FLUX TRANSPORT DYNAMOS

Flux transport dynamos are one specific variety of hydromagnetic dynamos models for the Sun and stars. They provide a modelling framework for the spatiotemporal evolution of the Sun's large-scale magnetic field (spatial scales commensurate with the solar radius), on long timecales (many months to century and millenia). Their two primary defining features are:

- the observed equator ward migration of sunspot source regions and poleward migration of surface fields are both driven by the "conveyor belt" action of the meridional flow;
- the cycle period is primarily set by the meridional flow speed.

Flux transport dynamos reproduce fairly well and robustly many observed solar cycle features, and are relatively well-constrained observationally. Their dynamical behavior, driven at least in part by a time-delay in the dynamo regenerative loop, also offers a natural explanation for the significant fluctuations observed in overall activity levels on decadal to millenial timescales.

Flux transport dynamos and the solar cycle

The solar magnetic field is the energy channel and dynamical engine driving solar activity, including all geoeffective solar eruptive phenomena. It is observed to evolve on a wide variety of spatial and temporal scales, the most prominent being the so-called solar cycle, first noted in the systematic variations of sunspot counts (see Figure 1), but now understood to be driven by a cyclic variation of the large-scale component of the solar magnetic field, involving polarity reversals occurring approximately every 11 years. This short cycle period points to a contemporaneously regenerated magnetic field, most likely by a hydromagnetic dynamo. This process is described by the equations of magnetohydrodynamics (MHD). Unfortunately, direct numerical simulations of the MHD equations are still a long way from reaching the physical parameter regime characterizing solar interior conditions. Current solar cycle models are therefore based on reduced forms of MHD fluid equations, such as mean-field hydromagnetic dynamo models, of which flux transport dynamos are one particularly promising variety.
A representative flux transport solar cycle model

Observations suggest that the dynamo loop for the large-scale magnetic field operates as a two-step process. The production of the strong toroidal (i.e. azimuthally-directed) magnetic component ultimately giving rise to sunspots is produced by shearing of the poloidal component by differential rotation. This is now believed to occur primarily deep in the solar interior, at or immediately beneath the core-envelope interface. The second step in the dynamo loop, producing a poloidal component with reversed polarity from the toroidal component, is more uncertain. One promising mechanism is the release into the solar photosphere of magnetic flux liberated by the decay of sunspots and active regions, with subsequent transport and accumulation at high solar latitudes (Babcock 1961; Leighton 1969; Wang et al. 1989). This process is actually observed, so in what follows the focus is placed on dynamo models relying on this poloidal field regeneration mechanism, because they are arguably the best exemplar of flux transport dynamos.

The basic operation of a flux transport dynamo based on the surface decay of sunspots is illustrated schematically on Figure 2. Sunspots currently emerging at the solar surface (B) are formed from the toroidal field $T_n$ currently being produced at cycle $n$ in the solar interior at (A). The decay products of these sunspots are carried by the combined diffusion-like action of the turbulent convective flow and the large-scale meridional circulation to high latitudes (C), where they will eventually lead to the reversal of the poloidal field $P_{n-1}$ having built up during the preceding sunspot cycle. Over time, this new poloidal field is transported...
into the interior, eventually ending up at (A) again, where the production of the subsequent cycle's toroidal field \(T_{n+1}\) can commence.

Figure 2: Schematic operation of a flux transport dynamo based on the surface decay of sunspots. The diagram shows a meridional quadrant of the solar interior, with representative meridional flow streamlines plotted in yellow. The flow is poleward at the surface and equatorward at the base of the envelope. The interface between the Sun's convective envelope and underlying radiative core is located a fractional radius 0.7, as indicated by the dotted line.

The form and magnitude of the differential rotation responsible for shearing the poloidal field in a toroidal component at the core-envelope interface is known from helioseismology. The meridional flow is directly measured at the solar surface, detected and measured by helioseismology in the outer half of the solar convective envelope, and relatively well-constrained by theoretical considerations further below. It serves as a "conveyor belt", setting the cycle period by transporting the poloidal component from its source region at the surface to the interior, and gradually displacing equatorward the toroidal field produced at the core-envelope interface (Wang et al. 1991; Choudhuri et al. 1995).

