Optimal Two-Time Scale Controller for a Missile Autopilot

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Abstract
In this paper, two-time scale method is used to control a 6-DOF missile autopilot. The mathematical model is developed using the singular perturbation theory. Structural properties of the system in terms of the slow and fast subsystems are established. An asymptotically optimal two-time scale controller was developed for missile. The two-time scale controller was compared to the optimal controller, and it was demonstrated that there is negligible degradation in performance.

Keywords
Missile Autopilot, Singular Perturbation, Two-Time-Scale, Kalman Filter, Optimal Control

I. Introduction

Over the years a number of authors have considered modeling, analysis and design of autopilots for atmospheric flight vehicles including guided missiles. In the majority of the published work on autopilot analysis and design, locally linearized versions of the model with decoupled airframe dynamics have been considered. This latter simplification arises out of the assumption that the airframe and its mass distribution are symmetrical about the body axes and that the yaw, pitch and roll motion about the equilibrium state, remain "small".

As a result, most of the autopilot analysis and design techniques, considered in open literature, use classical control approach, such as single input-single output transfer functions characterization of the system dynamics and bode, nyquist, root-locus and transient response analysis and synthesis techniques. These techniques are valid to cover a wider set of flight regimes and airframe configurations require autopilot design.

With the advent of fast processor it is now possible to take a more integrated approach to autopilot design. Modern optimal control techniques allow the designer to consider autopilots with high order dynamics (large number of states) with multiple inputs/outputs and to synthesis controllers such that the error between the demanded and the achieved output is minimized.

II. General Missile Six-DOF Model

It is often the case in control systems design that while the concept are valid for any system order, their actual application is restricted to lower order system models. This is because higher order system model design may require excessive computation. Such prohibitively high system order can be caused by the presence of small "parasitic" parameters, typically small time constants, masses, etc.

Several approaches exist at present to reduce the system order, such as retaining dominant modes or neglecting fast modes. In singular perturbation method, both slow and fast system modes are retained while control system analysis and design may be solved in two separate parts, one for the slow modes and the other for the fast.

The objective is to approximate the performance of the original system through the analysis and design of the slow and fast mode subsystems with separate time-scales. By approximating the system with its slow and fast modes, both the high dimensionality and stiffness difficulties of the original singularly perturbed system are alleviated while retaining, in some sense, an approximation to the original system performance.

A nonlinear model for a general missile is given by [6]:

$$\begin{aligned}
\dot{U} &= -\frac{\bar{q}s}{m}\Big(a_{c_A} + a_{c_A}|\alpha| + a_{c_A}|\beta| + a_{c_A}|\alpha\beta| + a_{c_A}|\delta_Q| \\
&\quad + a_{c_A}|\delta_R| + a_{c_A}|\delta_P| + a_{c_A}\delta_Q^2 + a_{c_A}\delta_R^2 + a_{c_A}|\delta_R\delta_Q|\Big) \\
&\quad - WQ + VR + \frac{F_{xg}}{m} \\
\dot{V} &= \frac{\bar{q}s}{m}\Big(a_{c_Y}\alpha + a_{c_A}\beta + a_{c_Y}\alpha^3 + a_{c_Y}\beta^3 + a_{c_Y}\delta_Q + a_{c_Y}\delta_R \\
&\quad + a_{c_Y}\delta_P\Big) - UR + WP + \frac{F_{yg}}{m} \\
\dot{W} &= -\frac{\bar{q}s}{m}\Big(a_{c_N}\alpha + a_{c_N}\beta + a_{c_N}\alpha^3 + a_{c_N}\beta^3 + a_{c_N}\delta_Q + a_{c_N}\delta_R \\
&\quad + a_{c_N}\delta_P\Big) - VP + UQ + \frac{F_{zg}}{m} \\
\dot{P} &= \frac{1}{I_x}\bar{q}sl\Big(a_{c_l}\alpha + a_{c_l}\beta + a_{c_l}\alpha^3 + a_{c_l}\beta^3 + a_{c_l}\delta_Q + a_{c_l}\delta_R \\
&\quad + a_{c_l}\delta_P\Big) \\
\dot{Q} &= \frac{1}{I_y}\bar{q}sl\Big(a_{c_m}\alpha + a_{c_m}\beta + a_{c_m}\alpha^3 + a_{c_m}\beta^3 + a_{c_m}\delta_Q + a_{c_m}\delta_R \\
&\quad + a_{c_m}\delta_P\Big) - \frac{I_x - I_z}{I_y}\times PR \\
\dot{R} &= \frac{1}{I_z}\bar{q}sl\Big(a_{c_n}\alpha + a_{c_n}\beta + a_{c_n}\alpha^3 + a_{c_n}\beta^3 + a_{c_n}\delta_Q + a_{c_n}\delta_R \\
&\quad + a_{c_n}\delta_P\Big) - \frac{I_y - I_x}{I_z}\times PQ
\end{aligned} \tag{1}$$

