Low and high cycle fatigue

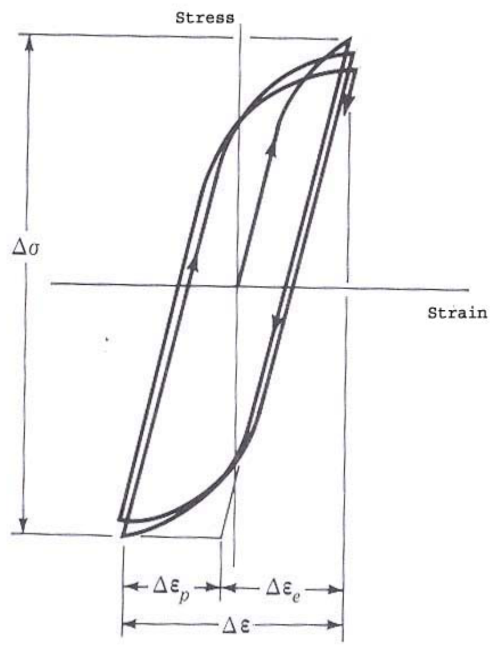
Instructional Objectives

At the end of this lesson, the students should be able to understand

- Design of components subjected to low cycle fatigue; concept and necessary formulations.
- Design of components subjected to high cycle fatigue loading with finite life; concept and necessary formulations.
- Fatigue strength formulations; Gerber, Goodman and Soderberg equations.

3.4.1 Low cycle fatigue

This is mainly applicable for short-lived devices where very large overloads may occur at low cycles. Typical examples include the elements of control systems in mechanical devices. A fatigue failure mostly begins at a local discontinuity and when the stress at the discontinuity exceeds elastic limit there is plastic strain. The cyclic plastic strain is responsible for crack propagation and fracture. Experiments have been carried out with reversed loading and the true stress-strain hysteresis loops are shown in **figure-3.4.1.1**. Due to cyclic strain the elastic limit increases for annealed steel and decreases for cold drawn steel. Low cycle fatigue is investigated in terms of cyclic strain. For this purpose we consider a typical plot of strain amplitude versus number of stress reversals to fail for steel as shown in **figure-3.4.1.2**.



***3.4.1.1F**- A typical stress-strain plot with a number of stress reversals (Ref.[4]).*

Here the stress range is $\Delta\sigma$. $\Delta\varepsilon_p$ and $\Delta\varepsilon_e$ are the plastic and elastic strain ranges, the total strain range being $\Delta\varepsilon$. Considering that the total strain amplitude can be given as

$$\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e$$

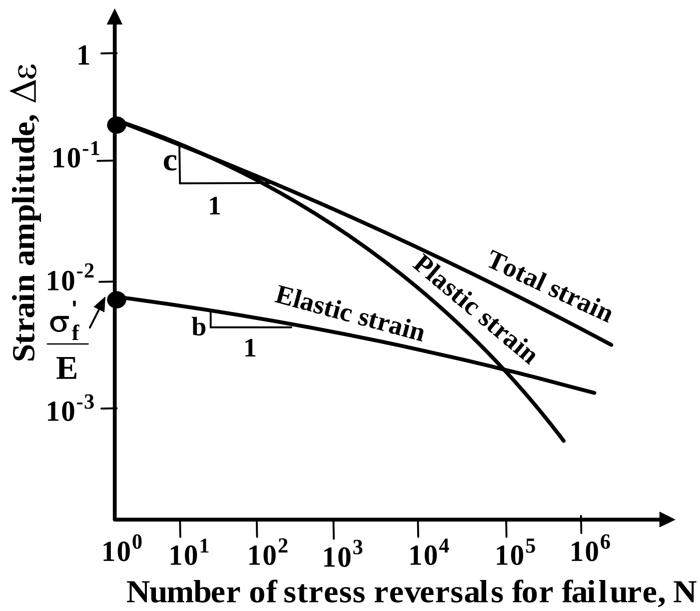
A relationship between strain and a number of stress reversals can be given as

$$\Delta\varepsilon = \frac{\sigma_f'}{E}(N)^a + \varepsilon_f'(N)^b$$

where σ_f and ε_f are the true stress and strain corresponding to fracture in one cycle and a, b are systems constants. The equations have been simplified as follows:

$$\Delta\varepsilon = \frac{3.5\sigma_u}{EN^{0.12}} + \left(\frac{\varepsilon_p}{N}\right)^{0.6}$$

In this form the equation can be readily used since σ_u, ε_p and E can be measured in a typical tensile test. However, in the presence of notches and cracks determination of total strain is difficult.



