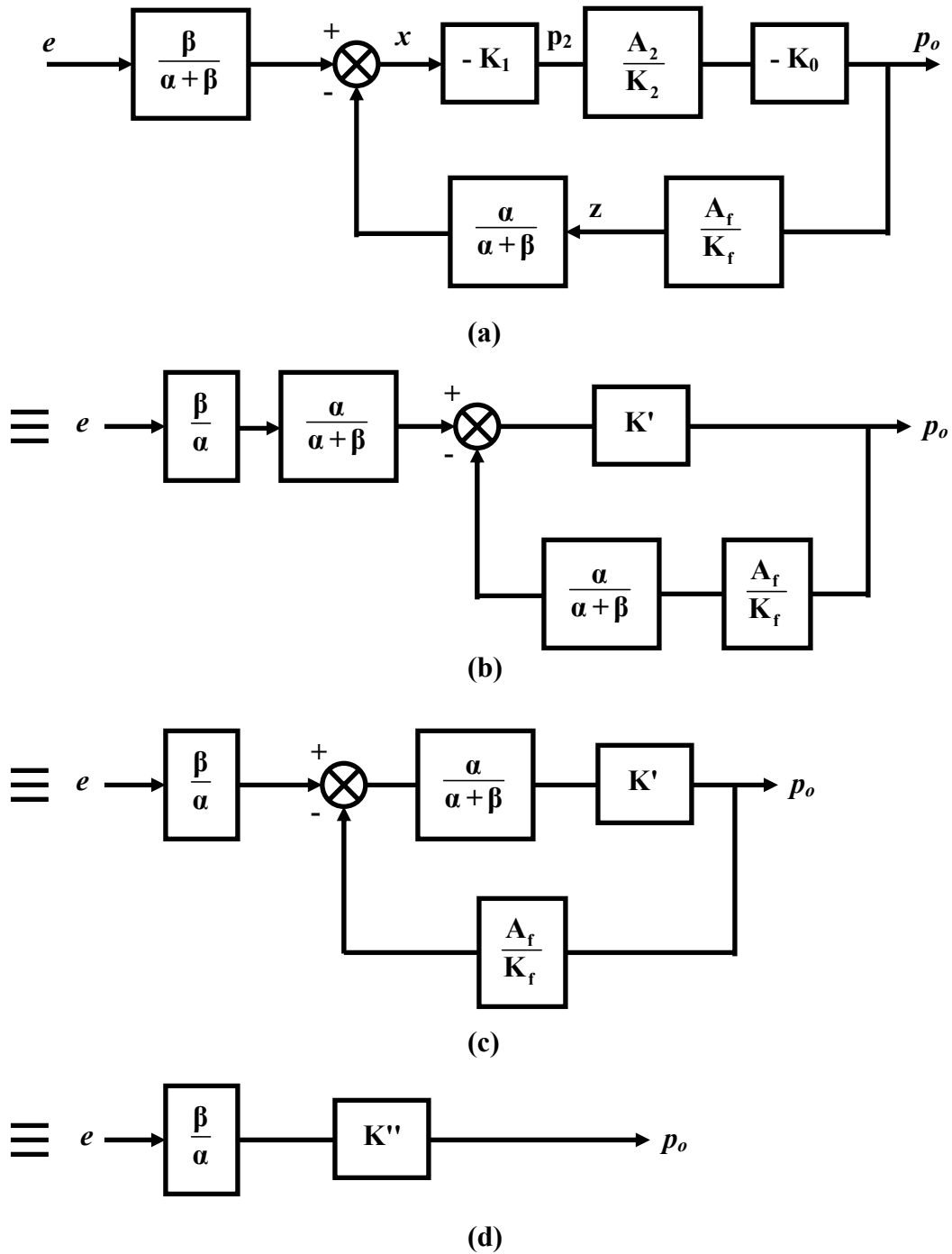


Fig. 3 Block diagram of the closed loop system.

$$K' = K_1 K_o \frac{A_2}{K_2}$$

and

$$K'' = \frac{\frac{\alpha}{\alpha + \beta} K'}{1 + \frac{\alpha}{\alpha + \beta} K' \frac{A_f}{K_f}}$$

