Plasticity

1.1 Plastic Deformation, and yield criteria:

1.1.1 States of stress

When a body is subjected to a stress below the yield strength, it will deform elastically. The moment the stress is removed, the body comes to initial position.

In contrast, when the body is stressed beyond the yield point, it will undergo permanent deformation. If it is a ductile material, it will plastically deform continuously with increase in stress applied.

If a certain object is subjected to uniaxial tensile load, it will start yielding – deforming plastically – when the stress reaches the uniaxial yield stress Y.

However, when the state of stress is triaxial, a single shear stress can not be used to predict yielding.

It is the combination of the three stress states which alone can predict yielding.

The relationship among the stresses which predict the yielding of a material is called yield criterion. The inherent assumptions involved in defining the yielding are: the material is isotropic & incompressible, Poisson's ratio equals 0.5 and the hydrostatic or mean stress does not cause yielding of the material. Porous materials like powder metallurgy alloys can be assumed compressible. They have Poisson's ratio less than 0.5.

Commonly, for ductile materials, there are two important yield criteria. They are von Mises yield criterion – also called distortion energy criterion and Tresca criterion also called Maximum shear stress theory.

The hydrostatic stress is given by:

$$\sigma_m = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$$

Total state of stress at a point can be represented as sum of hydrostatic and deviatoric stresses.

For plane stress, the deviatoric stress is given by: $\frac{\sigma_1-\sigma_2}{2}$ etc.

Yielding in normal materials is caused by the deviatoric stress

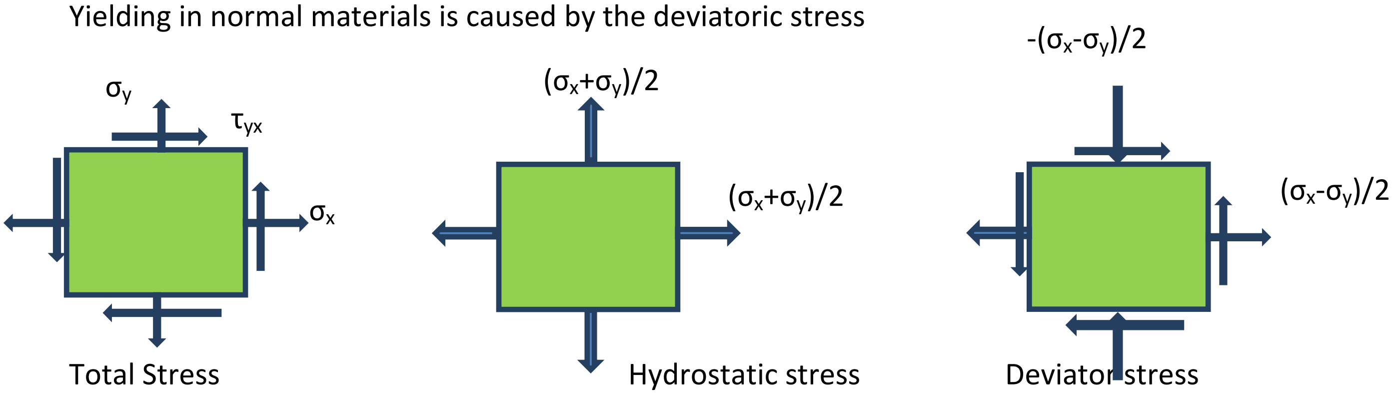


Fig.1.1.1: States of stress on a plane

From the above figures, we could understand that the given state of biaxial stress can be replaced by a sum of hydrostatic and deviatoric stresses. Hydrostatic stress, though does not influence the yielding, it does increase ductility of a material, when it is applied.

1.1.2 Yield criteria:

Commencement of plastic deformation in materials is predicted by yield criteria. Yield criteria are also called theories of yielding. A number of yield criteria have been developed for ductile and brittle materials.

Tresca yield criterion:

It states that when the maximum shear stress within an element is equal to or greater than a critical value, yielding will begin.

$\tau_{max} \geq k$

Where k is shear yield strength.

Or $\tau_{max} = (\sigma_1 - \sigma_3)/2 = k$ where σ_1 and σ_3 are principal stresses

Or $\sigma_1 - \sigma_3 = Y$

For uniaxial tension, we have k = Y/2

Here Y or k are material properties. The intermediate stress σ_2 has no effect on yielding.

