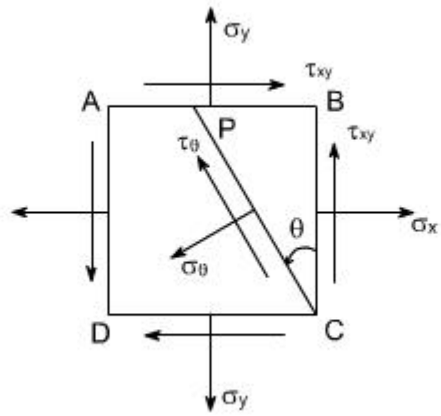
GRAPHICAL SOLUTION – MOHR'S STRESS CIRCLE

The transformation equations for plane stress can be represented in a graphical form known as Mohr's circle. This grapical representation is very useful in depending the relationships between normal and shear stresses acting on any inclined plane at a point in a stresses body.

To draw a Mohr's stress circle consider a complex stress system as shown in the figure



The above system represents a complete stress system for any condition of applied load in two dimensions

The Mohr's stress circle is used to find out graphically the direct stress σ and sheer stress τ on any plane inclined at θ to the plane on which σ_x acts.The direction of θ here is taken in anticlockwise direction from the BC.

STEPS:

In order to do achieve the desired objective we proceed in the following manner

(i) Label the Block ABCD.

(ii) Set up axes for the direct stress (as abscissa) and shear stress (as ordinate)

(iii) Plot the stresses on two adjacent faces e.g. AB and BC, using the following sign convention.

Direct stresses – tensile positive; compressive, negative

Shear stresses – tending to turn block clockwise, positive

– tending to turn block counter clockwise, negative

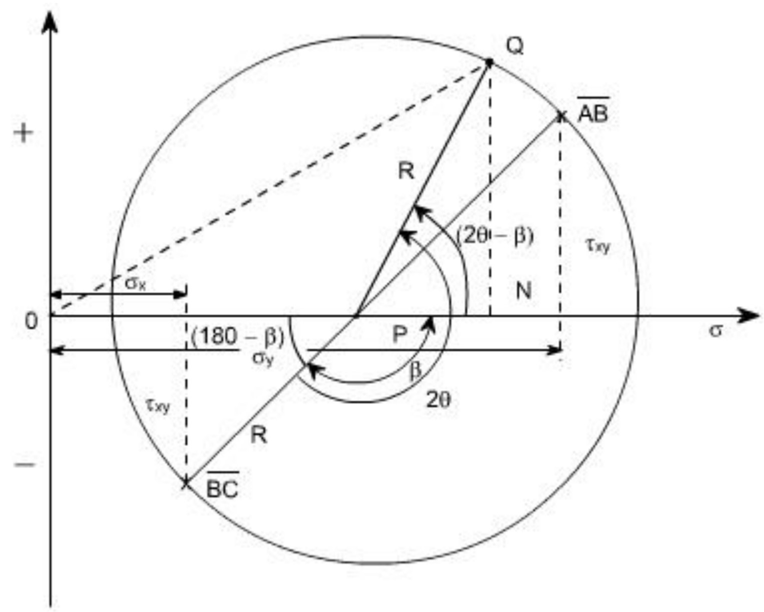
[i.e shearing stresses are +ve when its movement about the centre of the element is clockwise]

This gives two points on the graph which may than be labeled as $\overline{AB}$ and $\overline{BC}$ respectively to denote stresses on these planes.

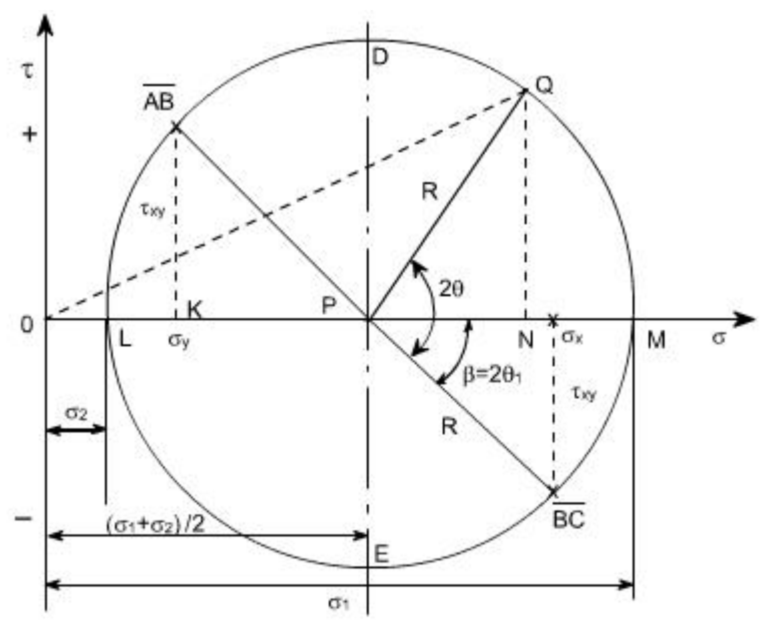
(iv) Join $\overline{AB}$ and $\overline{BC}$.

