Introduction to Damping in Free and Forced Vibrations

This chapter mainly deals with the effect of damping in two conditions like free and forced excitation of mechanical systems. Damping plays an important role in dying out the vibration amplitude effectively by absorbing the excitation energy.

Example -1.2 [1]: A machine can move in a vertical degree-of-freedom only. It is mounted elastically to a rigid foundation. Assume that the machine can be regarded as a point mass m and that the isolator is an ideal spring, the spring rate of which is *k*. What is the mounted resonance frequency of the machine in the following cases:

a) m = 10 kg,
 (i) *k* = 10 kN/m.
 (ii) *k* = 100 kN/m.
 (iii) *k* = 1 MN/m.

b) m = 100 kg,
 (i) *k* = 10 kN/m.
 (ii) *k* = 100 kN/m.
 (iii) *k* = 1 MN/m.

Solution-1: $\mu_0 = \sqrt{©/m}$, and $f_0 = \mu_0 / 2®$

a) (*i*) $f_0 = \sqrt{10 \cdot 10^3/10}\Big/2® = 5.03$ Hz.

(*ii*) $f_0 = \sqrt{100 \cdot 10^3/10}\Big/2® = 15.92$ Hz.

(*iii*) $f_0 = \sqrt{1 \cdot 10^6/10}\Big/2® = 50.3$ Hz.

b) (*i*) $f_0 = \sqrt{10 \cdot 10^3/100}\Big/2® = 1.59$ Hz.

(*ii*) $f_0 = \sqrt{100 \cdot 10^3/100}\Big/2® = 5.03$ Hz.

(*iii*) $f_0 = \sqrt{1 \cdot 10^6/100}\Big/2® = 15.92$ Hz.

Problem: 1.3 A machine mounted on vibration isolators is modeled as a single degree-of-freedom system. The relevant parameters are estimated to be as follows: mass m = 370 kg, spring rate *k* = 2 x 105 N/m, damping constant ¢ = 0.2 s^{-1}. Calculate the natural frequency of the mounted machine and the displacement amplitude of the machine, if it is excited at that frequency by a force with peak amplitude of 10 N.

Solution:

The parameters are the mass m = 370 kg, spring rate *k* = 2 .105 N/m and damping constant ¢ = 0.2 s-1.

The eigen-frequency is: $\mu_0 = \sqrt{©/m} = \sqrt{2 \cdot 10^5/370} \approx 23.2$ rad/s

Eigen-frequency: $f_0 = \frac{\mu_0}{2®} \approx 3.7$ Hz.

Let: $\mathbf{x}(t) = \mathbf{A}e^{i\mu t}$ $\mathbf{F}(t) = 10e^{(i\mu t + ¤)}$

So the modulus is: $|\mathbf{A}| = \frac{|10/m|}{\sqrt{(\mu_0^2 - \mu^2)^2 + (2¢\mu)^2}}$

which at $\mu = \mu_0$:

$$|\mathbf{A}| = \frac{10/m}{2¢\mu_0} \approx 0.029 \text{ m} = 2.9 \text{ mm}$$

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Introduction to damping:

Damping is a phenomenon by which mechanical energy is dissipated (usually converted as thermal energy) in dynamic systems. Vibrating systems can encounter damping in various ways like

- Intermolecular friction

- Sliding friction
- Fluid resistance

Three primary mechanisms of damping are as:

- Internal damping – of material
- Structural damping – at joints and interface
- Fluid damping – through fluid -structure interactions

Two types of external dampers can be added to a mechanical system to improve its energy dissipation characteristics:

- Active dampers – require external source of power
- Passive dampers – Does not required

MATERIAL (Internal) Damping

- Internal damping originates from energy dissipation associated with:
- microstructure defects (grain boundaries & impurities),
- thermo elastic effects (caused by local temperature gradients)
- eddy-current effects (ferromagnetic materials),
- dislocation motion in metals, etc.

Types of Internal damping:

- Viscoelastic damping
- Hysteretic damping

Damping estimation of any system is the most difficult process in any vibration analysis. The damping is generally complex and generally for mechanical systems it is so small to compute.