Von Mises criterion:

According to this criterion, yielding occurs when

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2 = 6k^2$$

For plane strain condition, we have: $\sigma_2 = (\sigma_1 + \sigma_3)/2$

Hence, from the distortion energy criterion, we have $\sigma_1 - \sigma_3 = \frac{2}{\sqrt{3}}Y$ Here, $\frac{2}{\sqrt{3}}Y$ is called plane strain yield strength. Von Mises criterion can also be interpreted as the yield criterion which states that when octahedral shear stress reaches critical value, yielding commences.

The octahedral shear stress is the shear stresses acting on the faces of an octahedron, given by:

$$\tau_{oct} = \frac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

According to Tresca criteria we know, $(\sigma_1 - \sigma_3)/2 = k$. Therefore,

$k = \frac{Y}{\sqrt{3}}$

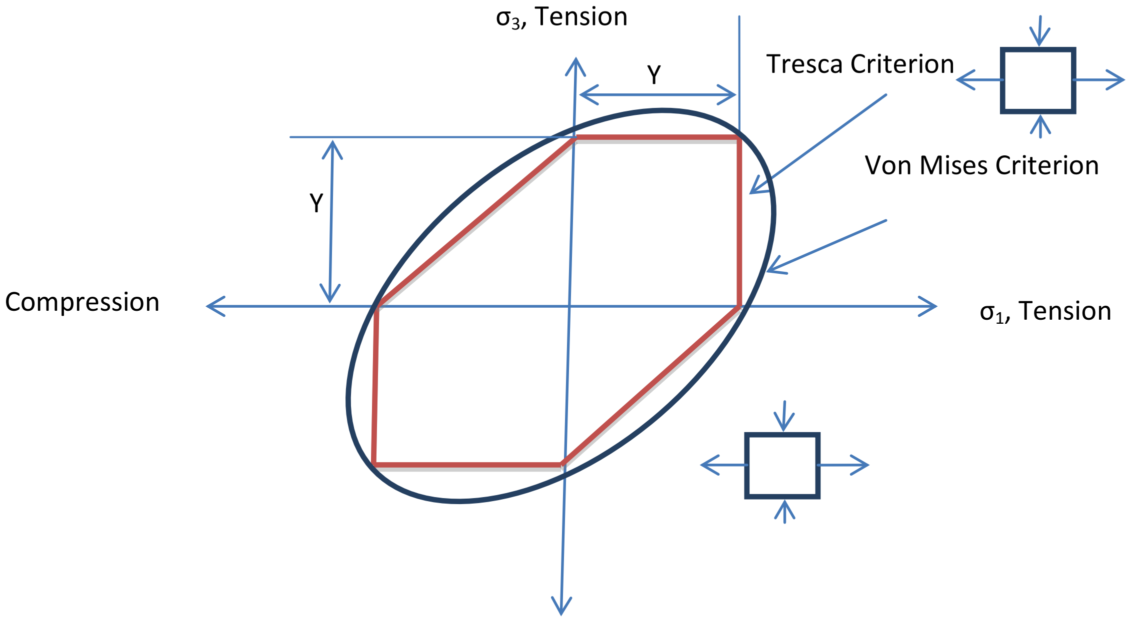


Fig. 1.1.2.1: Yield loci for the two yield criteria in plane stress

Von Mises yield criterion is found to be suitable for most of the ductile materials used in forming operations. More often in metal forming, this criterion is used for the analysis. The suitability of the yield criteria has been experimentally verified by conducting torsion test on thin walled tube, as the thin walled tube ensures plane stress. However, the use of Tresca criterion is found to result in negligible difference between the two criteria. We observe that the von Mises criterion is able to predict the yielding independent of the sign of the stresses because this criterion has square terms of the shear stresses.

1.3 Effective stress and effective strain:

Effective stress is defined as that stress which when reaches critical value, yielding can commence.

For Tresca criterion, effective stress is $\sigma_{eff} = \sigma_1 - \sigma_3$

For von Mises criterion, the effective stress is

$$\frac{1}{\sqrt{2}}\{[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]\}^{1/2}$$

The factor $1/\sqrt{2}$ is chosen such that the effective stress for uniaxial tensile loading is equal to uniaxial yield strength Y.

