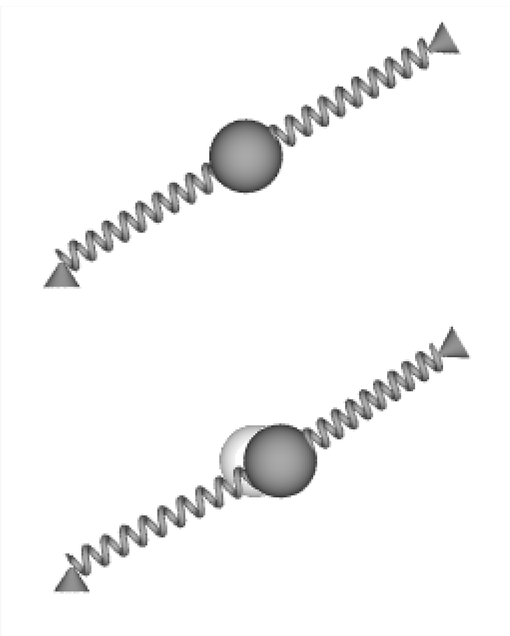
PHYSICS PRINCIPLES OF FOUCAULT PENDULUM

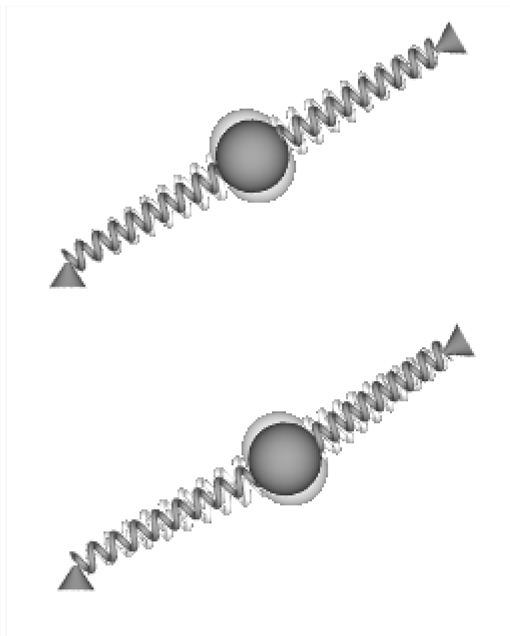
The assumptions for the purpose of simplifying the analysis.

- All of the mass of the bob is taken as concentrated in the midpoint of the bob
- The springs are taken as massless.
- The range of motion of the bob with respect to the rest state is taken to be within the limits of the small angle approximation.
- Only the forces exerted by the springs are considered.

Wheatstone pendulum forces



Picture 6. Image
Top: rest state.
Bottom: outward displacement of pendulum bob when platform is rotating.

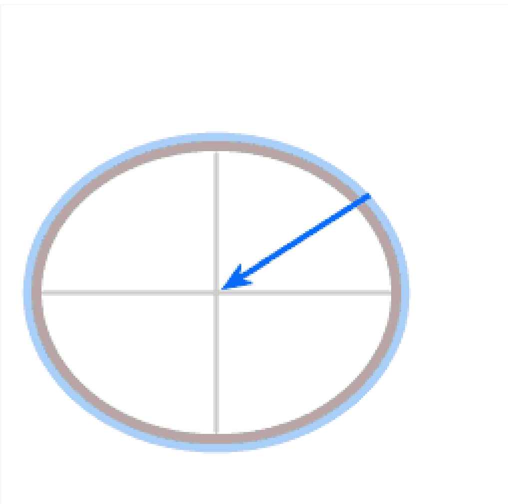


Picture 7. Image
Top: vibration pattern while platform is not rotating.
Bottom: vibration with rotating platform.

Images 6 and 7 show a detail of the Wheatstone pendulum, in different states of motion.

Image 6:
The top side of the image shows the shape of the springs when the Wheatstone pendulum is not in motion. Both springs are equally stretched, and they are aligned. The bottom side of the image illustrates the outward displacement of the pendulum bob when the platform is rotating. The inner spring is more extended than in the rest state, the outside spring is less extended than in the rest state, and thus the springs provide the required centripetal force to make the pendulum bob circumnavigate the central axis of rotation.

