

# Method for Determining the Probing Points for Efficient Measurement of Freeform Surface

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**Abstract**—In inspection and workpiece localization, sampling point data is an important issue. Since the devices for sampling only sample discrete points, not the completely surface, sampling size and location of the points will be taken into consideration. In this paper a method is presented for determining the sampled points size and location for achieving efficient sampling. Firstly, uncertainty analysis of the localization parameters is investigated. A localization uncertainty model is developed to predict the uncertainty of the localization process. Using this model the minimum size of the sampled points is predicted. Secondly, based on the algebra theory an eigenvalue-optimal optimization is proposed. Then a freeform surface is used in the simulation. The proposed optimization is implemented. The simulation result shows its effectivity.

**Keywords**—eigenvalue-optimal optimization, freeform surface inspection, sampling size and location, sampled points.

## I. INTRODUCTION

**I**NSPECTION and workpiece localization improve production efficiency. The first task in these technologies is to obtain the measurement data. Among the various sensing techniques available, mechanical contact probes such as coordinate measuring machine (CMM) touch probes and 3D topography measuring systems using structured light or fringe illumination are widely used in practical applications. CMMs with touch-triggered probes can provide high measurement accuracy at sub-micron level. However, the measurement speed is much lower than using a 3D vision system. A vision system can acquire thousands of data points over a large spatial range in a snapshot [1]. However, the achievable resolution is relatively low. Therefore, in practical applications, using one of the techniques means that the user has to suffer its limitations, e.g. the low speed with CMMs.

After the points are sampled, the researchers can use the specified algorithm to compute, such as fitting surface, computing tolerance and configuration. Due to the accuracy of the sampling device as well as the geometry and surface inaccuracy of the object, there exists certain measurement error with each sampled point. Thus, the positioning errors of coordinate data will result the positional (translational and rotational) error of the object derived from the sampled data.

Theoretically if we can measure all points on a object—the “real” measuring errors can be identified and analyzed, however, this is impossible or disadvantageous even if we can handle infinite points of a surface with infinite points.

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In summary, a sample survey is less expensive, less time consuming, and may even be more accurate than the complete enumeration [2]. This excessive sample size must be reduced to an acceptable number while maintaining the same high level of accuracy. Several possible sampling strategies have been addressed in the literature. These include: (a) uniform sampling, (b) simple random sampling, (c) stratified random sampling, (d) cluster sampling, (e) systematic sampling and other more esoteric schema. When we measure the point set in the machine reference frame using a computer-controlled coordinate machine, laser and computer vision, there are two questions that to be answered.

- 1 In order to get a reliable result, how many points should we measure on the object surface?
- 2 If a point number is specified, how should we plan the sampled points' location on the CAD model?

The remainder of the paper is organized as follows. In section 2, we review the existing coordinate-sampling strategies for inspection and workpiece localization, etc. In section 3, based on the uncertainty model, we present a method for the determination of the sampling size. In section 4, an optimization for location of sampling points is proposed. And after some simulation in section 5, we draw the conclusion in section 6.

## II. LITERATURE REVIEW

Many factors affect the sampling strategy, e.g., time, cost, manufacturing accuracy, tolerance specifications, the applied analysis algorithms and the object geometry. From a statistical viewpoint, each measurement data point contains a certain amount of geometrical information about the surface, and the quantity of information contained in the set of measurement data points depends on the number and locations of the measurement points.

How well the discrete sample points represent the sampled surface? Dimensional surface measurements have involved the use of deterministic sequences of numbers for determination of sample coordinates to maximize information collected. According to Woo and Liang [3], a two dimensional (2D) sampling strategy based on the Hammersley sequence shows a remarkable improvement of a nearly quadratic reduction in the number of samples when compared with the uniform sampling strategy, while maintaining the same level of accuracy. The HZ based strategy in 2D space was also suggested by Woo et al [4]. The only differences are that the total number of sample points in the HZ sequence must be a power of two and the binary representations of the odd bits are inverted. Also, Liang

et al. [5], [6] compared the 2D HZ sampling scheme to the uniform scheme and the SR theoretically and experimentally for roughness measurement with similar results. Lee et al. [7] demonstrated a methodology for extending the HM sequence for geometries such as circles, cones, and spheres. Kim and Raman [8] investigated different sampling strategies and different sample sizes for flatness measurements. Their findings were similar to others with regards to accuracy determination. Summerhays et al. [9] proposed new sampling patterns to guide form measurements of internal cylindrical surfaces with some success.