3.4.1.2F- *Plots of strain amplitude vs number of stress reversals for failure.*

3.4.2 High cycle fatigue with finite life

This applies to most commonly used machine parts and this can be analyzed by idealizing the S-N curve for, say, steel, as shown in **figure- 3.4.2.1** .

The line between 10^3 and 10^6 cycles is taken to represent high cycle fatigue with finite life and this can be given by

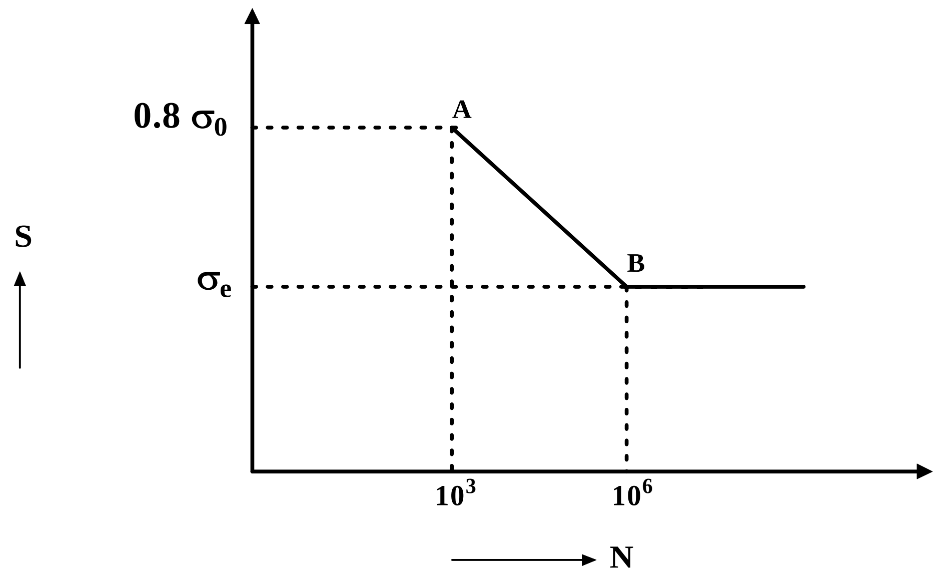
$$\log S = b \log N + c$$

where S is the reversed stress and b and c are constants.

At point A $\log(0.8\sigma_u) = b \log 10^3 + c$ where σ_u is the ultimate tensile stress

and at point B $\log \sigma_e = b \log 10^6 + c$ where σ_e is the endurance limit.

This gives $b = -\frac{1}{3}\log\frac{0.8\sigma_u}{\sigma_e}$ and $c = \log\frac{(0.8\sigma_u)^2}{\sigma_e}$



***3.4.2.1F**- A schematic plot of reversed stress against number of cycles to fail.*

3.4.3 Fatigue strength formulations

Fatigue strength experiments have been carried out over a wide range of stress variations in both tension and compression and a typical plot is shown in **figure-3.4.3.1**. Based on these results mainly, Gerber proposed a parabolic correlation and this is given by

$$\left(\frac{\sigma_m}{\sigma_u}\right)^2 + \left(\frac{\sigma_v}{\sigma_e}\right) = 1 \quad \text{Gerber line}$$

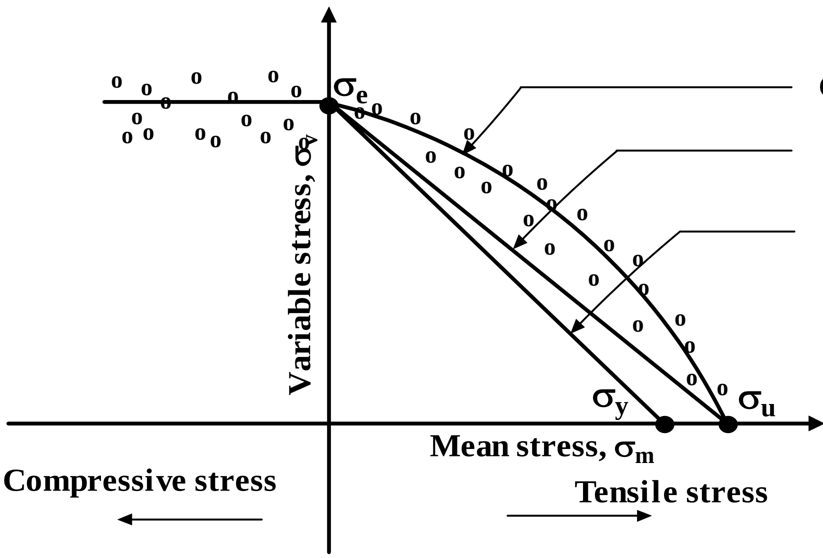
Goodman approximated a linear variation and this is given by

