Inertial Instruments and Inertial Navigation

Gimbals

Gimbals are essentially hinges that allow freedom of rotation about one axis. Gimbals often have superb bearings and motors to help achieve virtually frictionless behavior. Sensors in the bearings provide measurements of gimbal angles. Three gimbals allow freedom of rotation of a vehicle about three axes while a central platform remains stationary with respect to inertial space.

Gyros

A gyro is a spinning mass with relatively large angular momentum. We know that the rate of change of angular momentum is equal to the applied moment.

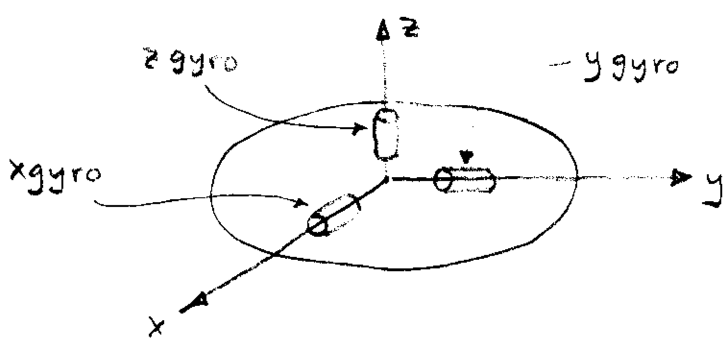
$$\frac{d}{dt_I}\bar{H} = \bar{M}$$

If no torque is applied then the angular momentum vector remains stationary with respect to inertial space. Gimbals allow a vehicle to rotate freely about a gyro so the gyro spin axis can provide a single axis direction that is stationary with respect to inertial space.

Restraining a gyro about an axis perpendicular to the angular momentum vector provides a means for measuring angular velocity with respect to inertial space. This device is called a rate gyro and is a common sensor for aiding in rate stabilization of vehicles (e.g., the D in a PD controller).

Inertial Platforms

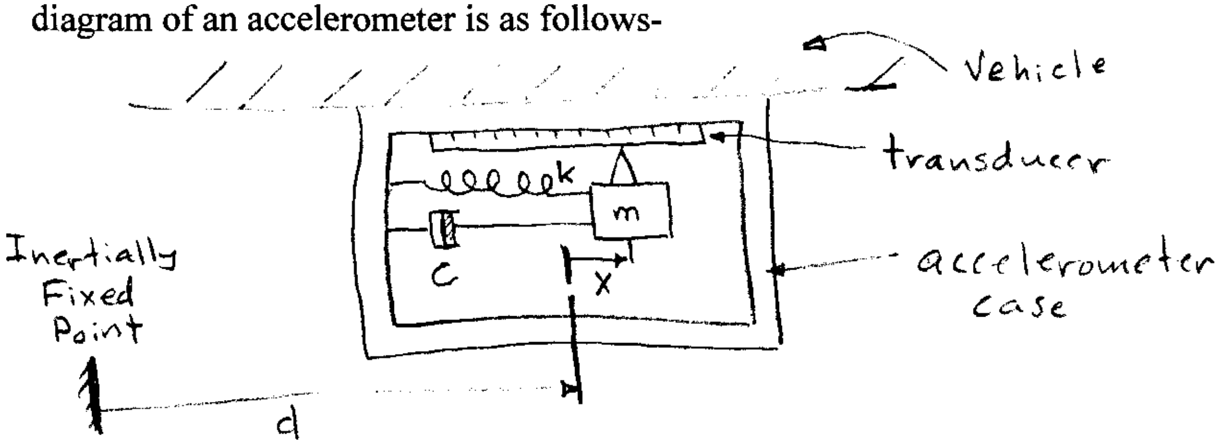
A gyro mounted on a platform can be used as a sensor in a feedback loop to stabilize the platform with respect to inertial space. This is called an inertially stabilized platform. As we will see, the inertially stabilized platform is an essential element of inertial navigation.



Applying torque to the gyro causes its spin vector (i.e., angular momentum vector) to move with respect to inertial space. Thus the inertially stabilized platform can be reoriented with respect to inertial space.