Dowling et al. [10] presented a survey of statistical issues in geometric feature inspection. Fitting and evaluation approaches, sampling design issues, and sources of measurement error were discussed. The incorporation of the knowledge of manufacturing processes was also suggested to improve the accuracy of geometric form inspection. Prakasvudhisarn [11] suggested guidelines for cones and conical frustum inspection by using three sampling sequences, HM, HZ, aligned systematic (AS) with various sample sizes. The sampled points were used to estimate the form error of the feature based on different fitting algorithms.

To help circumvent the adequacy of the data collection problems, Menq et al. [12] suggested a statistical sampling plan to determine a suitable sample size which can represent the entire population of the part surface with sufficient confidence and accuracy. Zhang et al. [13] proposed a feed-forward back-propagation neural network approach to estimate sample sizes of holes' measurements from various manufacturing operations. Machining processes, hole diameters, and tolerance bands were considered as influencing factors. Similarly, Lin and Lin [14] developed an algorithm based on the grey theory to predict the number of measuring points on the next workpiece for flatness verification by using data from the last four workpieces. Raghunandan and Rao [15] also reported a method to reduce sample size of flatness estimation by inspecting the first part in detail and using it as the reference for succeeding parts in a batch production.

After the sampling size determination of the points, where the points locate is another problem. Many errors will be considered. In [16], the authors reviewed error sources of sampling and different sampling strategies. It is noted that, in order to accurately measure part geometry, much higher sampling densities than those in the current practice must be incorporated.

For workpiece localization application, it is important to accurately recover the position and orientation of the workpiece subject to sampling errors. For this purpose, it is necessary to consider the geometric relations among the measured surfaces. It is known that a different set of sampling points will yield different transformation results [17]. Since sampling errors are inevitable, it becomes valuable to use a good sampling plan to ensure satisfactory recovery of Euclidean transformation.

Menq et al [17] estimated an upper bound of the transformation error with a normalized sensitivity measure. This measure serves as an index that reflects the joint effect of both the number of measurement points and the geometric attributes of measurement locations. Although such an index

has been proposed for a given part geometry, a method for the synthesis of the measurement points has yet to be developed. Canny and Paulos [18] proposed a simple example to show how the probing points affect the possible displacement region of the object. A method using hitting sets and set covers was developed to obtain near-optimal probe placements for any known polygonal object, while the exact optimal solution is NP-hard. For a sculptured workpiece, it is difficult to apply this sampling method. Cai et al [19] suggest an index used for planning fixture locators based on the variance of the resultant localization error. Nonlinear programming was used to minimize the variance. The method can only deal with one continuous surface. When there are many surfaces in consideration, there exists a combinatorial problem on the point number assignment of different surfaces.

For the application of medical image registration in [20], a 3-D model is constructed from images, using a sensor such as a computed tomographic (CT) scanner. In the synthesis procedure, a noise-amplification index is used to automatically generate near-optimal data configurations over a discrete point set [21]. Four methods were applied, namely, steepest-ascent hill climbing (SAH), near-ascent hill climbing (NAH), population-based incremental learning (PBIL), and a hybrid PBIL-hill-climbing approach. It was shown that the planning of 10-75 optimal points may need up to several hundred minutes of computational time. Such a method is suitable only for offline planning.

### III. DETERMINATION OF THE SAMPLING SIZE

Theoretically, if the sampled points data are the real coordinate of the surface and computing error of computer is not considered, the accurate result will be arrived at. However, errors exist in the sampled data. So many researchers figure out that more sampled points will result in more accurate the result. Sensitivity analysis of registration parameters to measurement errors is important. In general, measurement error, number of scanned data, and locations of scanned data all affect the determination of the localization results. If we can characterize the errors of localization parameters due to these factors, we can estimate the uncertainty of the localization parameters.

