OPTIMAL TRAJECTORIES TOWARDS NEAR-EARTH-OBJECTS USING SOLAR ELECTRIC PROPULSION (SEP) AND GRAVITY ASSISTED MANEUVER

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Abstract: The solar electric propulsion could be the best option for the transports of the future due to its high specific impulse when compared to the chemical propulsion. Electric propellants are being extensively used to assist the propulsion of terrestrial satellites for the maneuvers of orbit correction and as primary propulsion in missions toward other bodies of the solar system.
In this work the optimization of interplanetary missions using solar electric propulsion (SEP) and Gravity Assisted Maneuver to reduce the costs of the mission, is considered. The high specific impulse of electric propulsion makes a Gravity Assisted Maneuver 1 year after departure convenient. Missions for several Near Earth Asteroids will be considered. The analysis suggests criteria for the definition of initial solutions demanded for the process of optimization of trajectories.
Trajectories to the asteroid 2002TC70 are analyzed. Direct trajectories, trajectories with 1 gravity assisted at the Earth and with 2 gravity assisted with the Earth and either Mars are presented. Shall be analyzed missions with thrusters PPS1350 and the Phall 1 for performance comparison. An indirect optimization method will be used in the simulations.

Keywords: Astrodynamics, Celestial Mechanics, Space Trajectories.

1. INTRODUCTION

The future interplanetary missions will probably use the conventional chemical rockets to leave the sphere of influence of the Earth, and solar electric propulsion (SEP) to accomplish the other maneuvers of the mission.
Both NASA and ESA have launched spacecrafts which used SEP (Solar Electric Propulsion) as the primary propulsion system; NASA's DS1 and ESA's Smart-1 to the moon to comet Borrelly.
Indirect optimization methods are suitable for the low thrust trajectories that are used in simulations. A finite force is applied during a finite interval of time and it is necessary to integrate the state equation along the time to know its effect. Several results exist in the literature, starting with the works of Tsien (1953) and Lawden (1955). Other results and references can be found in Prado (1989), Prado and Rios-Neto (1993), Casalino and Colasurdo [1], Santos [2]. The most used method in this model is the so called "primer-vector theory", developed by Lawden (1953 and 1954) [4, 5], according to Prado [6, 7], Santos [8, 9]. In this paper, theory of optimal control is applied and a procedure based on the Newton's Method to decide the boundary problems is developed. The Pontryagin's Maximum Principle (PMP) is used to maximize the Hamiltonian associated to the problem and evaluates the optimal structure of the "switching function".
The spacecraft leaves the Earth's sphere of influence with a hyperbolic velocity whose optimal magnitude and the direction will be supplied by the optimization procedure. The initial mass is directly related to the magnitude of the hyperbolic velocity, assuming that a chemical thruster is used to leave a low Earth orbit (LEO). Out of the Earth's sphere of influence, the electric

propellants is activate and the available power is proportional to the square of the distance from the Sun; the propulsion is provided by one or two "PPS 1350 ion thrusters and Phall1 thrusters (UNB)".

2. DESCRIPTION OF THE PROBLEM

The spacecraft will be considered a point with variable mass m and the trajectory will be analyzed using the patched-conics approach. The time required by the spacecraft to leave the Earth's sphere of influence is neglected and, in this formulation, only equations of motion in the heliocentric reference system will be considered. The spacecraft is influenced by the Sun gravitational acceleration $\vec{g}(r)$ and the propulsion system that the vehicle implements, with a level of thrust T. With this formulation, a maneuver including an Earth's flyby can be used to gain energy and velocity, which provokes a discontinuity in the relative state variables of the velocity.

The variables are normalized using the radius of the Earth's orbit, the corresponding circular velocity, and the mass of the spacecraft in stationary orbit as values of reference.

The solar electric Propulsion system will be considered, therefore, the available power and thrust varies with the square of the distance from the sun.

In this problem, the thrust is the only control during the heliocentric arcs, and it will be optimized to get the minimum consumption, that is measured by the final mass of the spacecraft. Since the thrust appears linearly in the equation of motion, a bang-bang control, which consists of alternating ballistic arcs with arcs of maximum thrust, will be required. The trajectory is composed by a succession of ballistic arcs (zero-thrust) and arcs of maximum thrust, where the optimal direction will be supplied by the optimization procedure.

The boundary conditions are imposed in a satisfactory way at the junctions between trajectory arcs.

