PREDICTION OF PRESSURE DROP IN HELICAL COIL WITH SINGLE PHASE FLOW OF NON-NEWTONIAN FLUID

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Abstract: The pressure drop of single phase was studied experimentally with non-Newtonian fluids of Carboxy Methyl Cellulose (CMC). Single helical coil with five different helix angles were used in this study to identify the effect of helix angle on pressure drop. The effect of helix angle is significant in moderate and high generalized Dean’s number of laminar region. The effect is insignificant in low generalized Dean’s number range and turbulent region. Correlations were developed for predicting the frictional pressure drop in laminar and turbulent regions.

Key words: non-Newtonian fluid, rheological properties, generalized Reynolds number, Deans number, friction factor

INTRODUCTION:

Helical coils are used widely in processing industries for cooling and heating applications since the centrifugal forces experienced by the fluid acts to promote contact with the channel wall, thereby tending to insure good contact with the wall, and an enhanced heat transfer. Some of their main advantages over straight tubes are high heat and mass transfer coefficients, and space economy in terms of area per unit volume. Most of the fluids encountered in processing applications do not adhere to the Newtonian law of viscosity and hence these fluids known as Non-Newtonian fluids. Typical examples include plastics, suspensions, polymeric melts, foods, dyestuffs, and pharmaceuticals and the other multiphase mixtures such as foams, emulsions, etc.

Coils are used in chemical reactors, agitated vessels, and storage tanks to heat and cool process streams ranging from inorganic and organic chemicals to dairy products; Furthermore, such coils are also used for recovering heat from waste liquors, vapours and gases. Curved passage can be used to improve mass and heat transfer rates such as membrane blood oxygenators, in kidney dialysis devices, and in reverse osmosis units. When a fluid flows in a curved structure, there exists a secondary flow due to the action of centrifugal force as shown in Fig 1, and this has attracted much attention [1], [2],[3],[4],[5],[6],[7]. Such a secondary flow results in high frictional loss in curved pipes than that in straight pipes under similar conditions, such as flow rate, temperature, pressure.

Fig 1. Schematic diagram of secondary flow in helically coiled tube (P1 and P2 are two ends).

Experimental

Newtonian fluids: A number of equations have been proposed to calculate the axial pressure drop in coils of constant curvature for both laminar and turbulent conditions of single phase flow. Most of these equations endeavour to predict either the usual Fanning friction factor for coils, $f_c$, or the ratio of friction factors for the straight, $f_s$, and coil configuration, $f_c$. The single-phase friction factor can be obtained from the equation for fluid flow through a straight tube after replacing the Fanning friction factor $f_s$ by $f_c$.

$$\Delta P = \frac{2f_c \rho V^2}{d} \quad (1)$$

Beginning of the 20th century itself the theoretical work [2], dimensional analysis [3] was attempted to solve the flow of fluid through helical coil. Many correlations have been available to calculate the value of the friction factor $f_c$, and a selection of commonly used such correlations is given here. White (1932) [8] extended the Dean’s analysis and obtained a relationship for the pressure head loss in coils i.e.

$$f_c = f_s \left(1 - \left(1 - \left(\frac{11.6}{D_e}\right)^{0.45} - \left(\frac{1}{\sqrt{D_e}}\right)^{3.2}\right)\right)^{-1} \quad (2)$$

where $D_e = \text{Re} \ (d/D)^{1/2}$, the validity of the equation (2) is in the range $11.6 \leq D_e \leq 2000$ while [9],[10] obtained a relation for $f_c$ for $D_e > 1$ respectively

$$f_c = f_s \left[1 + 0.033 \left(\frac{1}{\log_{10} D_e}\right)^{1.0}\right] - (3)$$

$$f_c = f_s \left[1 + \left(0.09 D_e^{1.5} / (70 + D_e)\right]\right] (4)$$
The critical Reynolds number may be defined as the highest value of the Reynolds number for which the flow in helix is still in the viscous regime. Many researchers [6],[11],[12] have proposed the empirical equations for critical Reynolds number. The widely accepted equation for critical Reynolds number over the entire range of d/D values which is given below [12].

