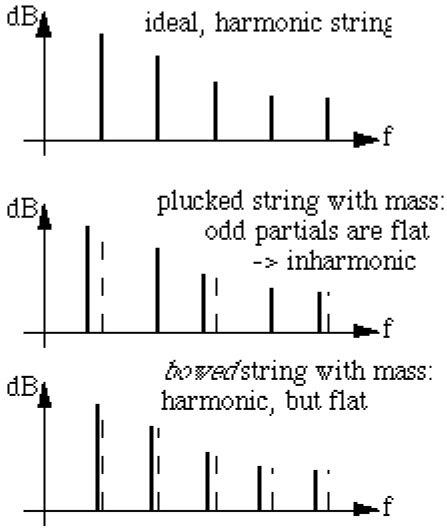
How harmonic are harmonics?

The spectrum of a single note from a musical instrument usually has a set of peaks at (approximately) harmonic ratios. That is, if the fundamental frequency is f, there are peaks at f, and also at (about) 2f, 3f, 4f, etc. For many, but not all, instruments, these ratios are close to exact: the second partial is at 2.00 times the first, etc. But why is it so? or rather, why is it sometimes so? The short answer is that bows, reeds and lips can make instruments harmonic, but that in struck or plucked instruments the harmonicity is more approximate. (There is more about how harmonics arise in string instruments and in flutes and other woodwinds.) We'll see why below, but first:

An experiment. Take a stringed instrument, and play the harmonic series on one of the open strings, as described on the page Waves in strings, reflections, standing waves and harmonics . Provided that the strings are reasonably new, the harmonics you play will be nearly exactly harmonic. Now pluck the open string in a normal way and you will hear a musical note with a clear pitch.

Now take a small piece of adhesive tape. Wind the tape carefully around the string at the midpoint. Pluck it again. You will probably notice that the pitch is a bit lower, but also less clear, and that the note sounds different: it may sound a bit like a bad quality bell. Then play the harmonics. You will find that the odd harmonics are all flat, but that the even ones have hardly changed. So this is no longer an exact harmonic series, and you can vary the departure from harmonicity by varying the size of the mass in the middle of the string. The two effects are related: when the series is harmonic, we hear a clear pitch, when it is far from harmonic, we usually do not hear a clear pitch, and the sound is more like that of a percussion instrument.



For the final part of the experiment, take a bow and bow the non-harmonic string. If the mass is not too large, and particularly if you have good bow control, you will be able to producea good musical note with a clear pitch. Why so? In the section Bows and Strings, we explain how the stick-slip action of the bow on the string produces a periodic vibration - that is a vibration that repeats itself exactly each cycle.

The sum of harmonic vibrations is a periodic vibration. This is one half of Fourier's theorem, and is easy to see. Let the fundamental frequency f have a period T. The second harmonic with frequency 2f has a period of T/2, so, after one vibration of the fundamental (after time T), the second harmonic has had exactly two vibrations, so the two waves are ready to start again with exactly the same relative position to each other, so they will produce the same combination that they did for the first cycle of the fundamental. The same is true for each harmonic nf, where n is a whole number. After time T, exactly n cycles of the nth harmonic have passed, and so all the harmonics are ready to start again for a new cycle. This is explained in more detail, and with diagrams, in What is a Sound Spectrum? The harmonic series is special because any combination of its vibrations produces a periodic or repeated vibration at the fundamental frequency f. Now the converse is also true: a periodic vibration has a harmonic spectrum. This is the other half of Fourier's theorem, but it is harder to show.

Strings and pipes are not inherently harmonic. An ideal, homogeneous, infinitely thin or infinitely flexible string has exactly harmonic modes of vibration. So does an ideal, homogeneous, infinitely thin pipe. Real stings and pipes do not. We saw in the experiment that adding a mass - making the string inhomogeneous - makes the string inharmonic. (By the way, worn or dirty strings are also inharmonic and harder to tune.

Washing them can help.)

Real strings are also inharmonic because they are not infinitely thin or flexible, and so do not bend perfectly easily at the bridge and the nut. This bending stiffness affects the higher modes more than the lower, so the harmonics are*stretched*. Solid strings are worse than wound strings, steel strings are worse than others, pianos - especially little pianos - are worse than harps. The inharmonicity disappears when the strings are bowed, but is more noticeable when they are plucked or struck. Because the bow's stick-slip action is periodic, it drives all of the resonances of the string at exactly harmonic ratios, even if it has to drive them slightly off their natural frequency. Thus the operating mode of a bowed string playing a steady* note is a compromise among the tunings of all of the (slightly inharmonic) string resonances. (For the technically minded, this phenomenon is due to the strong non-linearity of the stick-slip action. It is called mode locking.)

Real pipes are inharmonic because of their finite diameter: the end effects are frequency dependent. The pipes of musical instruments are complicated by departures from cylindrical or conical shape (valves and tone holes). Some of these complications are there to improve the harmonicity, but the results are rarely perfect. For any one note, however, the lip or the reed performs the same (strongly non-linear) role as the bow: the lip or reed undergoes periodic vibration and so produces a harmonic spectrum. Again, the operating mode of a brass or woodwind instrument playing a steady* note is a compromise among the tunings of all of the (slightly inharmonic) pipe resonances (mode locking again.)

"steady" here means over a very long time. Measurements of frequency are ultimately limited by the Uncertainty Principle. If you play a note for m seconds, the frequency of its harmonics cannot be measured with an accuracy greater than about 1/m Hz. If your spectrum analyser measures over only k seconds, it cannot measure more accurately than about 1/k Hz.

One final remark: the sound spectra of clarinets tend to have strong odd harmonics (fundamental, 3rd, 5th etc) and weak even harmonics (2nd, 4th etc), at least in their lowest register. This effect is discussed in "pipes and harmonics"and "flutes vs clarinets".

Source: http://www.phys.unsw.edu.au/jw/harmonics.html