The integration initiates when the spacecraft leaves the Earth's sphere of influence, at the position $\vec{r}_i = \vec{r}_\oplus(t_i)$ that coincides with the Earth's position, considering the velocity $\vec{v}_i$ free. The hyperbolic velocity is given by $\vec{v}_{\infty i} = \vec{v}_i - \vec{v}_\oplus(t_i)$, assuming that a rocket thruster is used to take the spacecraft from the Low Earth Orbit (LEO) with an impulsive maneuver and that the vehicle mass on LEO is specified. The increment of velocity (ΔV) demanded to provide the hyperbolic velocity is $\Delta V = \sqrt{v_{\infty i}^2 + v_e^2} - v_c$, where v_e and v_c are the escape and circular velocity at the LEO radius [2]. The initial mass at the exit from the Earth's sphere of influence is,

$$m_i = a - bV_\infty - cV_\infty^2 \qquad (1)$$

where,

$\varepsilon(1 - m_i)$ is the jettisoned mass of the exhausted motor, which is proportional to the propellant mass. The spacecraft intercepts the Earth and accomplishes Gravity Assisted Maneuvers (Santos et al., 2005) [7 - 9]. The position of the vehicle $\vec{r}_\pm = \vec{r}_\oplus(t_\pm)$ is constrained and the magnitude of the hyperbolic excess velocity $\vec{v}_{\infty\pm} = \vec{v}_\pm - \vec{v}_\oplus(t_\pm)$ is continuous $v_{\infty+}^2 = v_{\infty-}^2$ [2].

If the minimum height constraint on the *flyby* is requested, a condition on the velocity turn angle is added:

$$\vec{v}_{\infty+}^T \vec{v}_{\infty-} = -\cos(2\phi) v_{\infty-}^2 \qquad (2)$$

where,

$$\cos(\phi) = \frac{v_p^2}{\left(v_{\infty-}^2 + v_p^2\right)} \qquad (3)$$

v_p is the circular velocity at the low distances allowed for a planet.

$$\vec{v}_{\infty\pm} = \vec{v}_{i\pm} - \vec{v}_4 \qquad (4)$$

At the final point (subscript *f*), the position and velocity vectors of the spacecraft and the asteroid coincide,

$$r_f = r_A(t_f) \qquad (5)$$

$$v_f = v_A(t_f) \qquad (6)$$

The theory of optimal control provides the control law and the necessary boundary conditions for optimality.

3. OPTIMIZATION PROCEDURES

The objective is to use the theory of optimal control to maximize the spacecraft final mass.
Dynamical equations are,

$$\dot{\vec{r}} = \vec{v}$$
$$\dot{\vec{v}} = \vec{g}(\vec{r}) + \frac{\vec{T}}{m} \qquad (7)$$
$$\dot{m} = -\frac{\vec{T}}{c}$$

Applying the theory of optimal control, the Hamiltonian function is defined as (Lawden, 1954) [3, 2]:

$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t (\vec{g} + \frac{\vec{T}}{m}) - \lambda_m \frac{\vec{T}}{c} \qquad (8)$$

An indirect optimization procedure is used to maximize the payload. According to Pontryagin's Maximum Principle the optimal controls maximize H.
The nominal thrust T_o at 1 AU, and the electrical power are (Casalino [2], Santos [9]),

$$P_0 = \frac{T_0 c}{2\eta}$$
$$T_{Max} = \frac{T_0}{r^2} \qquad (9)$$

Optimal control theory provides differential equation for the adjoint equations of the problem (Euler-Lagrange).
Adjoint equations are,

$$\dot{\vec{\lambda}}_r^t = \vec{\lambda}_v^t \frac{\partial \vec{g}}{\partial \vec{r}} - S_f \frac{\partial T}{\partial \vec{r}} \qquad (10)$$

$$\dot{\vec{\lambda}}_v^t = -\vec{\lambda}_r^t \qquad (11)$$

$$\dot{\vec{\lambda}}_r = \lambda_v^t \frac{\vec{T}}{m^2} \qquad (12)$$

where, $G = \frac{\partial \vec{g}}{\partial \vec{r}}$.