Here,  $y_i$  denotes the sampled point data,  $x_i$  denotes the nominal point data from the nominal surface of the CAD. Given  $x_i$ , if we perturb the data set by a small transformation (including rotation and translation) and set this perturbed data set to be the sampled data set, the errors between the two sets can be modeled by the following equation.

$$\varepsilon_i = (\Delta R x_i + \Delta P - x_i) \cdot n_i, i = 1, 2, \dots, N \quad (1)$$

where  $N$  is the number of the element of  $x_i$ ,  $n_i$  is the corresponding normal vector of  $x_i$ ,  $\Delta R$  and  $\Delta P$  are small rotation and translation perturbations and can be represented as:

$$\Delta R = \begin{pmatrix} 1 & -\Delta\gamma & \Delta\beta \\ \Delta\gamma & 1 & -\Delta\alpha \\ -\Delta\beta & \Delta\alpha & 1 \end{pmatrix} \quad (2)$$

Where  $\alpha, \beta, \gamma$  are Euler angel. And

$$\Delta P = \begin{pmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \end{pmatrix} \quad (3)$$

From equation 1, 2 and 3, the relation will be arrived at

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_N \end{pmatrix} = \begin{pmatrix} -(n_1 \times x_1)^T & n_{1,x} & n_{1,y} & n_{1,z} \\ -(n_2 \times x_2)^T & n_{2,x} & n_{2,y} & n_{2,z} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -(n_N \times x_N)^T & n_{N,z} & n_{N,y} & n_{N,z} \end{pmatrix} \begin{pmatrix} \Delta\alpha \\ \Delta\beta \\ \Delta\gamma \\ \Delta p_x \\ \Delta p_y \\ \Delta p_z \end{pmatrix} \quad (4)$$

or in a compact format

$$\tilde{\varepsilon} = A\Delta\xi \quad (5)$$

Where A is the sensitivity matrix,  $\tilde{\varepsilon}$  is the measurement error in surface normal direction, and  $\Delta\xi$  is the transformation parameter error. Since A is a  $N \times 6$  matrix, equation 5 is equal to the below

$$\Delta\xi = [(A^T A)^{-1} A^T] \tilde{\varepsilon} \quad (6)$$

A first-order expansion of  $\Delta\xi_i$  will generate the following equation.

$$\Delta\xi_i = \sum_{j=1}^N \frac{\Delta\xi_i}{\Delta x_j} \varepsilon_j = [(A^T A)^{-1} A^T]_{row i} \tilde{\varepsilon} \quad (7)$$

If  $\Delta\xi_i$  and  $\varepsilon_i$  are normally distributed, then

$$\sigma_{\xi_j}^2 = \sum_{j=1}^N \frac{\Delta\xi_i^2}{\Delta x_j^2} s^2 \quad (8)$$

Where  $\sigma_{\xi_j}^2$  and  $s^2$  are the variances for  $\Delta\xi_i$  and  $\varepsilon_i$ , respectively. By multiply  $\sigma_{\xi_j}$  by a constant c (say, c=2 represents a 99.8% confidence level) the uncertainty of the transformation parameter  $\xi_i$  can be modeled as:

$$U = c\sigma_{\xi_i} \quad (9)$$

$$\xi_{i,(evaluated)} - c\sigma_{\xi_i} \leq \xi_{i,(ture)} \leq \xi_{i,(evaluated)} + c\sigma_{\xi_i} \quad (10)$$

It is obviously that the smaller the uncertainty number, the more accurate the transformation parameter is. The uncertainty number is determined by the tolerance. The sensitivity of the localization parameter to sampling error can be further defined as

$$S_{\xi_i} = \frac{\sigma_{\xi_i}}{s} \quad (11)$$

From equation 7, 8 and 11, we will get

$$S_{\xi_i} = \frac{\sigma_{\xi_i}}{s} = \sqrt{\sum_{j=1}^N \left(\frac{\Delta\xi_i}{\Delta x_j}\right)^2} = \sqrt{\sum_{j=1}^N \left\| [(A^T A)^{-1} A^T]_{row i} \right\|^2} \quad (12)$$

Form equation 4, it is obviously that A is a function of sampled data locations and the corresponding normal vectors. So A is a function mainly related to the digitized geometry. Now a standard sphere is taken as an example and the origin of the coordinate frame is at the center of the sphere.

When the sphere rotates along the three axes respectively, the configuration will not change. However, the perturbation in the translation along three axes will result in the configuration change. Assuming that the sampled point data are random distributed around the sphere, we will arrive at the following result:

$$\sigma_{\xi_i} = \frac{\sqrt{3}}{\sqrt{N}} \cdot s, i = 4, 5, 6 \quad (13)$$

The equation shows that the uncertainty of transformation parameters for a sphere is proportional to the standard deviation of the sampling error, and is inversely proportional to the squared root of the number of the sampled data.