If we assume

$$\frac{\alpha}{\alpha + \beta} K' \frac{A_f}{K_f} \gg 1 \quad (6)$$

then

$$K'' \approx \frac{K_f}{A_f}, \text{ and}$$

$$\frac{p_o}{e} \approx \frac{\beta K_f}{\alpha A_f} \quad (7)$$

It can be easily seen that the condition in (6) can be satisfied if the gains of the flapper nozzle amplifier and the air relay are high and the simplified relationship in (7) shows that p_o is proportional to e . Further the proportionality constant depends only on the area of the feedback bellows, the spring constant of the feedback bellows and the distances α and β only. The sensitivity is not dependent on the flapper nozzle and air relay characteristics as long as (6) is satisfied (their gains are indeed high, so it is not difficult to satisfy (6)). The proportional gain can be adjusted by adjusting the distances α and β .

Pneumatic Proportional plus derivative controller

The schematic arrangement of the pneumatic proportional controller is shown in Fig.1. The proportional plus derivative (PD) action can also be generated in a pneumatic controller by introducing a restrictor in the line towards the feedback bellows in Fig. 1. This particular arrangement is shown separately in Fig. 4, all other parts remaining same as in Fig.1. The area of opening of the restrictor is small, so that the time constant associated with changing the pressure inside the feedback bellows is appreciable. In order to explain the generation of PD action, we need to study in detail the performance of the section shown in Fig. 4. Since air inside the feedback bellows is confined, the compressibility of air needs to be considered.

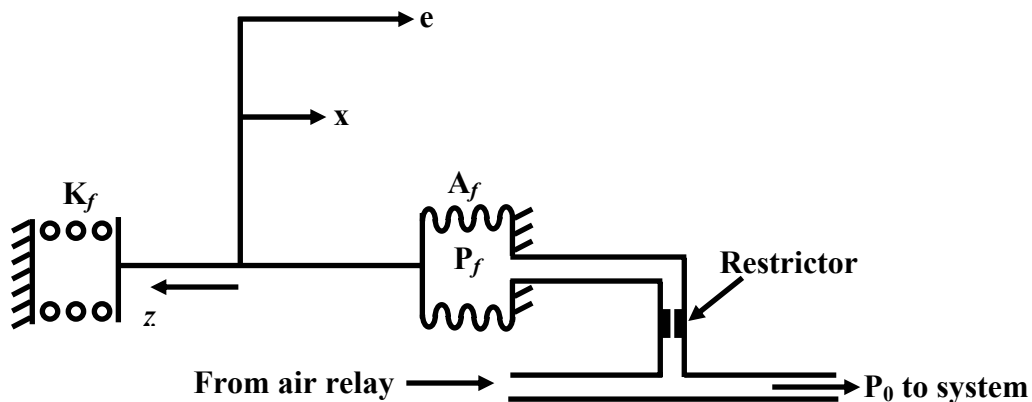


Fig. 4 Feedback arrangement for PD controller

Let P_o and P_f are the pressures before and after the restrictor respectively. The mass flow rate of the fluid G_s through the restrictor is proportional to the square root of the pressure difference between P_o and P_f . In general, we can write:

$$G_s = f(P_o, P_f)$$

The above nonlinear expression can be linearised by considering the incremental changes g_s, p_o and p_f of the values of variables G_s, P_o and P_f at the operating point as:

$$g_s = C_1 p_o + C_2 p_f \quad (8)$$

where,

$$C_1 = \frac{\partial G_{s,o}}{\partial P_{o,o}} \quad \text{and} \quad C_2 = \frac{\partial G_{s,o}}{\partial P_{f,o}}$$

are the slopes of the curves G_s vs. P_o and G_s vs. P_f at the operating point. These values can be obtained either experimentally, or from theoretical considerations.