Accelerometers

A second important inertial sensor is the accelerometer. A simplified diagram of an accelerometer is as follows-



Where

m=test mass

d=displacement of the vehicle from an inertially fixed point

x=displacement of the test mass from its rest point

x+d=displacement of the test mass from the inertially fixed point

x_t=transducer output signal

thus

$$m\left(\frac{d^2(d+x)}{dt^2}\right)_I = F_x = \text{total force applied to } m \text{ in the } x \text{ direction}$$

$$= -c\frac{dx}{dt} - kx$$

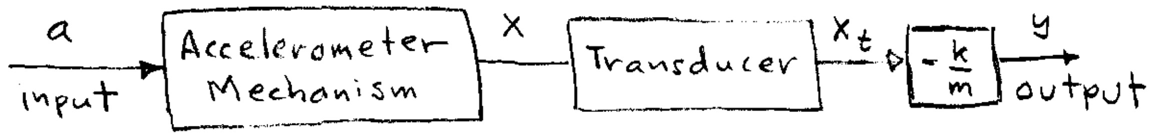
So the system differential equation is

$$m(\ddot{d}+\ddot{x}) + c\dot{x} + kx = 0$$

or

$$m\ddot{x} + c\dot{x} + kx = -ma \qquad a = \ddot{d}$$

which is a second order LTI system. Vehicle acceleration as the input and the output is the negative of indicated test mass displacement times k/m.



Note in particular that if the vehicle acceleration is constant then the steady state output is constant, thus producing an indication of that acceleration.

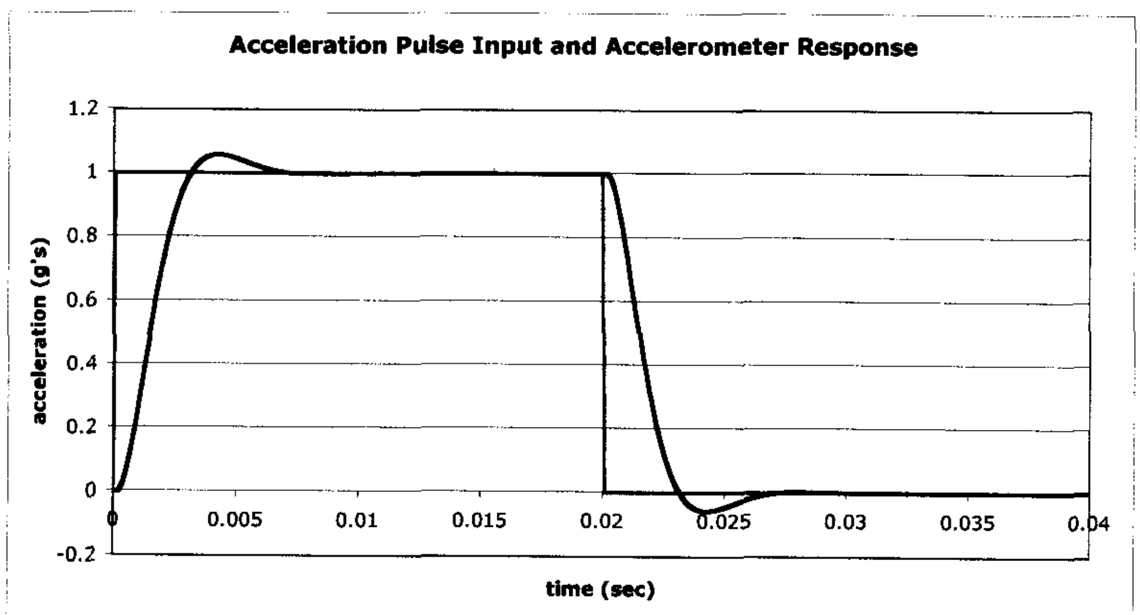
The undamped natural frequency and damping ratio of the accelerometer are

