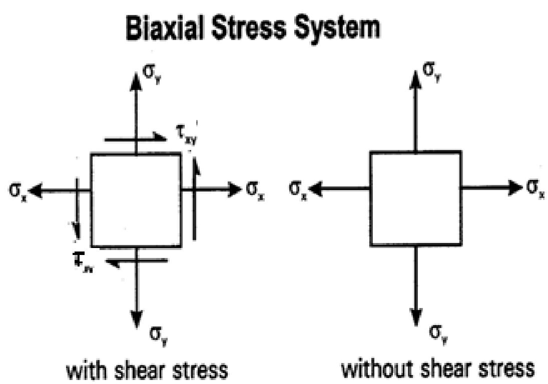
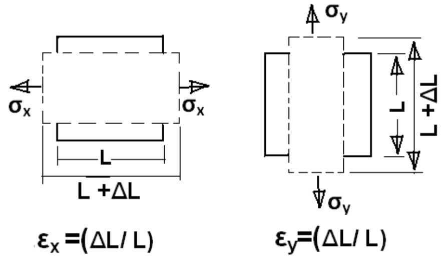
PLANE STRESS, PLANE STRAIN STRESS TENSOR AND BIAXIAL STRESS

Plane Stress
The type of stresses acting on a plane wherein the third direction does not exist is referred to as "Plane Stress". Only two normal stresses will be acting on the element with or without shear stress. σx and σy will be present and no σz.



e there are only two normal stresses is referred to as "Plane Strain".
Corresponding to stresses σ_x and σ_y there will be strains εx and εy acting on the element.



Plane Strain

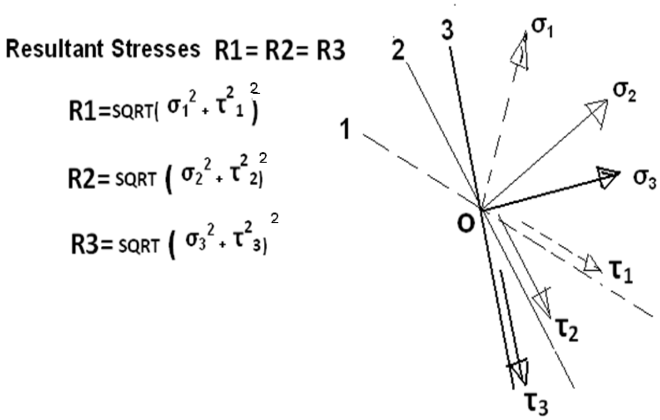
$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \sigma_{ij} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

With Shear Stress Without Shear Stress

Biaxial Stress System

Stress Tensor

At any given point in an element many planes may be drawn, but the resultant forces acting on theses planes are the same. Since the areas and inclinations of these planes are different, the normal and shear stresses on these planes are different.



In order to describe the stress completely
one has to specify the magnitude, direction, sense and also the surface on which it acts.
Hence, stress is generally referred to as "Tensor" or "Stress Tensor".
On an element there will be 3 direct stresses and 6 shear stresses acting.
Remember that we learnt that a force can be resolved into one normal stress and two tangential forces on a given plane, hence the corresponding stresses.
In tensor notation, these can be represented by the tensor σij or τij where i=x,y,z and j=x,y,z.

With shear stresses

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad \begin{aligned} \tau_{xy} &= \tau_{yx} \\ \tau_{yz} &= \tau_{zy} \\ \tau_{zx} &= \tau_{xz} \end{aligned}$$

σ_{ij} represents the state of the stress in a matrix form.

$$\sigma_{ij},\tau_{i,j} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad \begin{matrix} \tau_{xy} = \tau_{yx} \\ \tau_{yz} = \tau_{zy} \\ \tau_{zx} = \tau_{xz} \end{matrix}$$

σ_{ij} and $\tau_{i,j}$ represents the state of the stress in a matrix form.

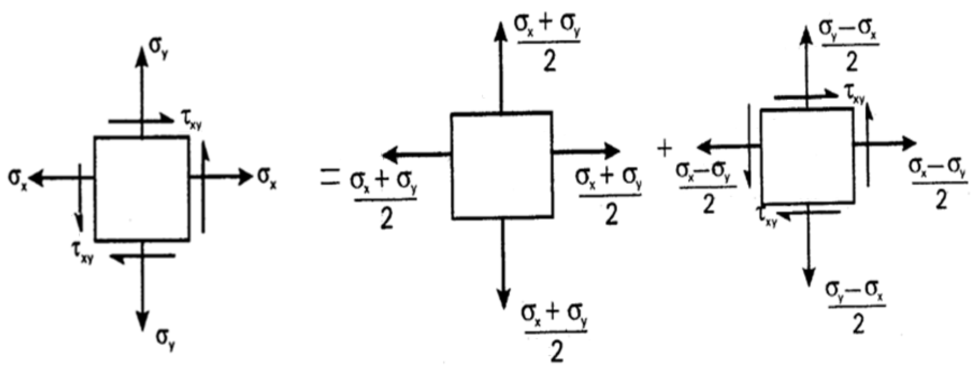
2D Biaxial Stress and Its Composition

Total Biaxial Stress can be split into two parts as shown below. The first part is referred to as Hydrostatic stress (σm) and Stress Deviator (σij).

Total Biaxial Stress = Hydrostatic Stress + Deviator Stress

σ = σm + σij

Hydrostatic Stress(σm) only causes elastic volume changes and does not cause plastic deformation.



Biaxial Stress System with shear stress — Hydrostatic Stress — Stress Deviator

Deviator Stress(σij) involves shear stress, causes plastic deformation, is very important in plastic deformation or mechanical working of metals.

It has been observed that the yield strength of metals is independent of hydrostatic stress, but the fracture strain is strongly influenced by hydrostatic stress.