Image 7:
The top side of the image shows the shape of the springs when the Wheatstone pendulum is vibrating, but not rotating around the central axis. Both springs are equally stretched, and at the equilibrium point they are aligned. The bottom side of the image shows that when the pendulum bob is vibrating while the platform is rotating the midpoint of the vibration is the outward displaced point of the pendulum bob.



Picture 8. Diagram
The forces in the case of a plumb line.
Blue arrow: the actual gravitational force, pointing towards the center of attraction.
Red arrow: the force exerted by the suspending wire.
Green arrow: the resultant force.

Foucault pendulum forces

Image 8 depicts the forces in the case of a plumb line. It's necessary to distinguish between true gravity and effective gravity. In the diagram the blue arrow represents true gravity, it is directed toward the center of gravitational attraction. On a non-rotating celestial body true gravity and effective gravity coincide, but in the case of a rotating celestial body (due to the rotation deformed to a corresponding oblate spheroid) some of the gravity is spent in providing required centripetal force. (For a quantitative discussion, see the 'differences in gravitational acceleration' section in the Equatorial bulge article.) Due to the rotation the plumb line swings wide. The plumb line's equilibrium point is the point where the angle between true gravity and the tension in the wire provides the required centripetal force.

Of course, since inertial mass is equal to gravitational mass a local gravimetric measurement cannot distinguish between effective gravity and true gravity. Nevertheless, for understanding the dynamics of the situation it is necessary to remain aware that at all times a centripetal force is required to sustain circumnavigating motion.

At 45 degrees latitude the required centripetal force, resolved in the direction parallel to the local surface, is 0.017 newton for every kilogram of mass. In the case of the 28 kilogram bob of the Foucault pendulum in the Pantheon in Paris that is about 0.5 newton of force. If you have a small weighing scale among your kitchen utensils then you can press down on it and feel how much force corresponds to a weight of 50 grams. (2.2 pounds corresponds to 1 kilogram.) Or you can calculate how much centripetal force you are subject to yourself, and then use the scale to feel how much force that is. If you weigh 75 to 80 kilogram you are looking at about 150 grams of force. The angle between true gravity and the direction of the plumb line provides that force.

The angle between true gravity and a plumb line at 45 degrees latitude is about 0.1 of a degree. For the Foucault pendulum in the Pantheon, with a length of 67 meters, the corresponding horizontal displacement is 0.11 meter. (Interestingly, for the pendulum in the Pantheon Foucault has documented that on occasion there was opportunity for uninterrupted runs of six to seven hours. As far as he could tell the precession rate remained the same. William Tobin writes in his biography of Foucault that according to calculations after 7 hours friction will have reduced the amplitude of the swing to about 0.1 meter.)

Decomposition of the force

1.50
0.20
Picture 9. Graphlet

Picture 9 is an interactive diagram. The red dot is draggable. The two horizontal lines are sliders.

The blue dot represents the central rotation axis. The green dot is circumnavigating the blue dot. The green vector represents the actually exerted force. For simplicity this force is treated as a perfect harmonic force; a force that is at every point proportional to the distance to the midpoint. The blue vector represents required centripetal force.

The brown vector is the green vector *minus* the blue vector. The grey dot represents the displaced midpoint of the vibration. The brown vector can be thought of as a *restoring force* towards the displaced midpoint of the vibration.

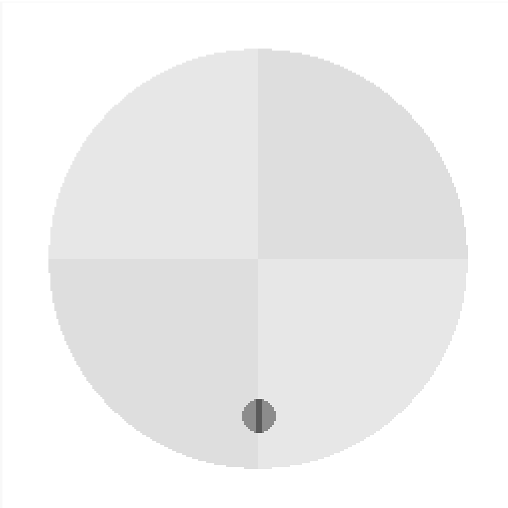
The sliders adjust the size of the required centripetal force and the actually exerted force respectively.