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{c}{2\sqrt{km}}$$

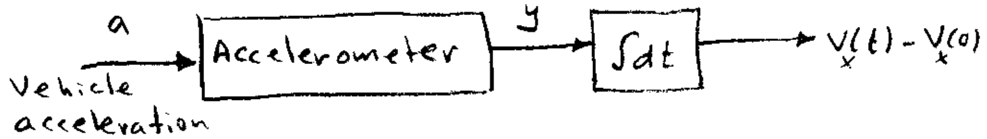
where the parameters c and k are controlled by the manufacturer. Typical values are

$$\omega = 10^3 \frac{rad}{sec} \qquad \zeta = 0.7$$

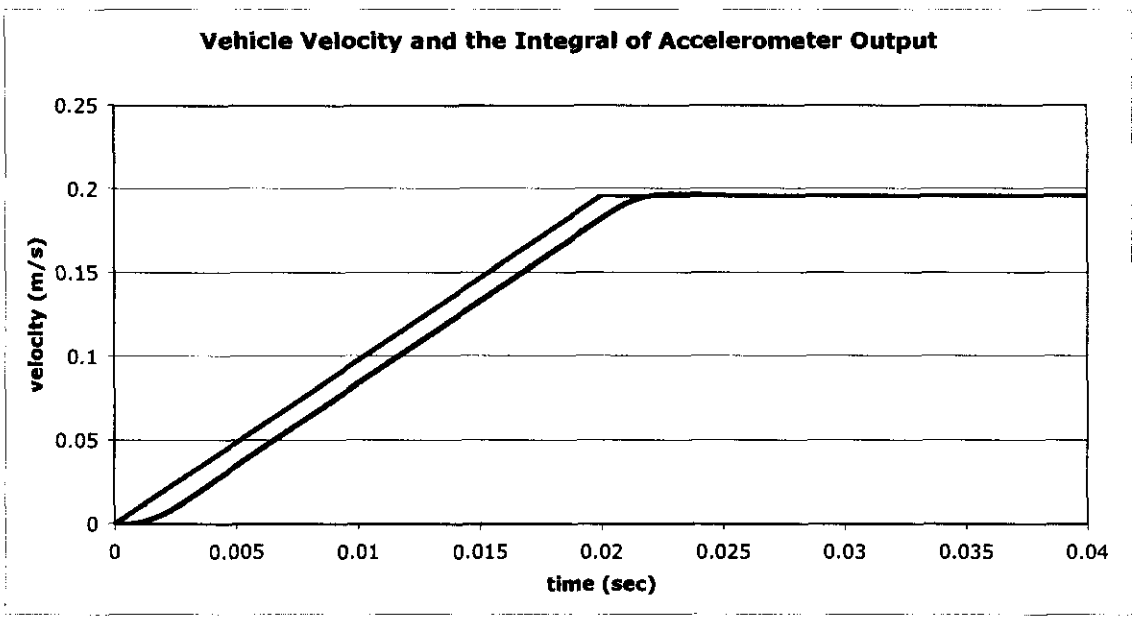
The following figure illustrates the response of such a system to a very short one “g” pulse of vehicle acceleration that is 20 milliseconds in duration.



Commonly vehicle velocity is desired so the accelerometer output is integrated over time



The following figure illustrates the vehicle velocity produced by the acceleration pulse shown above, compared with the time integral of the accelerometer output.



Note that, except for a small delay, the integral of the accelerometer output is a very good representation of vehicle velocity.

<u>Spacecraft System Applications of Inertial Systems</u>

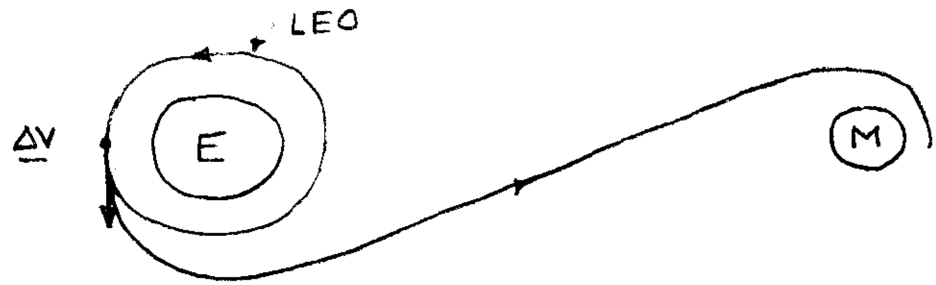
Space systems utilize inertially stabilized platforms in a number of ways