Optimal control: thrust direction and magnitude are,

$$\vec{T} \,//\, \vec{\lambda}_v$$
$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t G + \vec{T}\left(\frac{\lambda_v}{m} - \frac{\lambda_m}{c'}\right) \qquad (13)$$
$$S_f = \frac{\lambda_v}{m} - \frac{\lambda_m}{c'}$$

where,
c' - is the effective exhaust velocity of the rocket thruster;

$$T_{Max} = \begin{cases} \frac{T_0}{r^2} \rightarrow S_f > 0 \\ 0 \rightarrow S_f < 0 \end{cases} \qquad (14)$$

The necessary optimal conditions [5, 2] are:

$$\left(H_{j_-} + \frac{\partial \varphi}{\partial t_{j_-}} + \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_-}} \right) \delta t_{j_-} = 0 \qquad (15)$$

$$\left(H_{j_+} - \frac{\partial \varphi}{\partial t_{j_+}} - \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_+}} \right) \delta t_{j_+} = 0 \qquad (16)$$

$$\left(\lambda_{j_-}^t - \frac{\partial \varphi}{\partial \vec{x}_{j_-}} - \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_-}} \right) \delta \vec{x}_{j_-} = 0 \qquad (17)$$

$$\left(\lambda_{j_+}^t + \frac{\partial \varphi}{\partial \vec{x}_{j_+}} + \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_+}} \right) \delta \vec{x}_{j_+} = 0 \qquad (18)$$

Where:

$\vec{\chi}$: the vector collecting the constraining boundary conditions (see eq. 15 - 18)

$\varphi = m_f$

At the initial point:

1. $\vec{r}_0 = \vec{r}_\oplus$;
2. $m_o = 1 - bV_\infty - cV_\infty^2$
3. $(\vec{v}_0 - \vec{v}_\oplus)^2 = \vec{v}_{\infty 0}^2$;
4. Equations 16 and 18 provide optimal control with λ_{ro} and T_{ro} free;
5. the necessary condition optimal of the state is $\vec{\lambda}_{v0}$ (primer vector) be parallel to the hyperbolic velocity;

At flyby [2]:
1. the equations (15 and 16) are used to obtain the transversality conditions, that implicates in determining the arc time used;
2. in the equations (17 and 18) $\vec{\lambda}_{vi}$ is parallel to the hyperbolic velocity, before and after the

free flyby maneuver and the magnitude is continuous;

3. the states of Hamiltonian remain continuous through the flyby maneuvers;
4. when the minimum height constraint of the flyby is requested, a condition on the velocity turn angle is added (Eq. 2 and 3).

At the final point:

1. $\vec{\lambda}_{vf}$ is parallel to the hyperbolic velocity, $\vec{\lambda}_{rf}$ is parallel to the radius and $\vec{\lambda}^t_{rf}\vec{v}_f + \vec{\lambda}^t_{vf}\vec{g} = 0$;
2. the final values of $\vec{\lambda}_{mf}$ and H_f depends on the control model that was considered in the maneuver;
3. the adjoint variable $\vec{\lambda}_v$ is zero during the whole trajectory.

4. MISSION ASTEROID 2002TC70

4.1 NUMERICAL ANALYSIS WITH PPS1350 (ESA)

The characteristics of the spacecraft propulsion system that have been assumed are [11]:

1. the mass of the spacecraft with an altitude of 200 km in circular LEO is 2133.3 Kg;
2. specific impulse I_s = 1550s :
3. specific energy ε = 0.06;
4. T = 2 ·70 mN (thruster PPS 1350 used for the SMART-1 mission to the moon);
5. nominal thruster T_o=1 UA;
6. The time: *time* = 0 corresponds to the date 01/01/2000.

The necessary optimal condition were formulated in agreement with the problem; the bang-bang control was used in the formularization with limited power and constraint in the time of flight.

Name	2002TC70
Epoch	54200
a	1.369831
e	0.19691574
i	2.13932
Ω	161.89427
ω	134.84892
M	351.6336031
r_a	1.10009
r_p	1.639572

Table 1 – Keplerian Elements

4.1.1 Simulation without flyby

Using the optimization procedure we can find optimal trajectories, with the maximization of the spacecraft final mass (i.é., minimum fuel consumption). These trajectories depend on the mission objectives, for example, the performance depends on the mission time length. It is possible to reduce the time with some more spend of propellant.