(v) The point P where this line cuts the s axis is than the centre of Mohr's stress circle and the line joining $\overline{AB}$ and $\overline{BC}$ is diameter. Therefore the circle can now be drawn.

Now every point on the circle then represents a state of stress on some plane through C.



Proof:



Consider any point Q on the circumference of the circle, such that PQ makes an angle 2θ□with BC, and drop a perpendicular from Q to meet the s axis at N.Then OQ represents the resultant stress on the plane an angle θ to BC. Here we have assumed that $\sigma_x > \sigma_y$

Now let us find out the coordinates of point Q. These are ON and QN.

From the figure drawn earlier

$$ON = OP + PN$$

$$OP = OK + KP$$

$$OP = \sigma_y + 1/2\ (\sigma_x - \sigma_y)$$

$$= \sigma_y / 2 + \sigma_y / 2 + \sigma_x / 2 + \sigma_y / 2$$

$$= (\sigma_x + \sigma_y) / 2$$

$PN = R\cos(2\theta - \beta)$

hence $ON = OP + PN$

$= (\sigma_x + \sigma_y) / 2 + R\cos(2\theta - \beta)$

$= (\sigma_x + \sigma_y) / 2 + R\cos 2\theta \cos\beta + R\sin 2\theta \sin\beta$

now make the substitutions for $R\cos\beta$ and $R\sin\beta$.

$$R\cos\beta = \frac{(\sigma_x - \sigma_y)}{2};\ R\sin\beta = \tau_{xy}$$

Thus,

$ON = 1/2\ (\sigma_x + \sigma_y) + 1/2\ (\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$ (1)

Similarly $QM = R\sin(2\theta - \beta)$

$= R\sin 2\theta\cos\beta - R\cos 2\theta\sin\beta$

Thus, substituting the values of $R\cos\beta$ and $R\sin\beta$, we get

$QM = 1/2\ (\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta$ (2)

If we examine the equation (1) and (2), we see that this is the same equation which we have already derived analytically

Thus the co-ordinates of Q are the normal and shear stresses on the plane inclined at θ to BC in the original stress system.

N.B: Since angle $\overline{BC}$ PQ is 2θ on Mohr's circle and not θ it becomes obvious that angles are doubled on Mohr's circle. This is the only difference, however, as They are measured in the same direction and from the same plane in both figures.

Further points to be noted are :

(1) The direct stress is maximum when Q is at M and at this point obviously the sheer stress is zero, hence by definition OM is the length representing the maximum principal stresses σ_1 and $2\theta_1$ gives the angle of the plane θ_1 from BC. Similar OL is the other principal stress and is represented by σ_2

(2) The maximum shear stress is given by the highest point on the circle and is represented by the radius of the circle.

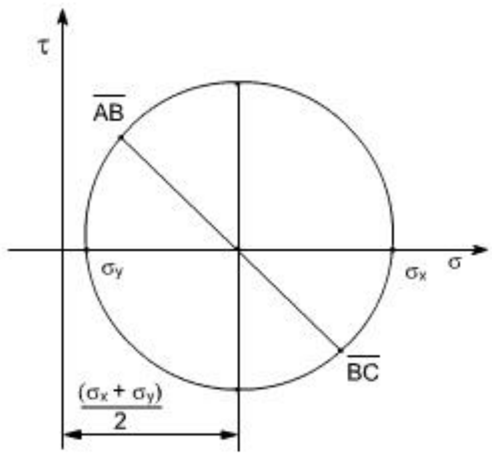
This follows that since shear stresses and complimentary sheer stresses have the same value; therefore the centre of the circle will always lie on the s axis midway between σ_x and σ_y . [since $+\tau_{xy}$ & $-\tau_{xy}$ are shear stress & complimentary shear stress so they are same in magnitude but different in sign.]

(3) From the above point the maximum sheer stress i.e. the Radius of the Mohr's stress circle would be

$$\frac{(\sigma_x - \sigma_y)}{2}$$

While the direct stress on the plane of maximum shear must be mid – may between σ_x and σ_y i.e

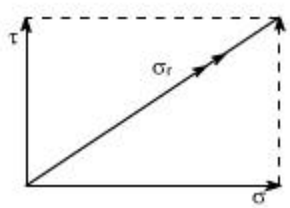
$$\frac{(\sigma_x + \sigma_y)}{2}$$



(4) As already defined the principal planes are the planes on which the shear components are zero.

Therefore are conclude that on principal plane the sheer stress is zero.

(5) Since the resultant of two stress at 90^0 can be found from the parallogram of vectors as shown in the diagram.Thus, the resultant stress on the plane at q to BC is given by OQ on Mohr's Circle.



(6) The graphical method of solution for a complex stress problems using Mohr's circle is a very powerful technique, since all the information relating to any plane within the stressed element is contained in the single construction. It thus, provides a convenient and rapid means of solution. Which is less prone to arithmetical errors and is highly recommended.

Source: http://nptel.ac.in/courses/Webcourse-contents/IIT-ROORKEE/strength%20of%20materials/homepage.htm