In these equations; U, V, W are the velocity components measured in the missile body axis. P, Q, R are the components of the body rotational rate. F_{xg}, F_{yg}, F_{zg} are the gravitational forces acting along the body axis and I_x, I_y, I_z are the missile moments of inertia. Also the aerodynamic force and moment coefficients are described in a polynomial form with respect to angle of attack α, angle of slideslip β, pitch fin deflection δ_Q, yaw fin deflection δR and the roll fin deflection δ_P. The coefficients of the polynomials describing the aerodynamic coefficients were derived by carrying out least squares fits on the aerodynamic data [4]. The variable s is the reference area and l is the reference length. For a regular missile; angle of attack α, angle of slideslip β and the other parameters like missile speed V_T, Mach number M and dynamic pressure q are defined as:

$$\alpha = \tan^{-1}\left(\frac{W}{U}\right) \tag{2}$$

$$\beta = \tan^{-1}\left(\frac{V}{U}\right) \tag{3}$$

$$V_T = \sqrt{U^2 + V^2 + W^2} \tag{4}$$

 ISSN : 2249-5762 (Online) | ISSN : 2249-5770 (Print)

$$M = \frac{V_T}{a} \qquad (5)$$

$$\bar{q} = \frac{1}{2}\rho V_T^2 \qquad (6)$$

After converting these equations to linear format and adapting to state space form, the state vector and the control vector are chosen to be:

$$x = [P\ Q\ R\ U\ V\ W]^T\ ,\ u = [\delta_Q\ \delta_R]^T \qquad (7)$$

III. Standard LQG Design

The Kalman filter gain matrix K is given by [13].

$$K = P_e C^T V^{-1} \qquad (8)$$

$$AP_e + P_e AP_e C^T V^{-1} CP_e + LWL^T = 0 \qquad (9)$$

The optimal state-feedback gain matrix G is given by:

$$G = -R^{-1} B^T P_c \qquad (10)$$

Where, Pc satisfies the following algebraic Riccatti equation:

$$PA + A^T P - (PB + N)R^{-1}(B^T P + N^T) = -Q \qquad (11)$$

So optimal control of the system with the cost function will take the form:

$$U = G\hat{X} \qquad (12)$$

Where the estimated states are obtained from Kalman filter.

IV. LQG Design using Two-Time Scale Method

Fig. 1, is a general missile coordinate system with related axis. In order to use two-time scale property of the system, equation (1) is partitioned as:

$$\dot{X} = A_{11}X + A_{12}Z + B_1U + L_1W \qquad (13)$$

$$\dot{Z} = \hat{A}_{21}X + \hat{A}_{22}Z + \hat{B}_2U + \hat{L}_2W \qquad (14)$$

$$Y = C_1X + C_2Z + \nu \qquad (15)$$

Where,

$$X = (x_1\ \ x_2\ \ x_3\ \ x_4)^T \qquad (16)$$

$$Z = (x_5\ \ x_6)^T \qquad (17)$$

The standard singular perturbation form of (13) and (14) is then given by:

$$\dot{X} = A_{11}X + A_{12}Z + B_1U + L_1W \qquad (18)$$

$$\varepsilon\dot{Z} = A_{21}X + A_{22}Z + B_2U + L_2W \qquad (19)$$

Where,

$$\hat{A}_{21} = \frac{A_{21}}{\varepsilon},\ \hat{A}_{22} = \frac{A_{22}}{\varepsilon},\ \hat{B}_2 = \frac{B_2}{\varepsilon},\ \hat{L}_2 = \frac{L_2}{\varepsilon} \qquad (20)$$

and ε is the singular perturbation parameter. Applying the standard singular perturbation approach to decouple the slow-fast subsystems, the slow and the fast dynamics is obtained [1].