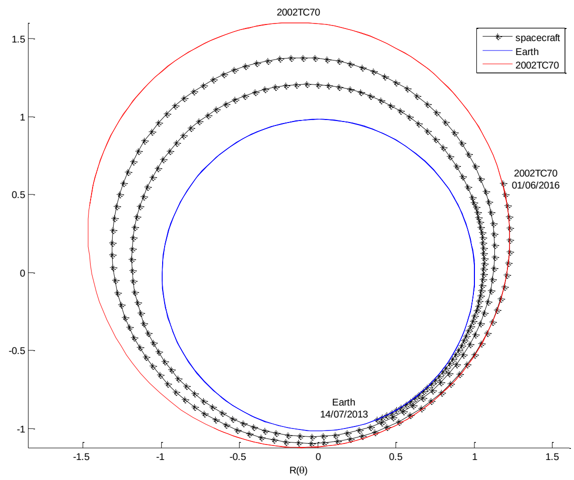

Figure 1 – Trajectory direct (without flyby) to the asteroid 2002TC70

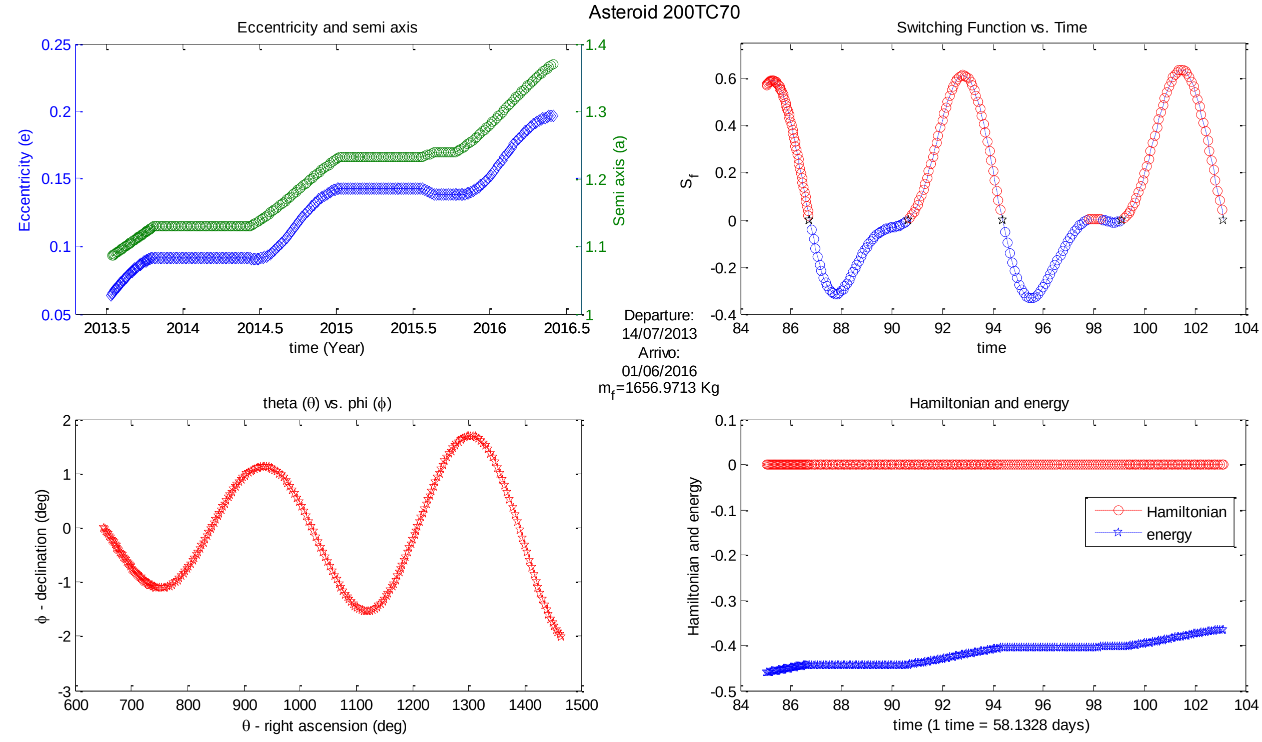

Figure 2 - Maneuver leaving from the Earth and arriving at the asteroid 2002TC.

The eccentricity (e) and semi-major axis (a); the structure of the switching function that shows the thrust arc and coast arc (Eq. 14); the right ascension as a function of the declination; the evolution of the Hamiltonian and energy orbits are visualized in Figure 2.

4.1.2 EGA mission

An Earth *flyby* can be used to vary the semi-major axis (a) and the eccentricity (e) in order to increase the apoapsis (r_a) (or reduce the periapsis r_p) or to vary the inclination (i) of the orbit.