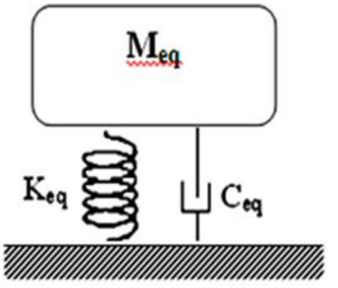


Fig 1.6 Spring – Mass – Damper system

$$M_{eq}\ddot{x} + C_{eq}\dot{x} + K_{eq}x = 0$$

There are three cases of interest. The discussion about these three cases is as follows:

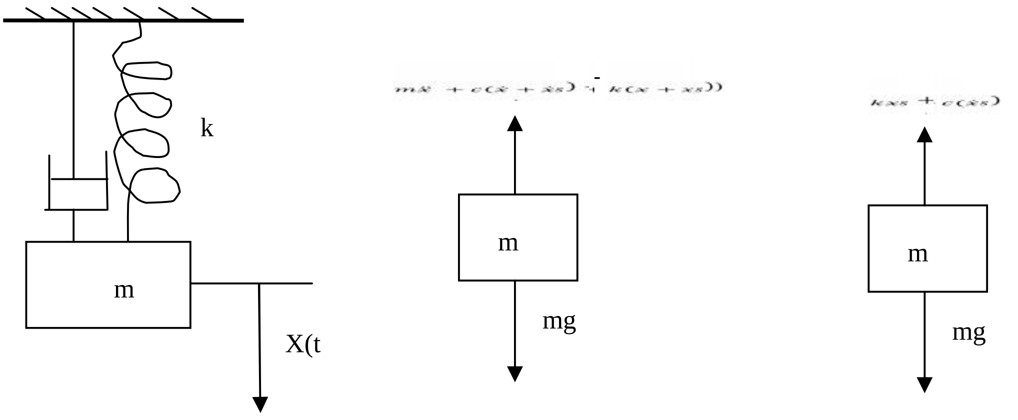
A. **<u>Under-damped Vibrations:</u>** This case occurs if the parameters of the system are such that ($0< \xi <1$). In this case the discriminate, $\omega_n\sqrt{\xi^2 - 1}$ becomes negative, and the roots of equation (1.11) becomes complex. Thus, the solution of eqn.(1.11) yields as follows.

$$x(t) = e^{-\zeta\omega_n t}(A\cos\omega\ \ t + B\sin\omega\ \ t) \qquad (1.12)$$

Or,

$$x(t) = Ce^{-\zeta\omega_n t}\sin(\omega\ \ t + \phi)$$

Where, A, B, C and ϕ are constant, their values may be determined by applying the initial conditions.



Free body diagram (FBD)

At $t = t_0,\ x = x_0, v = v_0$

$$A = x_0,$$

$$B = \frac{v_0 + \zeta\omega_n x_0}{\omega_d}$$

$$C = \frac{\sqrt{(v_0 + \zeta\omega_n x_0)^2 + (x_0\omega_d)^2}}{\omega_d}$$

$$\phi = \tan^{-1}\left(\frac{x_0\omega_d}{v_0 + \zeta\omega_n x_0}\right)$$

The underdamped response has the form as shown in Figure 1.7. It depicts that the amplitude of the vibrations are decaying with time.

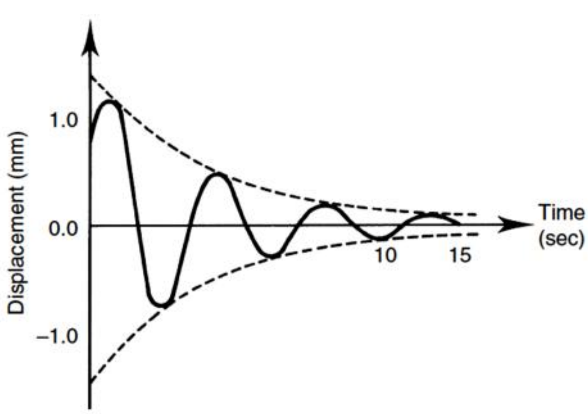


Fig. 1.7 Response of an underdamped system

B. <u>Over-damped Vibrations:</u>. This case occurs if the parameters of the system are such that $\xi > 1$. In this condition the discriminate, $\omega_n\sqrt{\xi^2 - 1}$ becomes negative, and the roots of equation (1.11) become negative real numbers. Thus the solution of eqn.(1.11) yields as follows.