The corresponding effective strain is defined as:

$$\varepsilon_{eff} = \frac{2}{3}\ (\varepsilon_1 - \varepsilon_3)$$

From von Mises criterion:

$$\text{Effective strain} = (\sqrt{2}/3)\{[(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2]\}^{1/2}$$

For Tresca:

Effective strain = $(2/3)(\varepsilon_1 - \varepsilon_3)$

For uniaxial loading, the effective strain is equal to uniaxial tensile strain.

Note: The constants in effective strain expressions, given above are chosen so that for uniaxial loading, the effective strain reduces to uniaxial strain.

Normal strain versus shear strain:

We know for pure shear: $\sigma_1 = -\sigma_3$ and $\sigma_1 = \tau$

Therefore from the effective stress equation of Tresca we get: Effective stress = $2\sigma_1 = 2\tau_1$

Similarly using von Mises effective stress, we have

Effective stress = $\sqrt{3}\sigma_1 = \sqrt{3}\tau_1$

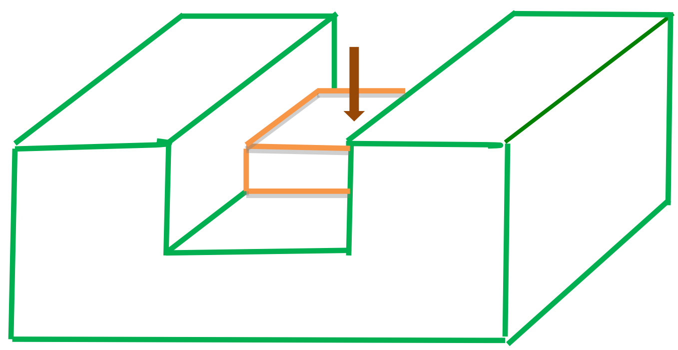


Fig. 4.3.1: A plane strain compression forging process

1.4 Flow rule:

Stress-strain relations for plastic deformation are given by the flow rules, as we can not assume linear relationship between them.

For triaxial stress the flow rules are given by:

$$d\varepsilon_1 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}(\sigma_1 - 0.5(\sigma_2 + \sigma_3)$$

$$d\varepsilon_2 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}(\sigma_2 - 0.5(\sigma_1 + \sigma_3)$$

$$d\varepsilon_3 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}(\sigma_3 - 0.5(\sigma_1 + \sigma_2)$$

For triaxial stress, we can represent the yielding by means of three dimensional representation called yield surface. The yield surface for von Mises criterion is a hollow cylinder which is oriented at equal angle with reference to the three reference axes x, y, z. Here we assume that each axis represents one of the three principal stresses.

Yielding commences when the triaxial stress acting on a body reaches the surface of the cylinder. If the material is a work-hardening material, the cylinder expands as more plastic deformation happens, due to increase in stress required for plastic flow. According to Drucker, the total strain vector should always be normal to the yield surface at any point which corresponds to a given state of stress. Further, it is known that the axis of the yield cylinder is the hydrostatic stress, σ_m. As the total strain is given to be normal to yield surface, the hydrostatic stress, which is normal to deviatoric stress. Deviatoricstress is acting along the direction of total strain vector. Therefore, mean stress can not cause yielding, because it is orthogonal to deviatoric stress.