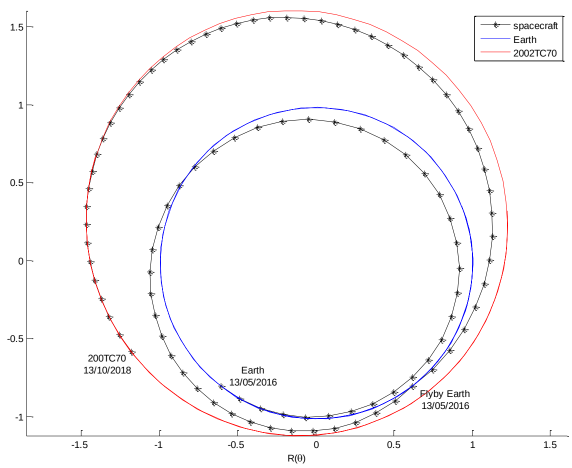

Figure 3 - Trajectory leaving the Earth and arriving to the asteroid, using EGA maneuvers.

Both the effects can be achieved if the line of nodes and the line of apsides are aligned (i.e. the argument of periapsis is close to ω ≈ 0, 180 or 360 degrees).

4.1.3 EMGA mission

The formulation allows the use of multiple flyby's searching for a better performance. The criterion of the choice of *flyby* in Mars or Venus is the semi-major axis (a) of the asteroid:

$$\begin{cases} a > 1 \rightarrow Mars\ Flyby \\ a < 1 \rightarrow Venus\ Flyby \end{cases} \qquad (19)$$

Mars flyby can be used to increase the periapsis (r_p) and in Venus to reduce the apoapsis (r_a).

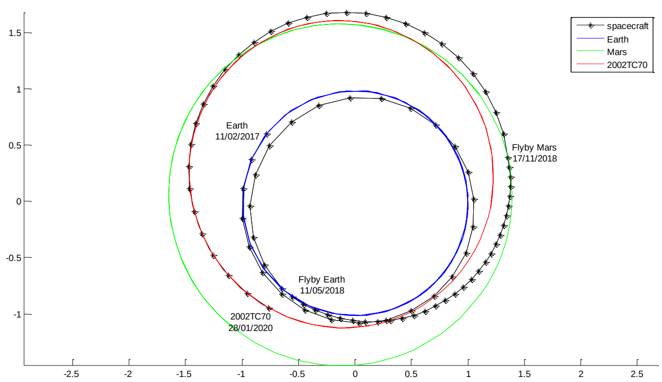

Figure 4 - Trajectory leaving the Earth and arriving to the asteroid, using EMGA maneuvers.