$$x(t) = Ae^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + Be^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t} \qquad (1.13)$$

Where A and B are constant and can be determined applying the initial conditions as given follows.

$$A = \frac{v_0 + \left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n x_0}{2\omega_n\sqrt{\zeta^2 - 1}}$$

$$B = \frac{v_0 + \left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n x_0}{2\omega_n\sqrt{\zeta^2 - 1}}$$

The response of an overdamped system is shown in Fig. 1.8. It may be seen that an over-damped system does not oscillate, but rather returns to its rest position exponentially. The over damping affects the system response as shown in Fig. 1.9.

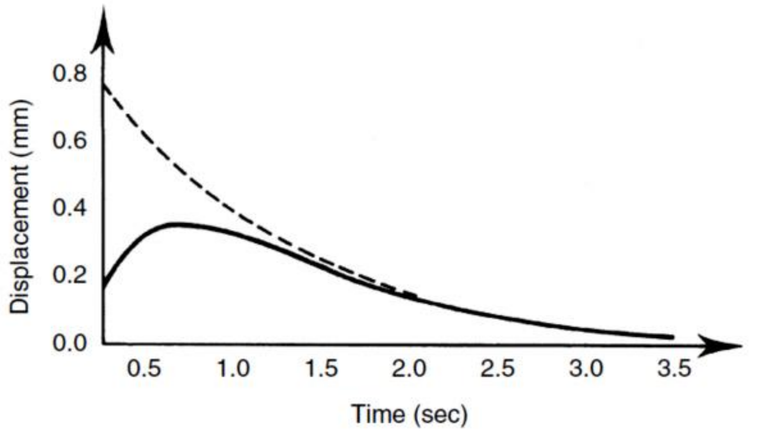


Fig. 1.8 Response of an overdamped system

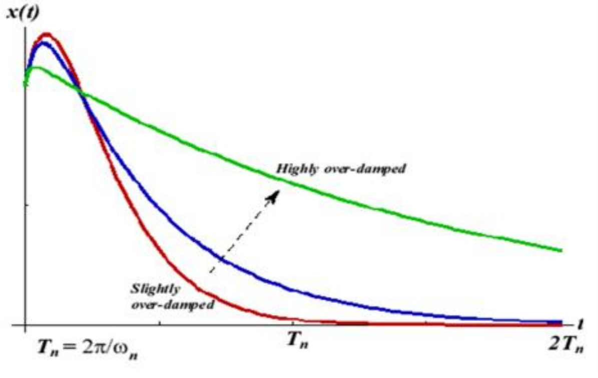


Fig. 1.9 Effect of Over-damping

C. **Critically damped**: When the value of damping coefficient becomes 1. It is known as critically damped system. In this condition discriminate, $\omega_n\sqrt{\xi^2 - 1}$ becomes zero, and the roots of equation (1.11) become negative repeated real numbers. Thus the solution of eqn. (1.11) yields as follows.

$$x(t) = e^{-\omega_n t}[(v_0 + \omega_n x_0)t + x_0] \qquad (1.14)$$

The response of a critically damped system is shown in Fig. 1.9.