-provide a reference for stabilizing and controlling vehicle attitude

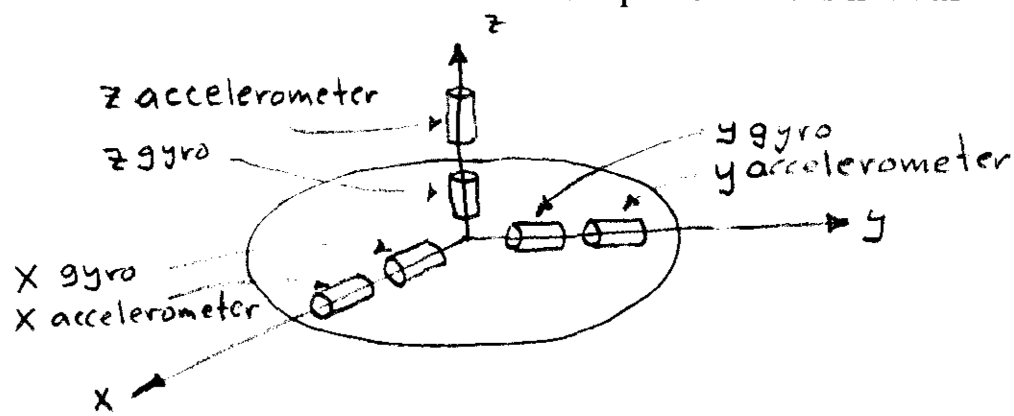
-stabilize sensors and point them in desired directions

-provide a stable reference for estimating changes in vehicle velocity

Consider the boost of a spacecraft from low earth orbit (LEO) onto a trajectory to the moon. Rocket motors must increase the vehicle velocity by the order of thousands of meters per second, but with a level of precision that is on the order of only meters per second. In other words the velocity change must be accomplished so that the error is only about 0.1% of the required boost in velocity.

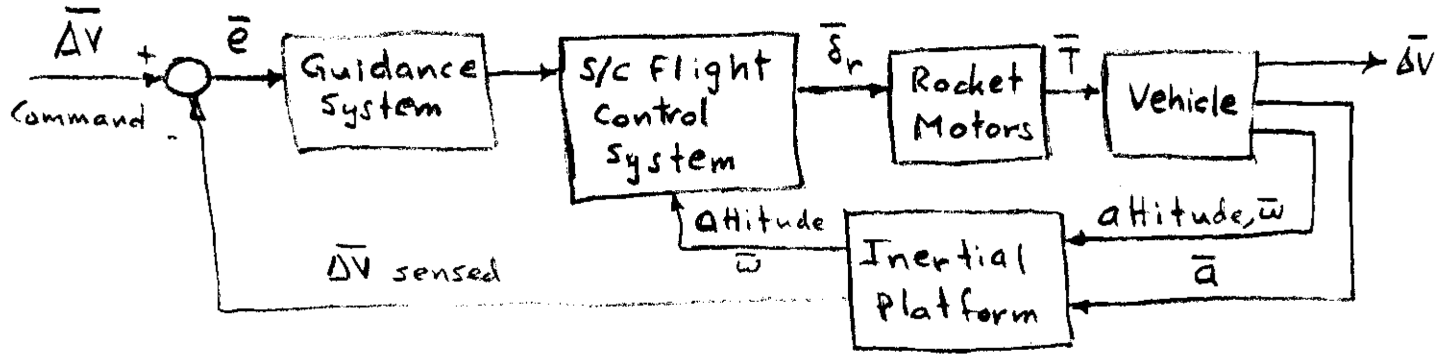


The boost velocity is achieved using an inertial platform as the primary sensor of the vehicle acceleration. The inertial platform looks like this-



The three gyros stabilize the platform orientation so that it is stationary with respect to inertial space. Each gyro is responsible for stabilization about one axis. The stable platform holds the accelerometers fixed in space so that they can sense acceleration. The three components of acceleration are then integrated to estimate the change in vehicle velocity vector.