In fact, this model can be generalized for any sampled geometry including free-form surfaces:

$$\sigma_{\xi_i} = \frac{K}{\sqrt{N}} \cdot s \quad (14)$$

Where K is a function of the sampled geometry. When the scanned geometry and area are fixed, it becomes a constant. In the previous sphere case, K is equal to  $\sqrt{3}$ ; but in other cases, say for example a free-form surface, K is unknown and needs to be calibrated.

#### IV. DETERMINATION OF THE SAMPLING LOCATION

In [22], a near-optimal sampling strategy is presented. The determinant of matrix is used as objective function of the optimization to select the optimal sampled point locations that can minimize the transformation errors. However, according to the theory of linear algebra, a matrix's eigenvalues reflect the magnitude change between the original vector and the vector transformed by the matrix. Although the matrix determinant is equal to the product of its entire set of eigenvalues, the maximized determinant cannot ensure that the matrix has a maximized eigenvalue eventually. Thus, this paper directly uses the eigenvalue as the objective function of the optimization.

Equation 5 states the relation between the transformation errors and the final computing result. From [23], there are the same relation between the sampled point set  $y_i$  and the nominal home points set  $x_i$ . Suppose that  $\delta y_i$  is the error along its normal vector in the CMM frame. We will have

$$\delta y = A\Delta\xi \quad (15)$$

where  $\delta y = \{\delta y_1, \delta y_2, \dots, \delta y_N\}$ , from equation 15, we get

$$\Delta\xi = [(A^T A)^{-1} A^T] \delta y \quad (16)$$

And  $|\Delta\xi|^2 = \delta y^T M \delta y$

Where  $M = A[(A^T A)^{-1} A^T]$

In workpiece localization, the transformation error will be considered, and the start point for manufacturing is another important issue. The relation will be formulated as

$$\delta f = W_f^T \Delta\xi \quad (17)$$

$$\delta f = W_f^T [(A^T A)^{-1} A^T] \delta y \quad (18)$$

$$\|\delta f\|^2 = \delta f^T \delta f = \delta y^T M \delta y \quad (19)$$

$$= \delta y^T [(A^T A)^{-1} A^T]^T W_f W_f^T [(A^T A)^{-1} A^T] \delta y \quad (20)$$

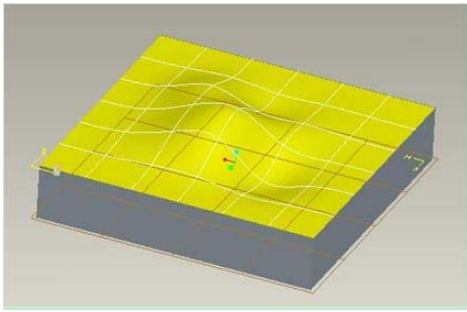


Fig. 1. Simulation model

Where  $M = [(A^T A)^{-1} A^T]^T W_f W_f^T [(A^T A)^{-1} A^T]$

Since M is a symmetric matrix, we can rewrite it as follows.

$$M = U^T \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \dots & \\ & & & \lambda_n \end{pmatrix} U \quad (21)$$

Where  $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$  are the eigenvalues of matrix M, and U is an orthogonal matrix. Substituting M into equation 20, we have

$$\begin{aligned} (\delta f)^2 &= V^T \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \dots & \\ & & & \lambda_n \end{pmatrix} V \\ &= \lambda_1 V_1^2 + \lambda_2 V_2^2 + \dots + \lambda_n V_n^2 \end{aligned} \quad (22)$$

where  $V_n = U(\delta y)$ .

Since U is an orthogonal matrix, we have

$$\|V_n\|^2 = V_n^T V_n = \|\delta y\|^2 \quad (23)$$

Suppose that  $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ , from equation 21, we can obtain the following

$$\lambda_1 \|\delta y\|^2 < \|\delta f\|^2 < \lambda_n \|\delta y\|^2 \quad (24)$$

From the above equation, for the same  $\delta y$ , the optimal location with the minimal locating error,  $\delta f$ , can be determined by minimizing  $max\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ . Thus, according to [22] we can first fix six points as the base point set and the maximum eigenvalue is computed. Then one or two new points interchange with points in the base point set. Its maximum eigenvalue is also computed. If the latter maximum eigenvalue is less than that of the base point set, the new points are recorded and selected as the candidate of the sampled points. If not, the points will be discarded.