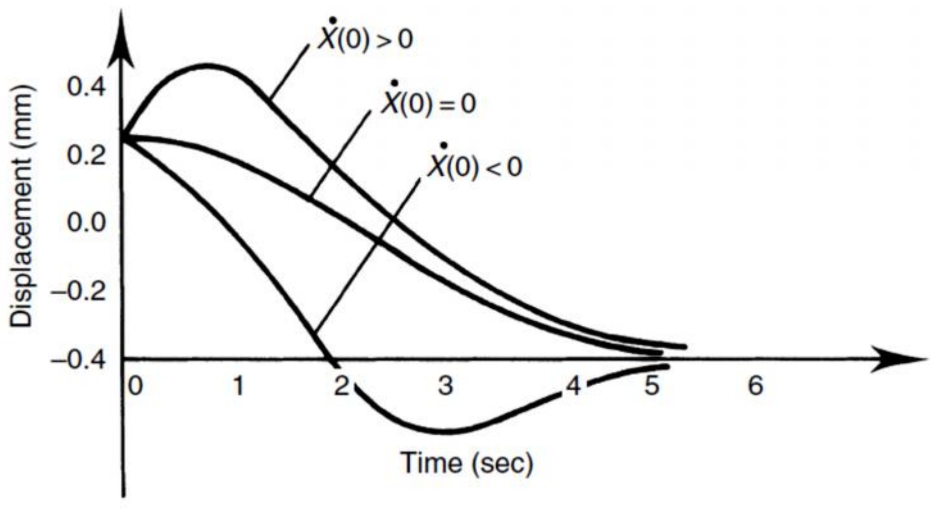


Fig. 1.10 Response of a critically damped system

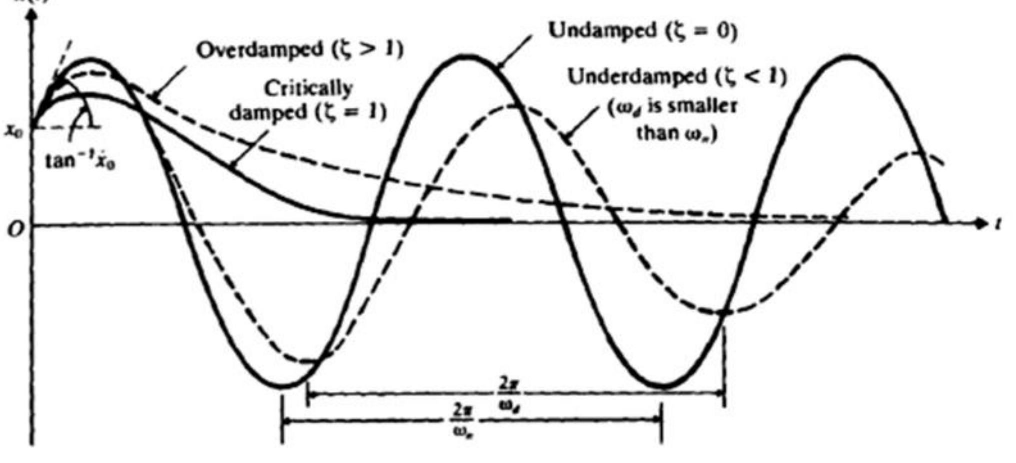
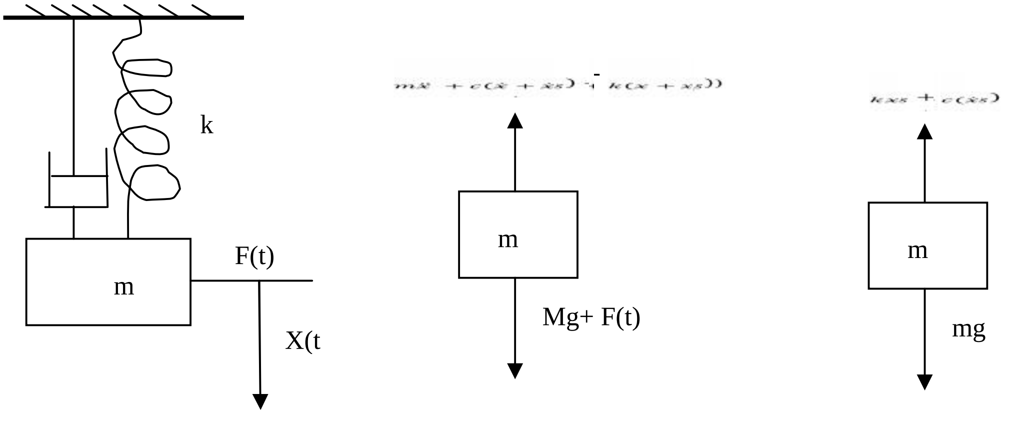