$$\left(\frac{\sigma_m}{\sigma_u}\right) + \left(\frac{\sigma_v}{\sigma_e}\right) = 1 \quad \text{Goodman line}$$

Soderberg proposed a linear variation based on tensile yield strength σ_Y and this is given by

$$\left(\frac{\sigma_m}{\sigma_y}\right)+\left(\frac{\sigma_v}{\sigma_e}\right)=1 \qquad \text{Soderberg line}$$

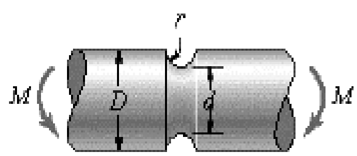
Here, σ_m and σ_v represent the mean and fluctuating components respectively.



***3.4.3.1F**- A schematic diagram of experimental plots of variable stress against mean stress and Gerber, Goodman and Soderberg lines.*

3.4.4 Problems with Answers

Q.1: A grooved shaft shown in **figure- 3.4.4.1** is subjected to rotating-bending load. The dimensions are shown in the figure and the bending moment is 30 Nm. The shaft has a ground finish and an ultimate tensile strength of 1000 MPa. Determine the life of the shaft.



r = 0.4 mm
D = 12 mm
d = 10 mm

3.4.4.1F

A.1:

Modified endurance limit, $\sigma_e' = \sigma_e C_1C_2C_3C_4C_5/ K_f$

Here, the diameter lies between 7.6 mm and 50 mm : $C_1 = 0.85$

The shaft is subjected to reversed bending load: $C_2 = 1$

From the surface factor vs tensile strength plot in **figure- 3.3.3.5**

For UTS = 1000 MPa and ground surface: $C_3 = 0.91$

Since $T \leq 450^oC$, $C_4 = 1$

For high reliability, $C_5 = 0.702$.

From the notch sensitivity plots in **figure- 3.3.4.2** , for r=0.4 mm and UTS = 1000 MPa, q = 0.78

From stress concentration plots in **figure-3.4.4.2**, for r/d = 0.04 and D/d = 1.2, $K_t = 1.9$. This gives $K_f = 1+q (K_t -1) = 1.702$.

Then, $\sigma_e' = \sigma_e x\ 0.89x\ 1x\ 0.91x\ 1x\ 0.702/1.702 = 0.319\ \sigma_e$

For steel, we may take $\sigma_e = 0.5\ \sigma_{UTS} = 500$ MPa and then we have $\sigma_e' = 159.5$ MPa.

Bending stress at the outermost fiber, $\sigma_b = \frac{32M}{\pi d^3}$

For the smaller diameter, d=0.01 mm, $\sigma_b = 305\ MPa$

Since $\sigma_b > \sigma_e^{'}$ life is finite.

For high cycle fatigue with finite life,

log S = b log N + C

where, $b = -\frac{1}{3}\log\frac{0.8\sigma_0}{\sigma_e'} = -\frac{1}{3}\log\frac{0.8x1000}{159.5} = -0.233$