Purpose of the decomposition

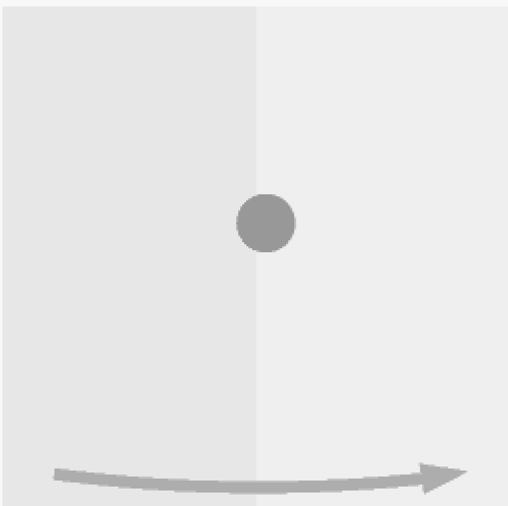
The derivation of the equation of motion further down in this article uses the components, not the actually exerted force. The advantage: in the equation of motion for the rotating coordinate system the required centripetal force term and the centrifugal term drop away against each other.

Exchange of angular momentum

Picture 10. Image
A Wheatstone pendulum setup with the attachment points aligned with the central axis of rotation.



Picture 11. Animation
Motion of the bob of the device depicted in Image 9.



Picture 12. Animation
Schematic representation of the motion of the bob as seen from a co-rotating point of view.

I will first discuss the motion pattern of the device depicted in image 10.

Let the platform be rotating counterclockwise. To underline the comparison with the Foucault pendulum I will call the the bob's motion in a direction tangent to the rotation 'east-west motion'.

Motion towards and away from the central axis

At every point in time, the bob has a particular angular momentum with respect to the central axis of rotation. When the bob is pulled closer to the central axis of rotation the centripetal force is doing mechanical work. As a consequence of the centripetal force doing work the angular velocity of the bob increases. (Compare a spinning ice-skater who pulls her arms closer to her body to make herself spin faster.) When the pendulum bob is moving away from the central axis the centripetal force is doing negative work, and the bob's angular velocity decreases. The animation illustrates that the effects during the halfswing towards the central axis and halfswing away from the central axis do not cancel each other; the combined effect is cumulative.

Velocity relative to co-rotating motion

The amount of centripetal force that is exerted upon the pendulum bob is "tuned" to the state of co-rotating with the system as a whole. I will refer to the velocity that corresponds to co-rotating with the system as a whole as 'equilibrium velocity'. During a swing of the pendulum bob in eastward direction the bob is circumnavigating the central axis faster than the equilibrium velocity, hence during an eastward swing the bob will swing wide. During a swing of the pendulum bob in westward direction the bob is circumnavigating the central axis slower than the equilibrium velocity, hence during a westward swing there is a surplus of centripetal force, which will pull the bob closer to the central axis. The animation illustrates that the effects during the halfswing from east-to-west and the halfswing from west-to-east do not cancel each other; the combined effect is cumulative.

Java simulation

To further illustrate the symmetries that underly the motion of animation 11 I have created the following 2D Java simulation:Circumnavigating pendulum

Transfer

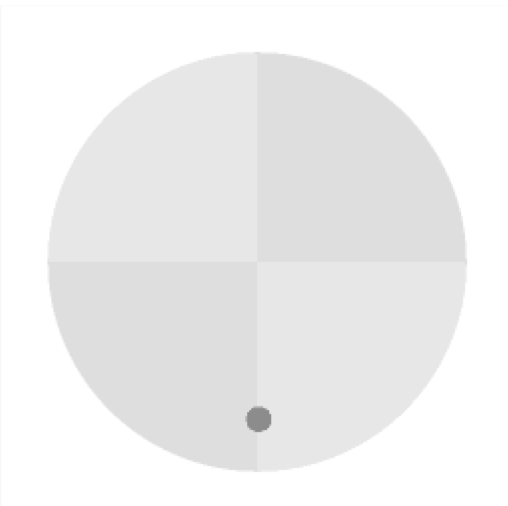
Swinging towards and away from the central axis is a vibration in radial direction. Swing in tangential direction superimposed on the circumnavigating motion is a vibration in tangential direction. The pendulum bob and the pendulum support are exchanging angular momentum all the time.