Fig. 1.11 Overall response of a mechanical system

FORCED RESPONSE

The preceding analysis considers the vibration of a component or structure as a result of some initial disturbance (i.e., v_0 and x_0). In this section, the vibration of a spring mass damper system subjected to an external force is examined. This external force may be of the form step function, impulse function, harmonic or ramp function.



(a) *(b)* *(c)*

Fig 1.12 (a) Schematic of a forced spring–mass–damper system, (b) free body diagram of the system in part (a), (c) free body diagram due to static condition.

In most of the situation the forcing function F(t), is periodic and having the following harmonic form.

$$F(t) = F_0 \sin\omega t$$

where, F_0 is the amplitude of the applied force and ω is the frequency of the applied force, sometimes called driving frequency.

From fig. 1.12, the equation of motion of a forced system may be expressed as follows.

$$m\ddot{x} + c\dot{x} + kx = F(t) = F_0 \sin\omega t \qquad (1.17)$$

The solution of this equation may be determined as;

x = Homogeneous solution (complemen func) + Particular solution

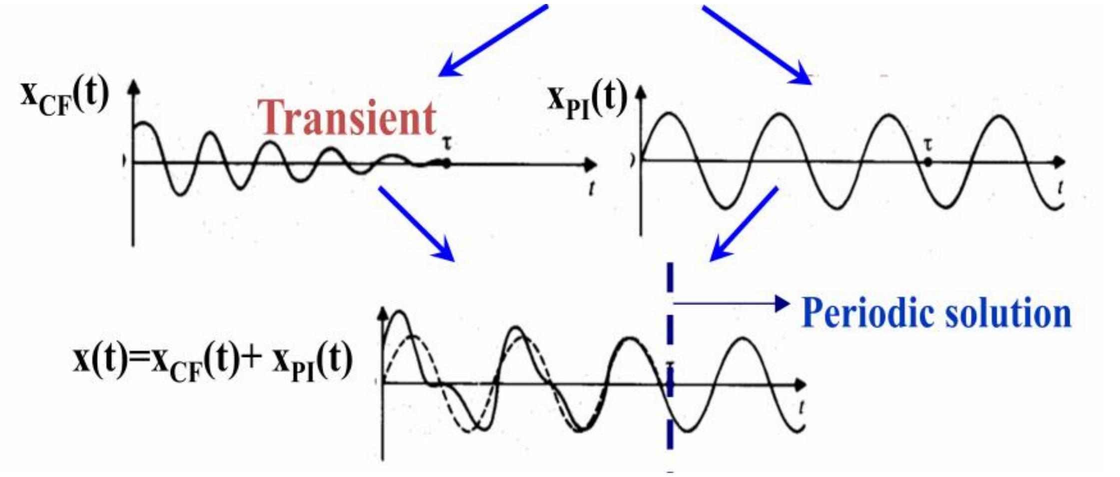


Fig. 1. 13 Complete solutaion (Transient (CF) and periodic (PI) solution)

The homogeneous solution can be easily obtained as the mathematical approach discussed for the solution of equation 1.11. Homogeneous solution and particular solution are usually referred to as the transient response and the steady state response sequentially. Physically, it is to assume that the steady state response will follow the forcing function. Hence, it is tempting to assume that the particular solution has the form

$$x_p(t) = X \sin(\omega t - \theta) \qquad (1.18)$$

Where, X = steady state amplitude

θ = phase shift at steady state

"Fundamentals of Sound and Vibrations" by KTH Sweden [1], this book is used under IITR-KTH MOU for course development.

Source:
http://nptel.ac.in/courses/112107088/2