\[ Re_{\text{critical}} = 2100(1 + 12(d/D)^{1/2}) \]  \hspace{2cm} (5)

For turbulent flow, several empirical equations for friction factor are available, correlating the values of pressure drop over a large range of Reynolds numbers. Most of these relations are based on either \( f_c/f_s \) or \( f_s \) and Dean’s number(De). The following [6] correlation was used widely.

\[ f_s = f_c + 0.01(d/D)^{1/2} \]  \hspace{2cm} (6)

**Non-Newtonian Fluids:**

Many Researchers have attempted ([13],[14]) non-Newtonian fluid flow in curved pipes. The effect of curvature on the flow of non-Newtonian fluids in helical coils is significant and can be represented as shown below [15].

For laminar region:

\[ 0.758 \leq n \leq 0.854; \quad \alpha = 3.14^0, 3.24^0 \]

Some [17] have studied the flow of water and several pseudoplastic polymeric solutions in helical coils of various curvature ratios with constant helix angle. Many researchers proposed the correlations for predicting the coil friction factor. The above correlations are widely accepted and predicted the experimental results well within the range of Deans number. Most of these correlations have not considered the effect of helix angle. The angle may vary from 0\(^0\) for circular helix to 90 \(^0\) for L bend. Some studies [18] focused on the effect of this helix angle on frictional pressure drop. Numerical simulations also tried to predict the flow behaviour [19],[20].

**Methods and Materials**

The experimental set up used in the present study is schematically shown in figure 2. The experimental setup consists of four sections consists of storage tank, liquid flow measuring devise, flow index measuring device, test section. The pressure drop was measured by using carbon tetrachloride and mercury filled ‘U’ tube manometers. The liquid flow rate was measured by using carbon tetrachloride and mercury measured by using carbon tetrachloride and mercury.

\[ \left( \frac{f_c}{f_s} \right) = 1.23(De)^{0.3} \]  \hspace{2cm} (7)

\[ 10 < De < 2000; 60 < Re < 6000; \]

\[ 0.7033 \leq n \leq 1.05 \quad (Re^\ast = \text{generalized Reynolds number}) \]

Critical Reynolds number is as stated in equation (5) provided Re is replaced by Re^\ast.

For turbulent region:

\[ f_s = \frac{0.079}{n^4} \left[ \frac{2.63}{Re^{10.57}} \right] + 0.012 \left( \frac{d}{D} \right)^{1/2} \]  \hspace{2cm} (8)

For 6000 < Re^\ast < 30000

These equations are qualitatively consistent with the theoretical analyses,[13],[14]. Few studies [16] have concentrated on the effect of curvature ratio on pressure drop and proposed the following correlation in laminar region (10 ≤ De ≤ 2300).

\[ \left( \frac{f_c}{f_s} \right) = 1 - 0.026(De)^{-0.675} \]  \hspace{2cm} (9)

measured by a calibrated rotameter. In the present study, five different test sections were used with five different helix angles. These five different sections were made using a hose pipe, of 25.3 mm in ID and 31 mm in OD. All these sections are wrapped on a wrought iron pipe, of 220 mm OD. Liquids were pumped through Centrifugal pump of 1 HP. Two pipeline viscometers (capillary viscometers) were used to measure the rheological properties of the non-Newtonian fluids. The inside diameter of the tube is ½”, 1” and the length across which pressure drop was measured is 2.4 m. The length to diameter ratios maintained to make the end effects negligible. The ½” tube was used for more viscous fluids while the 1” tube was used to measure the flow behaviour of less viscous fluids. In order to investigate the functional relationship between the friction factor and Reynolds number for non-Newtonian fluids and the other system parameters, the rheological properties were evaluated first. Then the flow experiments in the test coils were performed. The systematic procedure followed for obtaining the desired data is given below.