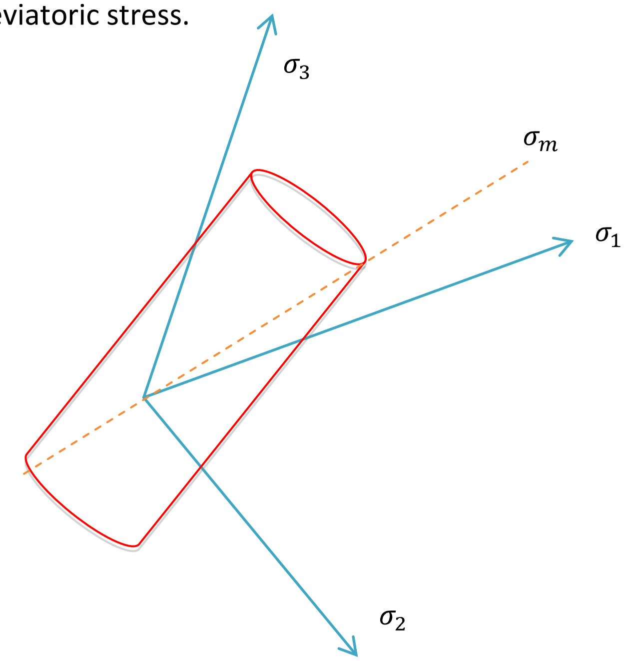
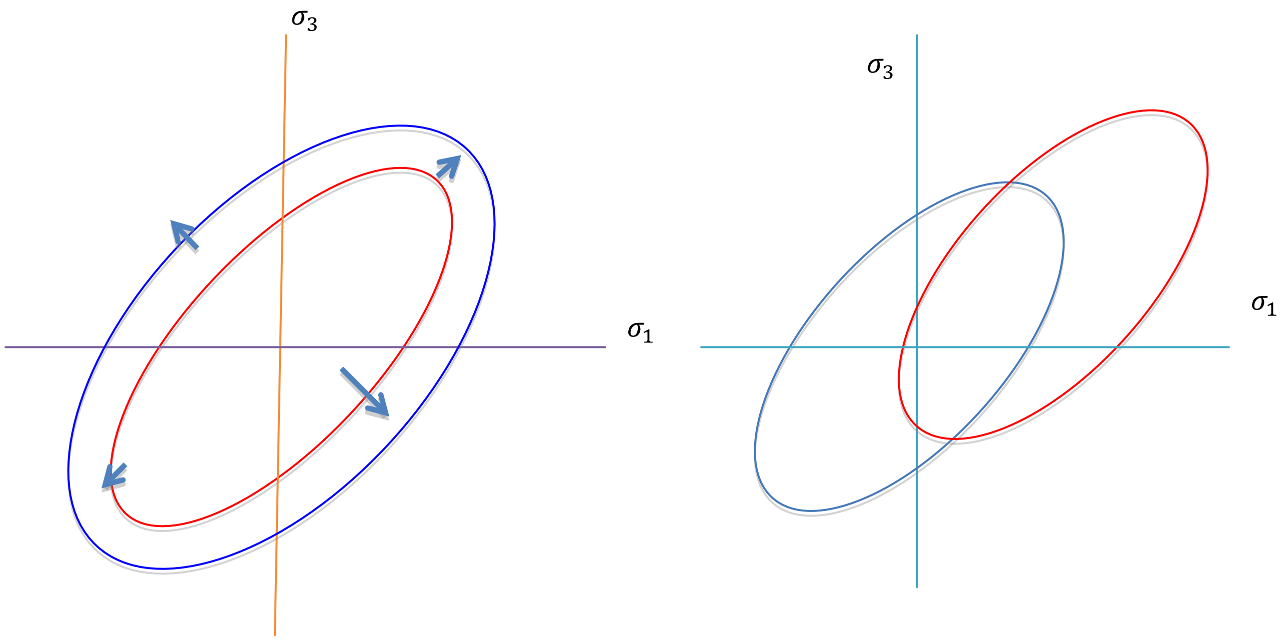


Fig. 1.4.1: Yield surface for a material which obeys von Mises yield criterion

1.5 Work hardening

In plastic deformationof some materials, the material becomes stronger after getting worked upon. Its yield strength increases after plastic working. This is known as work hardening. There are two ways of accounting for the work hardening. They are: Isotropic work hardening, in which yield strength increases uniformly in all directions. The yield locus gets stretched out uniformly all around. The other way is kinematic hardening in which the yield stress does not undergo any increase. On the other hand the yield locus gets shifted in the direction of strain. Predominantly, in plasticity we tend to account isotropic hardening alone.



Isotropic work hardening *Kinematic work hardening*

Fig. 1.5.1: Effect of two types of work hardening on yield locus

Example: Consider a body which is stressed so that it yields. A shear stress of 200 MPa is acting on octahedral plane. What would be the yield strength of the material under tension and shear?

Solution:

We are given the octahedral shear stress. The octahedral shear stress is given by:

$$\tau_{oct} = \frac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

Taking $\sigma_2 = \sigma_3 = 0$ and solving for σ_1, we get:

$\sigma_1 = 424.26$ MPa. This is the tensile yield strength value required.

According to von Mises criterion, we have the relation between tensile and shear yield strengths as:

k = Y/$\sqrt{3}$ = 244.96 MPa. This is the required shear yield strength.

Source:
http://nptel.ac.in/courses/112106153/9