In fact, $C_1 = -C_2$; this can be ascertained from the fact that if P_o and P_f both change from the operating point by the same amount (so that $p_o - p_f = 0$) there is no change in the pressure drop, and so there will also be no change in mass flow rate and $g_s = 0$. From (8). We obtain $C_1 = -C_2$ and (8) can be rewritten as:

$$g_s = C_1 (p_o - p_f) \quad (9)$$

Further, the pressure P_f inside the feedback bellows can also be obtained from the expression:

$$P_f = \frac{MRT_f}{V_f} \quad (10)$$

where M = Mass of the gas inside the bellows

V_f = Volume of the gas inside the bellows

T_f = Temperature of the gas (constant)

Let m and v_f be the changes in mass of the gas and volume of the gas from the operating point corresponding change in pressure by p_f from the operating point.

From (10), one can also obtain the linearised expression around the operating point as:

$$\begin{aligned} p_f &= \frac{\partial P_{f,o}}{\partial M_o} m + \frac{\partial P_{f,o}}{\partial V_{f,o}} v_f \\ &= C_3 m - C_4 v_f \end{aligned} \quad (11)$$

where C_3 and C_4 are constants. The negative sign associated with C_4 is due to the fact that increase in volume causes decrease in pressure.

Now the change in volume inside the bellows is due to the displacement of the free end, and

$$v_f = A_f z \quad (12)$$

Again the force balance condition at the feedback bellows gives:

$$K_f z = p_f A_f,$$

$$\text{or,} \quad p_f = \frac{K_f}{A_f} z \quad (13)$$

Equating (11) and (13), one can obtain:

$$m = \frac{1}{C_3} \left[\frac{K_f}{A_f} + C_4 A_f \right] z \quad (14)$$

Differentiating the above equation, we obtain:

$$g_s = \frac{dm}{dt} = \frac{1}{C_3} \left[\frac{K_f}{A_f} + C_4 A_f \right] \frac{dz}{dt}. \quad (15)$$

Again equating (9) and(15),

$$\frac{1}{C_1 C_3} \left[\frac{K_f}{A_f} + C_4 A_f \right] \frac{dz}{dt} = p_o - p_f = p_o - \frac{K_f}{A_f} z \quad (\text{from (13)}).$$

$$\text{Or, } \frac{K_f}{A_f} \left[\frac{1}{C_1 C_3} \left(1 + \frac{C_4 A_f^2}{K_f} \right) \frac{dz}{dt} + z \right] = p_o.$$

Taking the Laplace Transformation, we can have:

$$z(s) = \frac{(A_f / K_f)}{1 + s\tau_d} p_o(s), \quad (16)$$

where,

$$\tau_d = \frac{1 + C_4 A_f^2 / K_f}{C_1 C_3}$$

Compare (16) with the relationship obtained z and p_o in (4) in a proportional controller as:

$$z = \frac{A_f}{K_f} p_o.$$

It is clear that the introduction of the restrictor in the feedback bellows introduces a time constant in the feedback path. Further, by varying the restrictor area, C_1 can be changed (refer (8)), thus changing the time constant τ_d . Other parts in the block diagram for P-controller shown in Fig. 3 remains the same.

For the sake of simplicity, let us assume the link lengths $\alpha = \beta$. In that case we can develop the simplified block diagram for the system shown in Fig. 1 with the modified feedback bellows configuration shown in Fig. 4. The simplified block diagram can be expressed as shown in Fig.5, wherefrom we obtain,

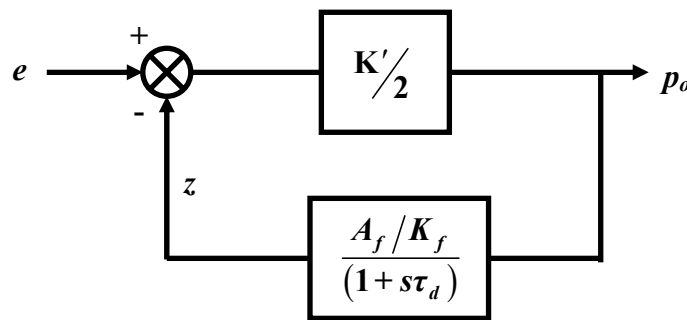


Fig. 5 Block diagram of the P-D controller.