Non-Newtonian (CMC) liquids were used as test fluids. The viscometer data in terms of flow rate and pressure drop, from therotameter and manometer respectively, were converted to wall shear stress – nominal wall shear rate form and were plotted on a log-log scale. This method was used to determine the...
values of the apparent consistency index \( (K') \) and flow behaviour index \( (n') \). While conducting the experiments, it was observed that the temperature of the fluid increases due to the pumping action. This effect was appreciable for high concentration solutions owing to their high viscosity. A study of flow behaviour changes with temperature was also made to assess the extent of this phenomenon. The helix angle of each test section was measured by using a vernier calipers. The pitch for each test section is constant and has values of 31.2 mm, 45.17 mm, 76.5 mm, 90.1 mm, 113.9 mm and using this principle the coil angle was measured \((2.6, 3.7, 6.3, 7.4, 9.4)\). The densities of the solutions were measured by using specific gravity bottles.

There was little effect of temperature on the flow behaviour index; on the other hand, it had more pronounced effect on consistency index as seen in the aforementioned figures. The flow behaviour index and consistency constant decreased with increasing temperature. The degrees of non-Newtonian behaviour increase with increasing polymer concentration.

Typical rheological properties of different concentrations of CMC solutions are tabulated in Table 1 at \( 293 - 299^\circ K \) and Table 2 shows the effect of temperature on these properties.

![Fig 2. Schematic experimental setup](image)

**Results and Discussion**

**Rheological Properties of non-Newtonian fluids:** The CMC solution was pumped through capillary viscometer and measured the flow rate and pressure drop. These are transformed into useful parameters as given in equation (10).

\[
\frac{d\Delta P}{4L} = K \left( \frac{8V}{d} \right)^n
\]  

These data plotted on a log-log graph determine the consistency index \( K \) and flow behaviour index \( n \). The effect of temperature on rheological properties is shown in Fig 3a and 3b for CMC solutions.

![Fig 3a. Variation of Rheological properties of CMC solution](image)

![Fig 3b. Effect of temperature on flow behavior index and flow consistency index](image)

**Table 1:** Rheological properties of CMC at 293 - 299 \(^\circ\) K

<table>
<thead>
<tr>
<th>S. No.</th>
<th>CMC con.</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( K ) (Pa.s(^n))</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2%</td>
<td>1006</td>
<td>2.103</td>
<td>0.568</td>
</tr>
<tr>
<td>2</td>
<td>1.8</td>
<td>1005</td>
<td>1.1004</td>
<td>0.638</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>1004</td>
<td>0.391</td>
<td>0.736</td>
</tr>
<tr>
<td>4</td>
<td>1.25</td>
<td>1002</td>
<td>0.113</td>
<td>0.831</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>999</td>
<td>0.056</td>
<td>0.863</td>
</tr>
<tr>
<td>6</td>
<td>0.75</td>
<td>998</td>
<td>0.0315</td>
<td>0.922</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>997</td>
<td>0.007</td>
<td>0.951</td>
</tr>
<tr>
<td>8</td>
<td>0.25</td>
<td>996</td>
<td>0.0067</td>
<td>0.976</td>
</tr>
</tbody>
</table>

**Table 2:** Effect of temperature on rheological properties of CMC solution

<table>
<thead>
<tr>
<th>S. No.</th>
<th>CMC con.</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( K ) (Pa.s(^n))</th>
<th>( n )</th>
<th>Temp ((^\circ) K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2%</td>
<td>1006</td>
<td>2.103</td>
<td>0.568</td>
<td>296</td>
</tr>
<tr>
<td>2</td>
<td>2%</td>
<td>1005</td>
<td>0.854</td>
<td>0.645</td>
<td>305</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>1005</td>
<td>1.1004</td>
<td>0.638</td>
<td>297</td>
</tr>
<tr>
<td>4</td>
<td>1.8</td>
<td>1005</td>
<td>0.8543</td>
<td>0.688</td>
<td>305</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
<td>1004</td>
<td>0.391</td>
<td>0.736</td>
<td>297</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>1002</td>
<td>0.24</td>
<td>0.753</td>
<td>305</td>
</tr>
</tbody>
</table>
Dimensionless pressure gradients are usually expressed as friction factors and velocities are expressed as Reynolds number. The relation between pressure gradient and mass flux is expressed in dimensionless form as a relation between the friction factor and Reynolds number. These relations also can express as plots such as Moody plot. Depends on fluid nature this may be partitioned into three regions: laminar, transition and turbulent. Present study has been restricted to Laminar and turbulent flow of Non-newtonian fluid flow in straight pipe and helical coil.