Of course, in the case of a Foucault pendulum the entire Earth is effectively the pendulum support. Obviously since the Earth is so much heavier than the pendulum bob the Earth's change of angular momentum is utterly negligable. But as a matter of principle change of (angular) momentum is always in the course of an *exchange* of (angular) momentum.

Animation 11 shows a remarkable symmetry: the direction of the bob's oscillation with respect to inertial space remains the same, just as in the case of a polar Foucault pendulum!

In the case of a polar Foucault pendulum the fact that the direction of the plane of swing remains the same is due to straightforward conservation of momentum. In the case depicted in image 10 and animation 11 there is a precession relative to the rotating system that precisely cancels the system's rotation. In the section in which I discuss the mathematical treatment I show how that occurs.

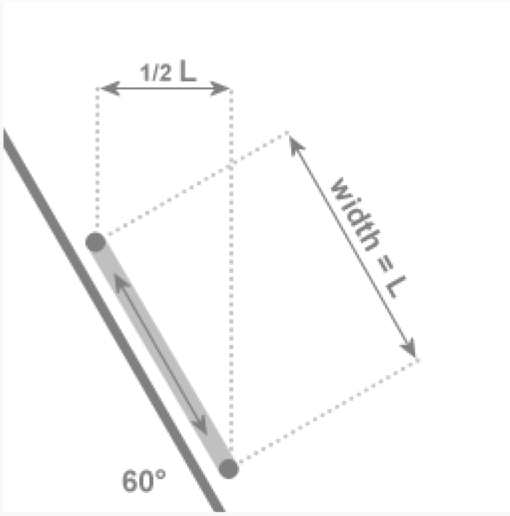
The sine of the latitude



Picture 13. Animation
Motion of the bob of the Wheatstone pendulum as seen from straight above the platform. Positioning of the springs (the springs are not shown) corresponds to 30 degrees latitude.

Animation 13 shows the case of the Wheatstone pendulum when it is set to model a Foucault pendulum at 30 degrees latitude. At 30 degrees latitude a full precession cycle takes two days. Compared to the case represented in animation 10 the precession relative to the co-rotating system is slower.

The closer to the equator a Foucaut pendulum is located, the slower its precession relative to the ground it is suspended above. At the equator the plane of swing is completely co-rotating with the rotation of the overall system.



Picture 14. Image
In the case of an angle of 60 degrees with the central axis of rotation: If the two extremal points of the swing are a distance of L apart, then the motion towards and away from the central axis covers a distance of 1/2 L

Image 14 illustrates why the precession is slower at lower latitudes.

The amount of precession (relative to the co-rotating system) is determined by the exchange of angular momentum. At 30 degrees latitude: when the swing from one extremal point to the opposite extremal point covers a distance of L, then the motion towards (or away from) the central axis is over a distance of 1/2 L. So in the case of a setup at 30 degrees latitude the force is half as effective, and the precession is half the rate of a Foucault setup near the poles.

Java applets

I have created a 3D simulation that models the Foucault pendulum

Applicability

The ratio of vibrations to rotations
In actual Foucault setups the ratio of vibrations to rotations is in the order of thousands to one. In the examples given in this article the ratio of vibrations to rotations is in the order of ten or twenty to one. Interestingly, this difference is inconsequential. In both ranges the physics principles are the same; the considerations that are presented are valid for all vibration-to-rotation ratio's. However, below a ratio of around 10 to 1 and approaching 1 on 1 the motion associated with the rotation and the motion associated with the vibration blur

into each other so much that there is no meaningful Foucault effect to be observed.

The amplitude of the vibration
Images 6 and 7 depict a case where the amplitude of the vibration is slightly smaller than the outward displacement of the bob; in the usual setup the amplitude of the vibration is larger than the displacement, much larger. Does this matter? It does not matter in the following sense: one of the properties of the Foucault pendulum is that its precession does not depend on the amplitude of the swing; when the amplitude of a Foucault pendulum decays the rate of precession remains the same. The Java simulations Foucault pendulumand Foucault rod illustrate the property that the rate of precession is independent from the vibration amplitude.

Source : http://www.cleonis.nl/physics/phys256/foucault_pendulum.php