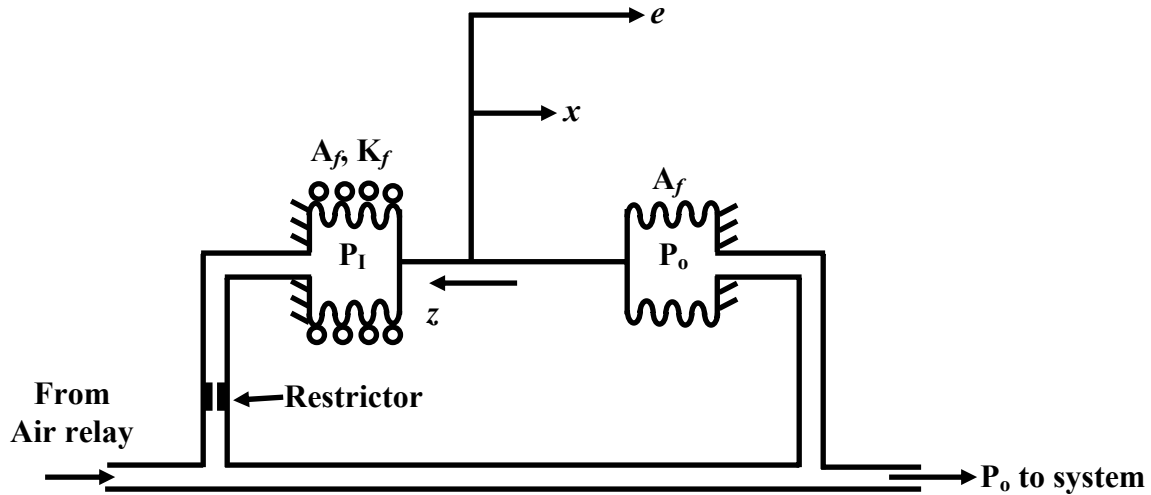


Fig. 6 Feedback arrangement for PI controller

$$\frac{p_o(s)}{e} = \frac{K'/2}{1 + \left(\frac{K'A_f}{2K_f} \right) / (1 + \tau_d s)} = \frac{\frac{K'}{2}(1 + \tau_d s)}{\left(1 + \frac{K'A_f}{2K_f} \right) + \tau_d s} \quad (17)$$

Further, if we assume that,

$$(K'A_f)/(2K_f) \gg \tau_d \text{ and also } \gg 1, \quad (18)$$

then (17) simplifies to:

$$\frac{p_o(s)}{e} \approx \frac{K_f}{A_f} (1 + \tau_d s) \quad (19)$$

that is similar to the transfer function of a standard P-D controller as: $K_p(1 + \tau_d s)$. Thus by simple introduction of a restrictor in the line connecting the feedback bellows can transform the pneumatic P-controller in Fig. 1 to a P-D controller.

Note that with a lagged feedback signal (as seen from the transfer function of the feedback block) the closed loop transfer function provides a net lead, with the same time constant as the lag time constant of the feedback path.

Pneumatic Proportional Plus Integral Controller

Proportional plus Integral (P-I) action can also be generated in a pneumatic controller in a similar way as discussed above for P-D controller, but by adding an additional feedback bellows with restrictor to the left of the link as shown in Fig. 6. There is no restriction in the air line connecting the bellows on the right hand side. There is also a spring surrounding the rhs bellows. We can also develop the equations here for the integrating bellows corresponding to (11)-(13) as:

$$p_I = C_3 m - C_4 v_I \quad (20)$$

$$v_I = -A_f z \quad (21)$$

$$(p_o - p_I)A_f = k_f z \quad (22)$$

Proceeding in a similar way, one can obtain the expression relating z and p_o as:

$$z(s) = \frac{\frac{A_f}{C_1 C_3 K_f} s}{1 + \left(\frac{1 + C_4 A_f^2 / K_f}{C_1 C_3} \right) s} p_o(s) \triangleq \frac{K_I s}{1 + \tau_I s} p_o(s) \quad (23)$$