The following diagram illustrates the essential elements of the feedback system used to accomplish this function.

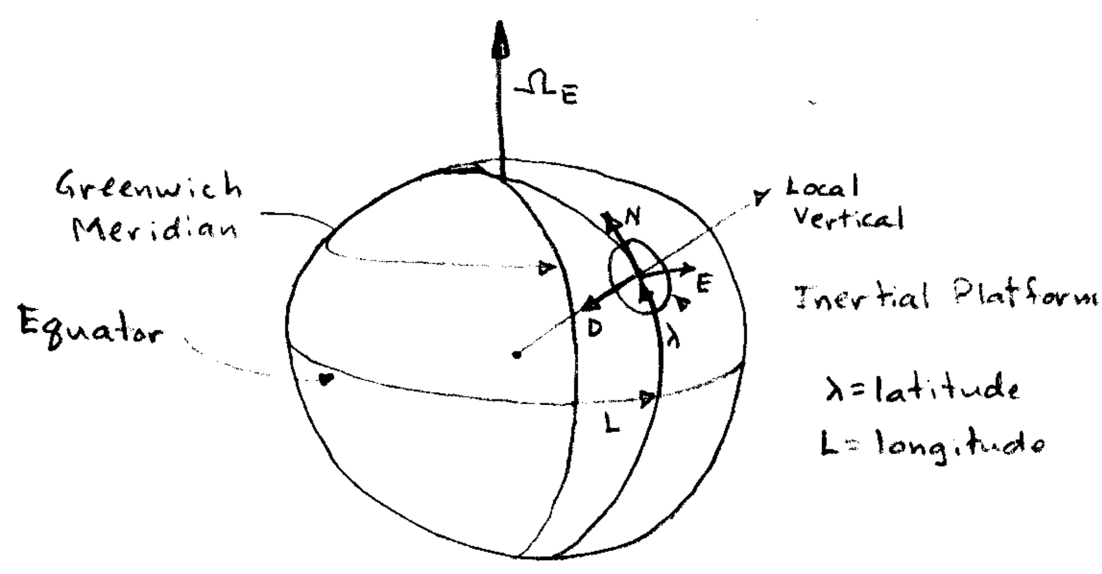


Note the inner loop that controls vehicle attitude and the outer loop that controls the velocity change imparted by the rocket motors. The inner loop utilizes both attitude and angular velocity measurements, from the inertial platform gyros, to stabilize and guide the vehicle. The outer loop utilizes integrated accelerometer outputs to achieve the desired velocity change.

Aircraft System Applications of Inertial Systems

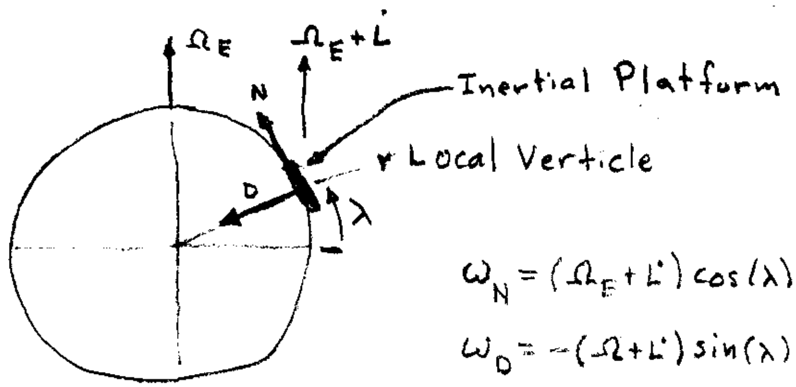
Modern aircraft systems use inertial platforms to implement combined inertial and GPS navigation systems. Typically the inertial system accomplishes the navigation function itself, especially over short time intervals. However, gyro and accelerometer drift and bias errors tend to degrade performance over time, so a GPS receiver serves to periodically correct these errors. In effect the inertial system serves as the short term (high bandwidth) sensor and the GPS serves as the long term (low bandwidth) sensor.