### V. SIMULATION RESULT

In this section, a model of freeform surface is used to demonstrate the simulation results. The surface is a bicubic B-spline surface, which is shown in figure 1. Its dimension is  $15 \times 10$ cm. The behavior of equation 14 is first examined. After taking different sampling on the surface randomly, a  $N(0.005, 0.01)$  noise is added to these points and result of the application is denoted as actual point set. Then the relation

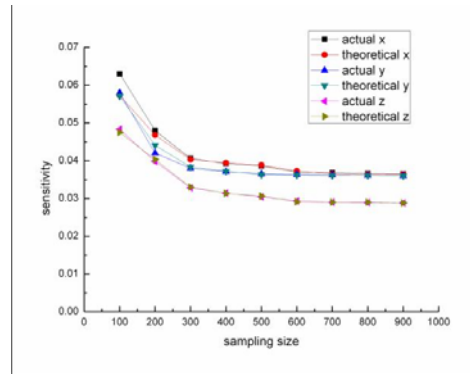


Fig. 2. Sensitivity of translation versus sampling size

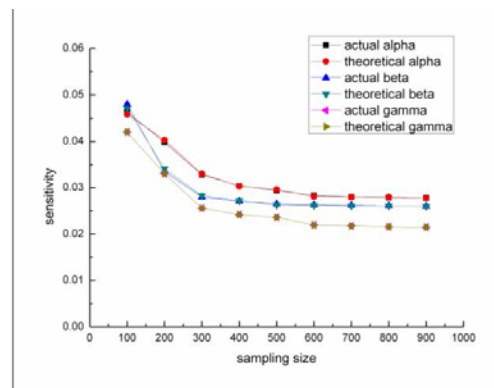


Fig. 3. Sensitivity of rotation versus sampling size

between the six localization parameters and the number of sampled is plotted. It is obviously that the localization parameters of actual point set match more well with that of the theoretical point set when the size of the sampled points increases.

From figure 2 and 3, when the sampled point size exceeds 200, these curves match quite well. It means that K is constant in the two cases. Before the determination of the sampling size, K must be estimated. By sampling a reasonable number of sampled points, we can construct the sensitivity matrix A. From equation 12, the maximum sensitivity number can be calculated. Then the K can be estimated as  $K = S_{max} \sqrt{N}$ . Here, we use 300 points to calculate the sensitivity numbers of the six localization parameters. Table 1 lists the six sensitivity numbers. The largest sensitivity number is in the y direction. So  $K = 0.0406 \times \sqrt{300} = 0.7032$ . The standard deviation of errors in the surface normal direction of the 300 points is 0.128 mm. Based on the tolerance, if we want to control the localization accuracy to be within 0.008 mm, then  $\delta_y = \frac{U}{2} = \frac{0.008}{2} = 0.004$ . So the sampling size will be arrived at

$$N = \left( \frac{0.7031}{0.004/0.128} \right)^2 = 506 \quad (25)$$

TABLE I SENSITIVITY OF THE SIX LOCALIZATION PARAMETERS

	Alpha	Beta	Gamma	X	Y	Z
Sens	0.00313	0.0276	0.0232	0.0371	0.0406	0.0331

After the sampling size is determined, we sample the surface

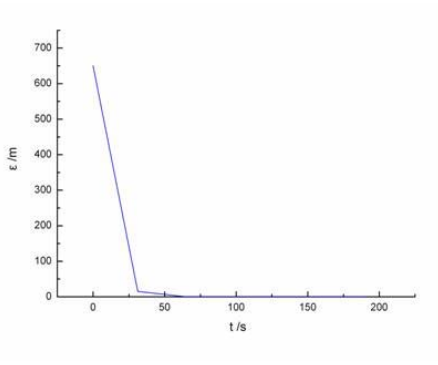


Fig. 4. Total error versus computing time

with 1200 points. Since regions near surface boundaries are not desirable for sampling and the probing radius is 3-mm, we leave 4-mm margin from every boundary of the surface to simulate the sampling. As the method in [22], six points are generated randomly. Or if some points are significant, they can be chosen into the six points. The maximum eigenvalue of the matrix M whose elements are decided by the six points is computed and denoted as  $\lambda$ . Then an exchange between one of the six points and a candidate point of the 1200 points is done. After each exchange, the maximum eigenvalue of the matrix M decided by the new six points is computed and denoted as  $\lambda_0$ . If  $\lambda_0 > \lambda$ , the candidate point will be deleted. Otherwise, it is saved and used in the next procedure. When the number of those saved points is equal to the pre-determined size, a transformation acts on those saved points to get the theoretical sampling points. The transformation is:

$$R = \begin{bmatrix} 0.9564 & -0.2571 & 0.1386 \\ 0.2580 & 0.9661 & 0.0121 \\ -0.1371 & 0.0242 & 0.9903 \end{bmatrix}, P = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \quad (26)$$

Where  $\alpha = 5^\circ$ ,  $\beta = 8^\circ$ ,  $\gamma = 10^\circ$ ,  $P_x = 2$ ,  $P_y = 2$ ,  $P_z = 4$ . The localization is implemented. And the Hong-Tan algorithm is used. The detail of localization and Hong-Tan algorithm is referred to [24]. After the localization process, the uncertainty was calculated using equation 12 again. The  $2\sigma_{\xi_i} = 0.0076$  mm is very close to what was required. And when we stop the algorithm, the function reached 0.000623 mm. the relation between the total error and time is showed in figure 4.

Let  $g_a = (p_a, R_a)$  and  $g_e = (p_e, R_e)$  be, respectively, the actual and estimated transformation. We define the rotational and translation errors be

$$\varepsilon_R = |\theta|$$

where  $e^{\hat{\omega}\theta} = R_e^T R_a$ ,  $\|\omega\| = 1$ , and

$$\varepsilon_p = \|p_e - p_a\|$$

Here the comparison is performed between the method proposed in this paper and the method proposed in [22]. We call our method as method a, and the method in [22] as method b. Figure 5 and figure 6 plot rotational and translation errors for each method as a function of the number of measurement points. From the figures we see that our method have better property than the method b.

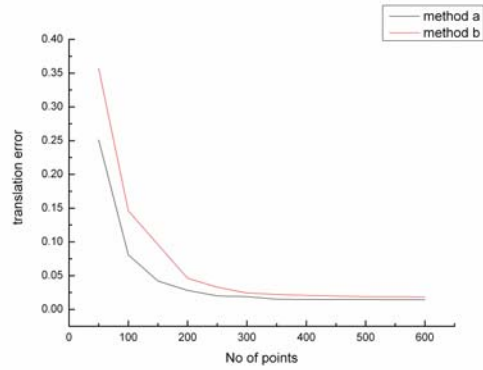


Fig. 5. Translation error for Optimization for sampling strategy in freeform surface inspection

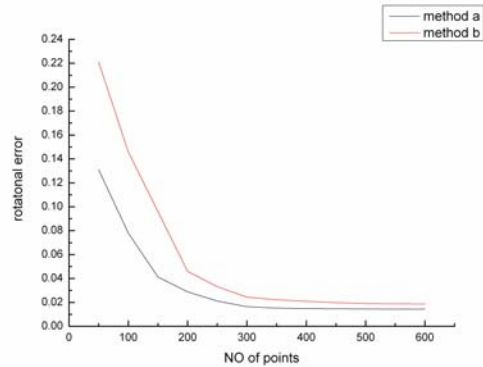


Fig. 6. Rotation error for Optimization for sampling strategy in freeform surface inspection

## VI. CONCLUSION

Inspection and workpiece localization etc have many important applications in the industry process. Since the algorithms applied in the fields are time consuming and the devices only sample discrete points on the workpiece surface, a good sampling strategy would help perform a reliable and economical decision. The sampling size and location of the points are addressed. To predict the minimum required size of the points, uncertainty analysis of the localization parameters is studied. The developed uncertainty model suggests that the uncertainty of the localization parameter is proportional to the standard deviation of the error of the sampled points and the geometric constant of the sampled surface, but inversely proportional to the squared root of sampled points. In linear theory, a matrix's eigenvalues reflect the magnitude change between the origin vector and the vectors transformed by the matrix. Although the matrix determinant is equal to the product of it entire set of eigenvalues, the maximized determinant cannot ensure that the matrix has a maximized eigenvalue eventually. Thus, an eigenvalue-optimal strategy for determining the locations is proposed. A freeform surface is used and the simulation result shows its effectivity.

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