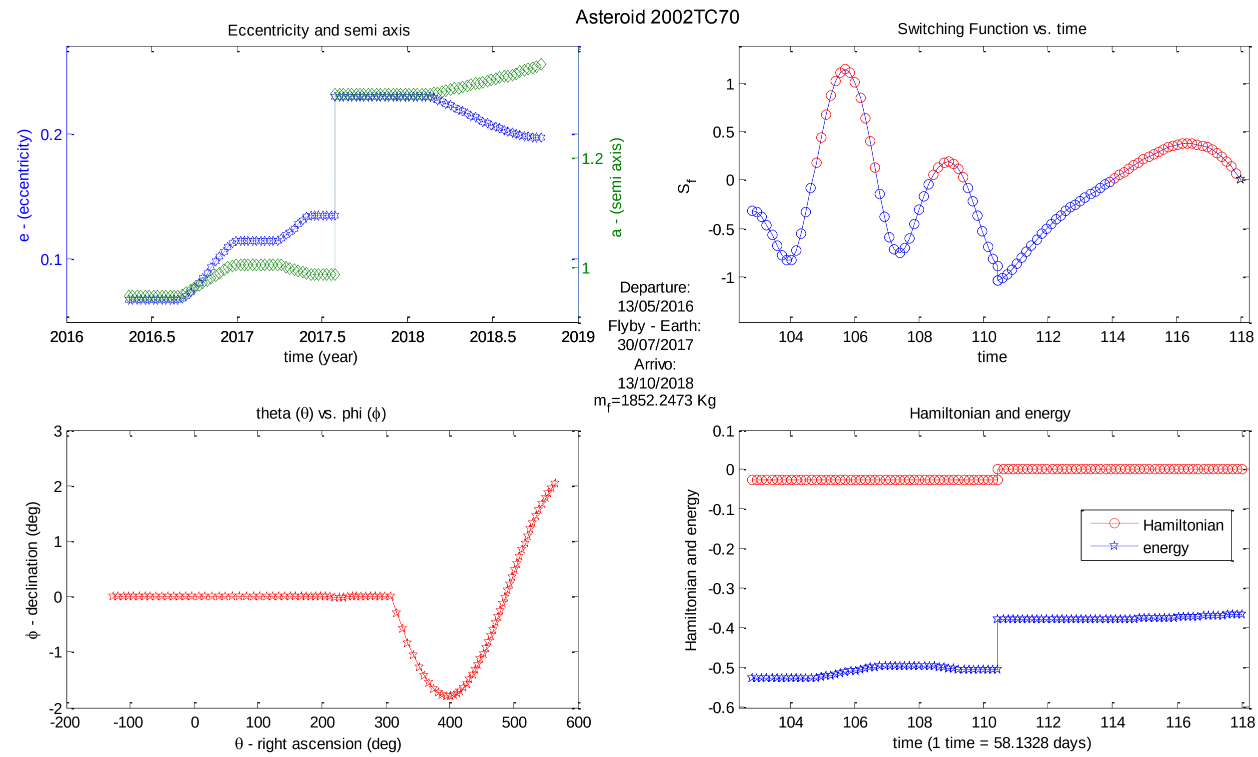

Figure 5 - Simulation with maneuver of Earth flyby to the asteroid 2002TC70 .

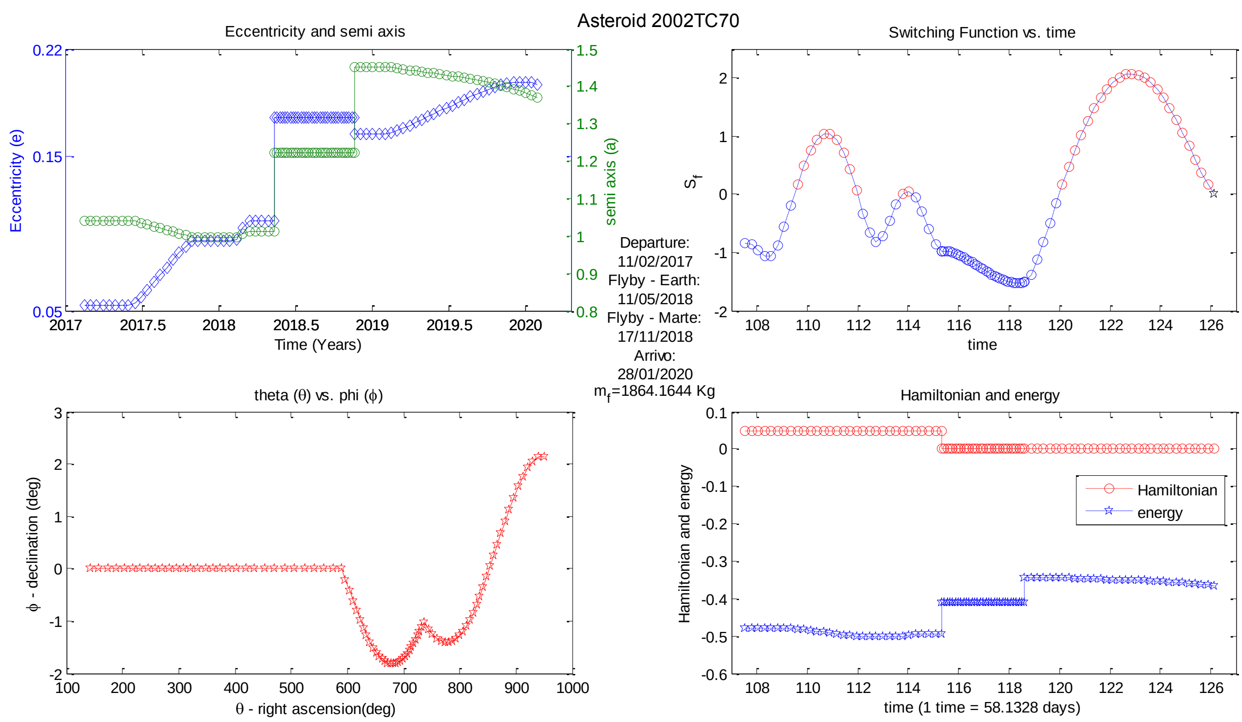

Figure 6 - Simulation with multiple flyby's: in the Earth and in Mars to the asteroid 2002TC70.