Reduction to a one-dimensional iterative map

One dynamically interesting aspect of the dynamo scenario illustrated on Figure 2 is that there is an unavoidable time delay built into the dynamo loop, as a consequence of the spatial segregation of the two magnetic source regions. The transport of the surface poloidal field to the core-envelope interface requires a time of the order of the circulation's turnover time, which is of a few decades according to current estimates. The destabilisation and buoyant rise of the toroidal component to the surface (red dashed arrow on Figure 2) is,
in comparison, a rapid process. This leads to a time-delay in the dynamo loop, commensurate with the circulation turnover time and thus cycle period: the poloidal field produced at the surface at cycle $n$ is not the source of poloidal field for the current cycle, but will serve as a source of toroidal field for a subsequent cycle a decade or two later.

The dynamical consequences of this long time delay can be explored with the following simple model (Durney 2000; Charbonneau 2001): Going with the scenario illustrated in Figure 2, assume that the toroidal field $T_n$ is linearly proportional to the poloidal field strength $P_{n-1}$ of the previous cycle. Production of the current cycle's poloidal field $P_n$, on the other hand, is understood to be a nonlinear function of the current toroidal field $T_n$, production of $P_n$ being possible only in a finite range of toroidal field strengths. Writing $P_n \propto T_n(1-T_n)$, and absorbing the proportionality constants into scaled forms of the amplitudes, leads to:

$$p_{n+1}=ap_n(1-p_n)$$

Equation (1) is a one-dimensional cubic iterative map for the amplitude $p_n$ of cycle $n$, where the map parameter $a$ plays the role of a dynamo number. As it is increased, the amplitude iterate undergoes a transition to chaos via a sequence of period-doubling bifurcations characteristic of such single-humped map (see Figure 3A).

![Figure 3](image_url)

Figure 3: Part (A) shows a bifurcation diagram for the one-dimensional iterative map embodying the time delay dynamics of flux transport dynamo models based on the surface decay of sunspots and active regions. The gray shading indicates the basin of attraction. Part (B) shows a bifurcation diagram constructed by measuring cycle amplitudes in a sequence of magnetic energy time series reconstructed by numerical 2D kinematic dynamo solutions of a conceptually similar solar cycle model (see Charbonneau et al. 2005). Magnetic energy is used here as a proxy of cycle amplitude.

The attractor has here a finite-sized basin of attraction, indicated by gray shading on Figure 3A. This can lead to intermittency in the presence of low-amplitude stochastic forcing. This is in fact expected in the present
context, with (observed) stochastic fluctuations in sunspot emergence rates leading effectively to fluctuations in the map parameter, and small-scale turbulent dynamo action producing small-scale magnetic fields, acting as a form of additive perturbation to the cycle amplitude. Consider then the following stochastically forced map:

\[ p_{n+1} = a_n p_n (1 - p_n) + e_n e_n \in [0, E] E \ll 1 \tag{2} \]

where the map parameter \( a_n \) and additive noise amplitude \( e_n \) are extracted anew at each iteration from some preset distribution. Time series of cycle amplitudes produced with this stochastically forced map are characterized by bursting phase \( (p_n \sim 1) \), corresponding to "normal" cyclic behavior, punctuated by quiescent epochs \( (p_n \ll 1) \) of irregular spacing and duration; these are the map's equivalent to Maunder Minimum epochs of strongly depressed sunspot counts (see Figure 1).

An interesting property of stochastically forced iterative maps such as the ones use here is that, upon being perturbed, relaxation to the attractor is oscillatory over much of the periodic and multiperiodic domain. This translates, in the time series of cycle amplitudes, into more-or-less regular alternance between higher-than-average and lower-than-average cycle amplitudes. Such a pattern is actually observed in the SSN time series, and is known as the Gnevyshev-Ohl Rule in the solar physics literature.

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