Often an inertial platform is used to track the local north, east and vertical (down) directions at the vehicle location. The following diagram illustrates this configuration



In effect the inertial platform provides a replica of the local horizon within the aircraft and serves as the reference for the horizontal situation indicator on the aircraft instrument panel. The inertial platform gimbals allow the inertial platform to remain stable, with respect to the local vertical, even as the vehicle may perform violent angular maneuvers about the stable inertial platform.

In order to maintain the local vertical the platform must rotate at an angular velocity that matches the sum of earth rate and the rate at which the vehicle moves over the Earth. For example, in order to match Earth rate plus the rate at which the vehicle moves in the east or west directions, the system must rotate about a vector pointing in the direction of the Earth's axis of rotation. Thus, if the vehicle is at some latitude λ then the north, and down gyros must be torqued so their axes rotate at a rate equal to the sum of Earth rate plus the rate of change of vehicle longitude, as shown in the following diagram.

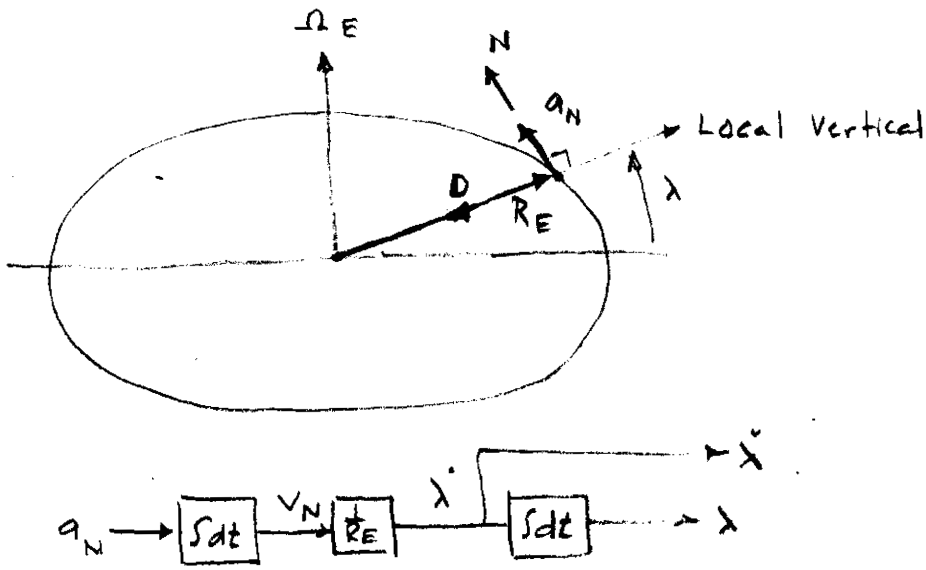


$$\omega_N = (\Omega_E + \dot{L})\cos(\lambda)$$

$$\omega_D = -(\Omega + \dot{L})\sin(\lambda)$$

Similarly, as the vehicle moves in the north or south directions the east gyro must be torqued so that the platform rotates at the rate of change of latitude.

Earth rate is well known to very high accuracy. However the inertial system must keep track of both position and velocity so that both the local vertical can be maintained and that latitude and longitude are available as system outputs to the vehicle crew. Accelerometers mounted on the inertial platform provide measurements of north, east and down accelerations. These are integrated to maintain north, east and down velocities, which in turn are integrated to maintain latitude, longitude and altitude of the aircraft. For example, north acceleration is integrated to maintain north velocity, which is divided by the local Earth radius to obtain latitude rate, which is again integrated to maintain latitude, as shown in the following diagram.



In addition, the attitude rate indication is used to torque the north and down gyros as indicated above. A similar function is performed on the east accelerometer outputs so as to maintain longitude.

Source:http://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec31.pdf