The closed loop block diagram for P-D controller (shown in Fig.5) now gets modified for P-I controller as shown in Fig.7. The overall transfer function can be obtained as:

$$\frac{p_o}{e}(s) = \frac{(K'/2)}{1 + \frac{(K'/2)K_I s}{1 + \tau_I s}} = \frac{(K'/2)(1 + \tau_I s)}{1 + (\tau_I + \frac{K'K_I}{2})s} \quad (24)$$

If $\frac{K'K_I}{2} \gg \tau_I$ and $\frac{K'K_I}{2} s \gg 1$, then

$$\frac{p_o}{e}(s) = \frac{1}{K_I s} (1 + \tau_I s) = \frac{\tau_I}{K_I} \left(1 + \frac{1}{\tau_I s} \right) \quad (25)$$

The above transfer function is clearly of a P-I controller. The time constant can be varied by varying the restrictor opening. The reader can now easily understand the action of a complete pneumatic P-I controller shown schematically in Fig. 4 of Lesson14.

Here again, the feedback path dynamics (given in (23)) is approximately of a differentiator. The effect is inverted in the closed loop, generating integral action, having a reset time equal to the lag time constant of the feedback path.

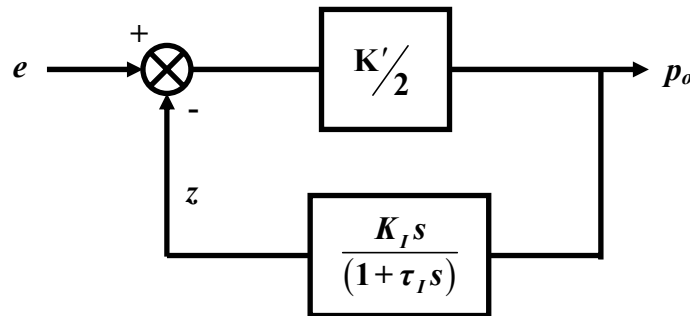


Fig. 7 Block diagram of the P-I controller.

Conclusion

In this lesson we have discussed the working principles of pneumatic P, PD and PI controllers. PID action can also be generated by combining Fig. 4 and Fig.6. The major issues of a pneumatic controller we have learnt are (i) the proportional gain of a pneumatic proportional controller can be adjusted by adjusting the segment lengths α and β of the link, (ii) the proportional gain is independent of the flapper nozzle and relay gains, (iii) the derivative and integral actions can be generated by putting restrictors in the feedback bellows and their time constants τ_d and τ_I can be adjusted by throttling the restrictors. While deriving the working relationship of these controllers, we have also learnt how to linearise the characteristics of a nonlinear linear device. This

knowledge reminds us that all the variables in the transfer functions developed are actually the changes from the nominal operating points and the linearised transfer function is valid for small deviations from operating points only.

Pneumatic control system is free from electromagnetic hazards and the problem of short circuit; thus can be used as an intrinsically safe system where the ambient may contain highly combustible vapours as in a petrochemical and natural gas processing unit. The major limitation of this type of systems are that they are slow in response. Regular maintenance for keeping the control elements free from dust and moisture is also necessary for proper operation of the controllers.

References

1. F.H. Raven: Automatic Control Engineering (4/e), McGraw-Hill, NY, 1987.
2. D.P. Eckman: Automatic process Control, Wiley Eastern, New Delhi, 1958.
3. K. Ogata: Modern Control engineering (2/e), Prentice Hall of India, New Delhi, 1995.

Review Questions

1. What is the function of air relay in Fig.1?
2. Sketch the schematic arrangement of a pneumatic proportional controller and draw the closed loop block diagram.
3. A high gain feedback system reduces the effect of parameter variations on the performance. Can you justify the statement with a pneumatic proportional controller?
4. Identify the element that you should vary for adjusting the derivative time τ_d .
5. A nonlinear expression: $Y = 2X_1^2 X_2$ is linearised over the operating point ($X_1 = 1, X_2 = 1$) and linearised expression is given by $y = C_1 x_1 + C_2 x_2$ (where y, x_1 and x_2 are the deviations from the operating point). Find the values of C_1 and C_2 .

Answer

Q5. $C_1 = 4, C_2 = 2$.

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