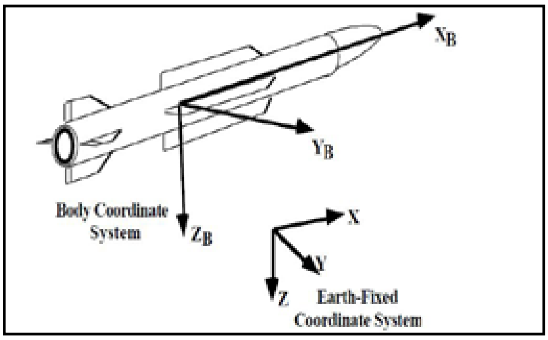

Fig. 1: A General Missile Coordinate System

A. The Slow Subsystem

$$\dot{X}_s = A_sX_s + B_sU_s + L_sW \qquad (21)$$

$$Y_s = C_sX_s + D_sU_s + E_sW + \nu \qquad (22)$$

$$X_s(0) = X^0 \qquad (23)$$

$$A_s = A_{11} - A_{12}A^{-1}_{22}A_{21} \qquad (24)$$

$$B_s = B_1 - A_{12}A^{-1}_{22}B_2 \qquad (25)$$

$$L_s = L_1 - A_{12}A^{-1}_{22}L_2 \qquad (26)$$

$$C_s = C_1 - C_2A_{22}^{-1}A_{21} \qquad (27)$$

$$D_s = -C_2A_{22}^{-1}B_2 \qquad (28)$$

$$E_s = -C_2A_{22}^{-1}L_2 \qquad (29)$$

The performance index J_s associated with slow subsystem is defined:

$$J_s = E\{\frac{1}{2}\int_0^\infty \{X_s^TQ_sX_s + X_s^TNU_s + U_s^TR_sU_s\}dt\} \qquad (30)$$

The control signal for the slow subsystem is:

$$U_s = G_s\hat{X}_s \qquad (31)$$

Where, $\hat{X}_s$ is the optimal estimated slow state X_s provided by the slow Kalman filter:

$$\dot{\hat{X}}_s = A_s\hat{X}_s + B_sU_s + L_S[Y - D_sU_s - C_s\hat{X}_s] \qquad (32)$$

The filter gain Ls is:

$$L_s = (P_sC_s^T + L_sE_s^T)V_s^{-1} \qquad (33)$$

And P_s is the stabilizing solution of the slow algebraic Riccatti equation:

$$[A_s - L_sE_s^tV_s^{-1}C_s]P_s + P_s[A_s - L_sE_s^tV_s^{-1}C_s]^t$$
$$+ L_s[I - E_s^tV_s^{-1}E_s]L_s^T - P_sC_s^tV_s^{-1}C_sP_s = 0 \qquad (34)$$

Where,

$$V_s = V + E_sE_s^t \qquad (35)$$

B. The Fast Subsystem

$$\varepsilon\dot{Z}_f = \hat{A}_{22}Z_f + \hat{B}_2U_f + \hat{L}_2w$$
$$Y_f = C_2Z_f +$$
$$Z_f(0) = Z^0 + A_{22}^{-1}A_{21}X \qquad (36)$$

The performance index J_f is:

$$J_f = E\{\frac{1}{2}\int_0^\infty \{Z^TQZ_f + Z_f^TNU_f + U_f^TRU_f\}dt\} \qquad (37)$$

The feedback control signal for fast sub-system is given by:

$$U_f = G_f\hat{Z}_f \qquad (38)$$

Where, $\hat{Z}_f$ is the optimal estimated fast state (Z_f), provided by the fast Kalman filter:

$$\varepsilon\dot{\hat{Z}} = A_{22}\hat{Z} + B_2U_f + L_f[Y_f - C_2\hat{Z}] \qquad (39)$$

The filter gain Lf is:

$$L_f = P_fC_2^TV^{-1} \qquad (40)$$

And P_f is the stabilizing solution of the fast algebraic Riccati equation

$$A_{22}P_f + P_f A_{22}^t + L_2 W L_2 - P_f C_2 V^{-1} C_2 P_f = 0 \quad (41)$$

The composite control signal is the sum of the slow and fast control signals

$$U_c = U_s + U_f = G_s \hat{X} + G_f \hat{Z} \quad (42)$$

If U_C applied to the system the resulting performance index J_c is near optimal in the sense

$$J_c = J_s + J_f = J_o \quad (43)$$

Where J_o is the optimal value of the performance index.

V. Application on Missile Autopilot

The slow-fast controller design presented in section 4 is now applied to the missile autopilot model in section II. The A, B and C matrices of missile state space model and consonant matrices Q and R of cost function are given bellow.

$$Q = \mathrm{diag}\left(\left[100,100,100,\ 1e-8, 2e1, 2e16\right]\right) \quad (44)$$

$$R = 1e10 \times \mathrm{diag}\left(\left[1,1\right]\right) \quad (45)$$

$$A = \begin{bmatrix} -0.322 & 0.064 & 0.0364 & -0.9917 & 0.0003 & 0.0008 \\ 0 & -1 & 1 & 0.0037 & 0 & 0 \\ -30.6492 & 0 & -2.6784 & 0.6646 & -0.7333 & 0.1315 \\ 8.5395 & 0 & -0.0254 & -2.4764 & -0.0319 & -0.0620 \\ 0 & 0 & 0 & 0 & -20.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20.2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 10.2 & 0 \\ 0 & 10.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -57.2958 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (46)$$

In order to observe the performance of each of the above controllers in the presence of external disturbances, a step command is considered. Such a particular disturbance simulates both the slow and the fast modes in the system. In addition, a white noise signal with a power spectral density has been selected as a disturbance signal to model the missile motor vibrations.