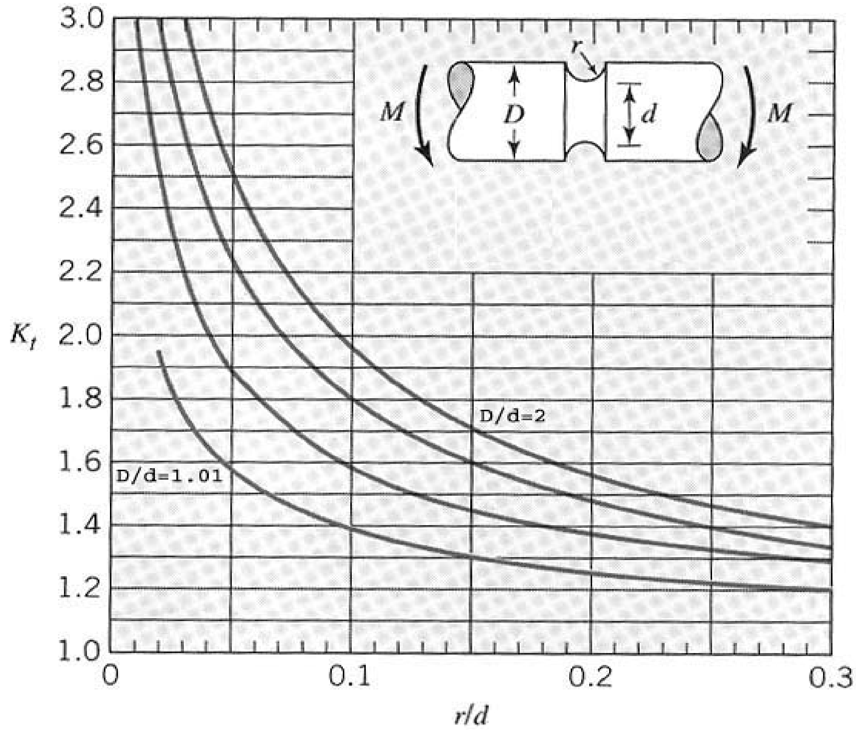
$$c = \log\frac{(0.8\sigma_u)^2}{\sigma_e'} = \log\frac{(0.8x1000)^2}{159.5} = 3.60$$

Therefore, finite life N can be given by

$N=10^{-c/b}\ S^{1/b}$ if $10^3 \leq N \leq 10^6$.

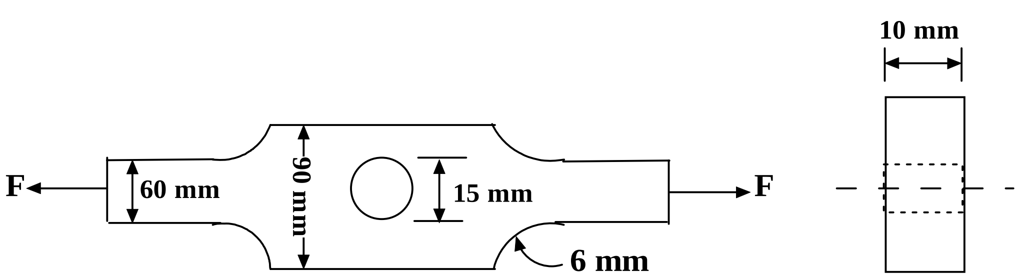
Since the reversed bending stress is 306 MPa,

N = $2.98x\ 10^9$ cycles.



3.4.4.2F (Ref.[5])

Q.2: A portion of a connecting link made of steel is shown in **figure-3.4.4.3** . The tensile axial force F fluctuates between 15 KN to 60 KN. Find the factor of safety if the ultimate tensile strength and yield strength for the material are 440 MPa and 370 MPa respectively and the component has a machine finish.



3.4.4.3F

A.2:

To determine the modified endurance limit at the step, $\sigma_e' = \sigma_e C_1C_2C_3C_4C_5/ K_f$ where

C_1 = 0.75 since d ≥ 50 mm

C_2 = 0.85 for axial loading

C_3 = 0.78 since σ_u = 440 MPa and the surface is machined.

C_4 = 1 since T≤ 450^{o}C

C_5 = 0.75 for high reliability.

At the step, r/d = 0.1, D/d = 1.5 and from **figure-3.2.4.6**, K_t = 2.1 and from **figure- 3.3.4.2** q = 0.8. This gives K_f = 1+q (K_t -1) = 1.88.

Modified endurance limit, $\sigma_e' = \sigma_e$x 0.75x 0.85x 0.82x 1x 0.75/1.88 = 0.208 σ_e

Take $\sigma_e = 0.5\ \sigma_u$. Then σ_e' = 45.76 MPa.

The link is subjected to reversed axial loading between 15 KN to 60 KN.

This gives $\sigma_{max} = \frac{60x10^3}{0.01x0.06} = 100\,\text{MPa}$, $\sigma_{min} = \frac{15x10^3}{0.01x0.06} = 25\,\text{MPa}$

Therefore, σ_{mean} = 62.5 MPa and σ_v = 37.5 MPa.

Using Soderberg's equation we now have,

$\frac{1}{F.S} = \frac{62.5}{370} + \frac{37.5}{45.75}$ so that F.S = 1.011

This is a low factor of safety.