**Straight Pipe Friction factor:** The Reynolds number was modified as generalized Reynolds number for non-newtonian fluids incorporating the effect of flow behaviour and flow consistency index. Variation of friction factor with generalized Reynolds number is shown in Fig 4 for Laminar flow region. As the generalized Reynolds number increases the friction factor decreased and turbulent region the friction factor almost stabilized with increase in generalized Reynolds number and shown in Fig 5. Present authors tested the theoretical relation of friction factor and Reynolds number in Laminar region as specified in Equation (11) and plotted the predicted and experimental results in Fig 6. The predicted friction factor is in well agreement with experimental data with an RMS error of 19% while the predicted friction factor in turbulent region was having an RMS error of 21% with standard correlation of smooth pipes given in equation (12).

\[
f_s = \frac{16}{Re'} \quad \ldots (11)
\]

\[
f_s = 0.0014 + \frac{0.125}{Re^{0.36}} \quad \ldots (12)
\]

**Helical Coil friction factor:** For Helix angle of 3.7, the experimental data of pressure drop was converted to coil friction factor by using the equation (1) and modified Dean’s number. In laminar region these experimental values are compared with the existing correlations[6],[15],[16] by replacing the De with modified De. As seen the present results are at variation with the findings [15], but are relatively closed to that of [6],[16]. Furthermore there does not appear to be any effect of n.

Fig 4. Variation of friction factor with generalized Reynolds number in Laminar flow conditions

Fig 5. Variation of friction factor with generalized Reynolds number in Turbulent flow conditions

Fig 6. Comparison of estimated and experimental friction factors

Fig 7. Comparison of Experimental results with Literature Correlations for same Helix angle.
**Effect of Helix angle**: To identify the effect of Helix angle on coil friction factor in laminar region, the authors have selected various angles of helix angle, the data was represented in the below Fig’s 8 to 10. The effect of helix angle is significant at moderate to high generalized Reynolds number (Re') in the laminar region while at low Re' the effect is negligible for various concentrations of CMC solution ranging from 0.25% to 2%.

It is clearly evident that the existing correlations need to modify/incorporate the effect of helix in the correlation. The turbulent results are shown in the Fig 11, the helix angle effect is negligible. Probably the flow is stabilizing in the turbulent region. Present author has proposed a correlation to account the Helix angle in the laminar region and without helix angle in turbulent region and the equations (13) and (14) are given below.

\[
\frac{f_c}{f_s} = 0.015(De')^{0.75} + (\sin \alpha)^{0.25} \quad \text{(13)}
\]

\[
\frac{f_c}{f_s} = 6.2(De')^{-0.2} \quad \text{(14)}
\]

The predicted values are in excellent agreement with the experimental values and shown in Fig 12. The RMS error for laminar region is 24% while turbulent it is 16%.
CONCLUSIONS:

The following conclusions can be drawn from the study:

a) The effect of Temperature on Flow consistency index and flow behaviour index was identified.

b) The friction factor and generalized Reynolds number relations in straight tube was successfully followed the theoretical relation in laminar region while turbulent region the usual correlation of Newtonian fluids followed.

c) The effect of helix angle on friction factor was identified and modified the existing correlations to incorporate the effect of helix angle in moderate and high Re’ in laminar region.

d) The helical coil effect is negligible in low Re’ of Laminar and Turbulent region.

Nomenclature

d- diameter of coil (m)
D- diameter of Helix (coil support) (m)
De- Dean number (Re(d/D)0.5)
f- coil friction factor
f- linear pipe or straight friction factor
K‘ -flow consistency index, (Pa.s^n)
n’ - flow behaviour index
P- pressure drop (Pa)
Re –Reynolds number (dv/ν)
Re‘-Generalized Reynolds number (d^3√(ν/1-K’))
V – velocity(m/s)

Greek letters
θ- Helix angle
ρ- density (kg/m^3)
ν- viscosity (pa.s)

REFERENCES