The outputs are important in LQG controller design, since in such a design, by using the optimal estimator, the measurable output must provide a good and proper estimate for the modes (x_1, x_2, x_3, x_4). Computing the controller gain matrix (12) for the standard LQG design, we get:

G =

126.3528	-292.0671	-13.3157
55.0274	-38.5518	-9.6537
6.6940	-2.2063	1.1021
-0.1769	-0.1769	0.0415

(47)

Applying this controller to the full order system, and computing the cost function, we obtain:

$$J_o = 17.2580 \quad (48)$$

In order to apply the singular perturbation theory, first, let's check the two-time-scale property of the system. The singular perturbation parameter e is computed:

$$\left|\ddot{e}_s\right|_{max} / \left|\ddot{e}_f\right|_{min} \cong 0.1365 \quad (49)$$

The calculations of the controller gains matrices using the singular perturbation theory for both the slow and the fast subsystems result in the following

G_s =

98.4589	58.5791	-18.5473	45.4864
-15.4122	-3.1109	1.4898	-0.5445
0.1683	-1.2377	-1.2401	0.0057

(50)

G_f =

-58.2471	-14.5716
-1.3151	-0.1546

(51)

And the composite cost function is computed to be:

$$J_c = 18.4424 \quad (52)$$

Design of the controller using the singular perturbation theory entails cost increase that is negligible, and clearly justifies the utilization of this approach. The simulation results of both design approaches are compared graphically. Fig. 2, shows the step response of full order missile autopilot design, and fig. 3, shows the step response of reduced system using singular perturbation theory, that is too close to each other and shows the efficiency of this work.

Although we use a six degree of freedom missile model in this work, but using singular perturbation theory for expanded problem with further dimensions can give better and greater advantages.

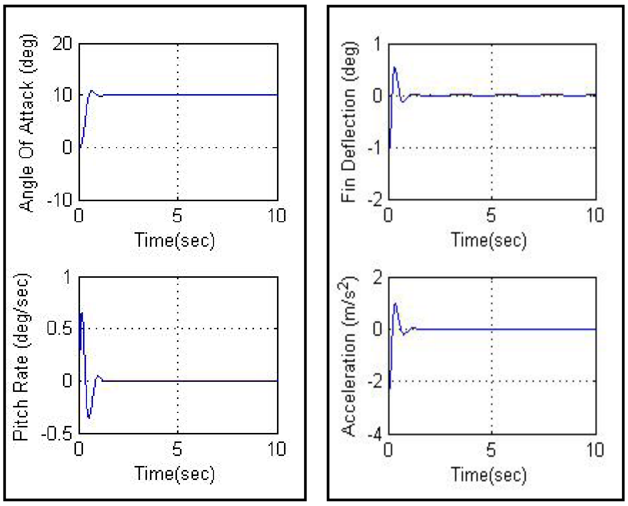

Fig. 2: Angle of Attack, Pitch Rate, Fin Deflection and Lateral Acceleration Response of Original Full Order Missile Autopilot

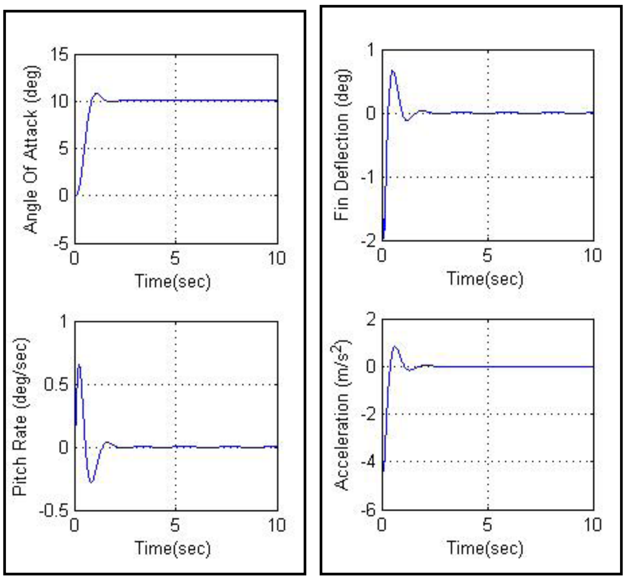

Fig. 3: Angle of Attack, Pitch Rate, Fin Deflection and Lateral Acceleration Response of Reduced Order Composite Control of Missile Autopilot

VI. Conclusion

The model of a missile autopilot can be put in singularly perturbed form to design LQG controller. The responses were concluding by using observer with good match between full and reduced systems. In spite of the simplified structure of these strategies, the resulting missile autopilot performance is comparable to the performance with the full order feedback design.

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