Consider now the endurance limit modification at the hole. The endurance limit modifying factors remain the same except that K_f is different since K_t is different. From **figure- 3.2.4.7** for d/w= 15/90 = 0.25, K_t = 2.46 and q remaining the same as before i.e 0.8

Therefore, K_f = 1+q (K_t -1) = 2.163.

This gives σ_e' = 39.68 MPa. Repeating the calculations for F.S using Soderberg's equation , F.S = 0.897.

This indicates that the plate may fail near the hole.

Q.3: A 60 mm diameter cold drawn steel bar is subjected to a completely reversed torque of 100 Nm and an applied bending moment that varies between 400 Nm and -200 Nm. The shaft has a machined finish and has a 6 mm diameter hole drilled transversely through it. If the ultimate tensile stress σ_u and yield stress σ_y of the material are 600 MPa and 420 MPa respectively, find the factor of safety.

A.3:

The mean and fluctuating torsional shear stresses are

$\tau_m = 0$; $\tau_v = \dfrac{16x100}{\pi x (0.06)^3}$ = 2.36 MPa.

and the mean and fluctuating bending stresses are

$\sigma_m = \dfrac{32x100}{\pi x (0.06)^3}$ = 4.72 MPa; $\sigma_v = \dfrac{32x300}{\pi x (0.06)^3}$ = 14.16 MPa.

For finding the modifies endurance limit we have,

C_1 = 0.75 since d > 50 mm

C_2 = 0.78 for torsional load

= 1 for bending load

C_3 = 0.78 since σ_u = 600 MPa and the surface is machined (**figure-3.4.4.2**).

C_4 = 1 since $T \leq 450^oC$

C_5 = 0.7 for high reliability.

and K_f = 2.25 for bending with d/D =0.1 (from **figure- 3.4.4.5**)

= 2.9 for torsion on the shaft surface with d/D = 0.1 (from **figure-3.4.4.6**)

This gives for bending $\sigma_{eb}' = \sigma_e$x 0.75x1x 0.78x 1x 0.7/2.25 = 0.182 σ_e

For torsion $\sigma_{es}' = \sigma_{es}$x 0.75x0.78x 0.78x 1x 0.7/2.9 = 0.11 σ_e

And if $\sigma_e = 0.5\ \sigma_u$ = 300 MPa, σ_{eb}' =54.6 MPa; σ_{es}' = 33 MPa

We may now find the equivalent bending and torsional shear stresses as:

$\tau_{eq} = \tau_m + \tau_v \dfrac{\tau_y}{\sigma_{es}'}$ = 15.01 MPa (Taking $\tau_y = 0.5\ \sigma_y$ = 210 MPa)

$$\sigma_{eq} = \sigma_m + \sigma_v \frac{\sigma_y}{\sigma'_{eb}}$$ = 113.64 MPa.

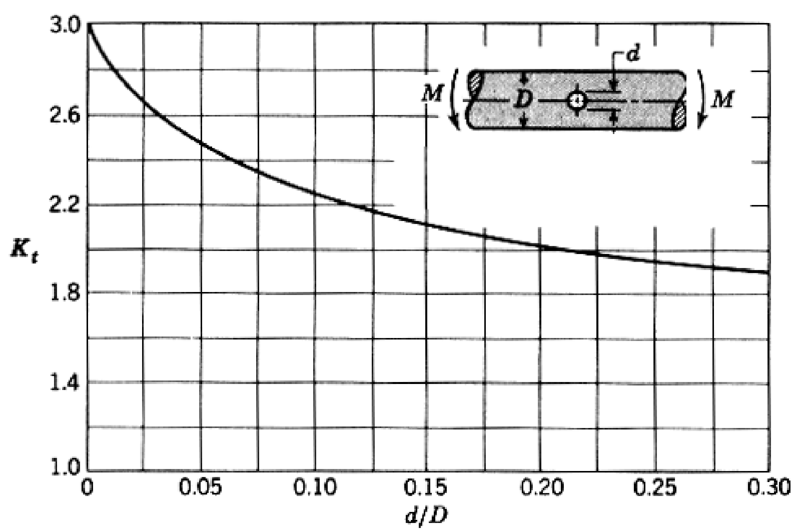
Equivalent principal stresses may now be found as

$$\sigma_{1eq} = \frac{\sigma_{eq}}{2} + \sqrt{\left(\frac{\sigma_{eq}}{2}\right)^2 + \tau_{eq}^2}$$

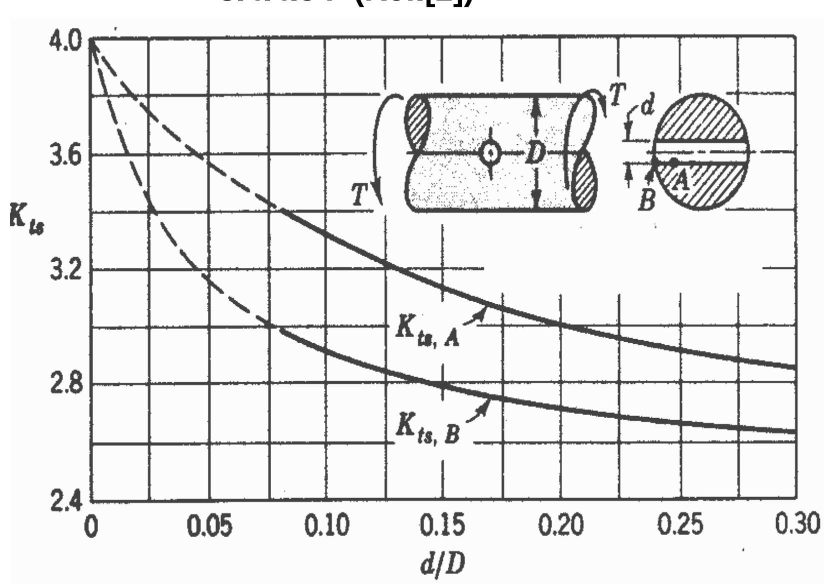
$$\sigma_{2eq} = \frac{\sigma_{eq}}{2} - \sqrt{\left(\frac{\sigma_{eq}}{2}\right)^2 + \tau_{eq}^2}$$

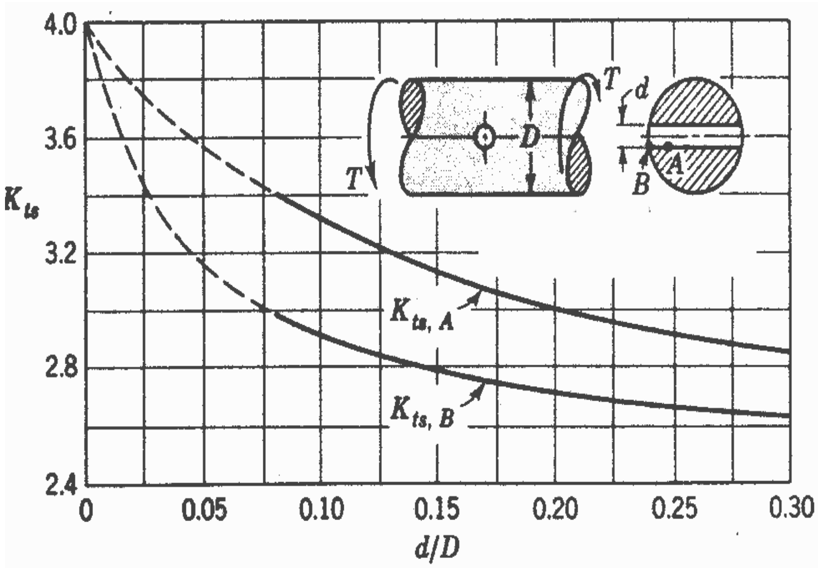
and using von-Mises criterion

$$\sigma_{eq}{}^2 + 3\tau_{eq}^2 = 2\left(\frac{\sigma_y}{F.S}\right)^2$$ which gives F.S = 5.18.



3.4.4.5 F (Ref.[2])





3.4.4.6 F (Ref.[2])

3.4.5Summary of this Lesson

The simplified equations for designing components subjected to both low cycle and high cycle fatigue with finite life have been explained and methods to determine the component life have been demonstrated. Based on experimental evidences, a number of fatigue strength formulations are available and Gerber, Goodman and Soderberg equations have been discussed. Methods to determine the factor of safety or the safe design stresses under variable loading have been demonstrated.

3.4.6 Reference for Module-3

1) Design of machine elements by M.F.Spotts, Prentice hall of India,1991.
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4) Mechanical engineering design by Joseph E. Shigley, McGraw Hill, 1986.
5) Fundamentals of machine component design, 3rd edition, by Robert C. Juvinall and Kurt M. Marshek, John Wiley & Sons, 2000.