FREE VIBRATIONS OF NON-HOMOGENEOUS TAPERED SQUARE PLATE WITH BI-DIRECTIONAL TEMPERATURE VARIATIONS

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Abstract- An analysis is presented to the study the free vibrations of non- homogeneous clamped square plate with bi-directional temperature variations. It is assumed that thickness of the plate varies exponentially in one direction. Non-homogeneity of plate material is assumed to arise due to parabolic variation of density along x-direction. Rayleigh-Ritz technique on the basis of classical plate theory is applied to solve the fourth order differential equation of motion. Numerical values of frequency are calculated with the help of Mathematica (Software) and are presented in tabular and graphical forms for different values of thermal gradient, taper constant and non-homogeneity constant.

Keywords: *Thermal effect, Non-homogeneity, Frequency, Taper constant, Thickness.*

I. INTRODUCTION

Vibrations occur in every machine & mechanical structures due to the forces which directly affect the efficiency and reliability of machines parts. A lot of energy waste in the form of kinetic energy used in vibrations. Sometimes thermal plants or nuclear plants shut down due to failure of turbines (due to uncontrolled vibrations) which directly affect the economy, safety, and employment system and time management of the nation. Therefore, engineers & scientists are extensively interested to know about the vibrational behaviour of vibrating system before finalizing any design of the structure. So, this is the need of the hour to make authentic & reliable mathematical models for the help of researchers so that they can used them in practical manner and make much better designs of machines with more efficiency and durability.

In modern technologies, where most of machines work under the influence of temperature, tapered plates are frequently used to make the parts of mechanical structures. A lot of research work about the effect of temperature & thickness variation on the vibrations of different plates under different conditions has been done.

Number of researchers has worked on the effect of one dimensional temperature variation on the vibration of various shapes of plates whose thickness varies in one or two directions but very few works on two dimensional temperature variations.

De & Debnath [1] studied the thermal effect on axi-symmetric vibrations of a circular plate of exponentially varying thickness and density. Thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness studied by Gupta, Johri and Vats [2]. Gupta and Sharma [3] evaluated the thermal effect on vibration of non-homogeneous trapezoidal plate of linearly varying thickness. Effect of thermal gradient on transverse vibration of non homogeneous orthotropic trapezoidal plate of parabolically varying thickness with boundary condition clamped-simply supported-clamped-simply supported by the help of Rayleigh Ritz technique studied by Gupta & Sharma [4]. Imrak & Gerdemeli [5] discussed the problem of isotropic rectangular plate with four clamped edges. Kumar & Lal [6] investigated the vibrations of non homogeneous orthotropic rectangular plates with bilinear thickness variation resting on Winkler foundation. Khanna, Kaur & Kumar [7] studied the effect of varying poisson ratio on temperature-thickness coupling problem of non-homogeneous rectangular plate by the help of Rayleigh Ritz technique. Recently Khanna & Sharma [8] discussed the free vibrations of non-homogeneous tapered square plate with bi-linear temperature variations. They calculated the frequency for four sides clamped non-homogeneous square plate on the basis of classical plate theory. Leissa [9] provided different models on the vibrations of plates. Maurizi & Laura [10] analyzed the vibration on clamped rectangular plates of generalized orthotropy. Nagaya [11] discussed the vibrations and dynamic response of visco-elastic plates on non-periodic elastic supports. Singh & Jain [12] studied the free asymmetric transverse vibration of polar orthotropic annular sector plate with thickness varying parabolically in radial direction.

Here, Rayleigh Ritz technique is applied to calculate the frequency for the first two modes of vibrations for non-homogeneous tapered clamped square plate with different values of thermal gradient, taper constant and non-homogeneity constant by the help of latest software 'Mathematica'. Numeric results are presented in graphical and tabular form for various combinations of parameters.

II. FOURTH ORDER DIFFERENTIAL EQUATION OF MOTION

Fourth order Differential equation of motion for square plate of variable thickness is

$$\begin{bmatrix} D_1\left(W_{,xxxx}+2W_{,xxyy}+W_{,yyyy}\right)+2D_{1,x}\left(W_{,xxx}+W_{,xyy}\right)+ \\ 2D_{1,y}\left(W_{,yyy}+W_{,yxx}\right)+D_{1,xx}\left(W_{,xx}+\nu W_{,yy}\right)+ \\ D_{1,yy}\left(W_{,yy}+\nu W_{,xx}\right)+2(1-\nu)D_{1,xy}W_{,xy} \end{bmatrix}-\rho p^2 lW=0 \qquad (2.1)$$

Where ',' indicates the differentiation with respect to suffix variable.

Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = El^3/12(1-\nu^2) \tag{2.2}$$

where E, ν & l are young modulus, poisson ratio & thickness of plate respectively.

III. ASSUMPTIONS

To solve equation of motion, it is considered that the thickness variation of the plate exponential in x-direction:

$$l = l_0 e^{\frac{\beta x}{a}} \tag{3.1}$$

where β is taper constant & $l = l_0$ at $x = y = 0$

To make easy but flexible calculation, first six terms are considered in the expansion of $\exp(\beta x/a)$ in equation (3.1). Hence

$$l = l_0[1+\frac{(\beta x/a)}{1!}+\frac{(\beta x/a)^2}{2!}+\frac{(\beta x/a)^3}{3!}+\frac{(\beta x/a)^4}{4!}+\frac{(\beta x/a)^5}{5!}] \tag{3.2}$$

Assuming the density of non-homogeneous square plate varies parabolic in x-direction i.e.

$$\rho = \rho_0(1+\alpha_1(x/a)^2) \tag{3.3}$$

where α_1 is the non-homogeneity constant.

Assuming the square plate of engineering material has a steady two dimensional temperature distribution i.e. linear in x-direction & parabolic in y-direction temperature variations as

$$\tau = \tau_0(1-(x/a))(1-(y/a)^2) \tag{3.4}$$

where τ denotes the temperature excess the temperature above the reference temperature at any point on the plate, τ_0 denotes the temperature at any point on the boundary of plate and "a" is the length of a side of a square plate.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form,

$$E = E_0(1-\gamma\tau) \tag{3.5}$$

where, E_0 is the value of the young modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of E with τ. The modulus of variation (3.5) becomes

$$E = E_0\{1-\alpha(1-(x/a))(1-(y/a)^2)\} \tag{3.6}$$

where, $\alpha = \gamma\tau_0, (0 \le \alpha < 1)$ is thermal gradient.

Now put the value of E and l from equation (3.6) and

equation (3.2) in equation (2.2), one obtain

$$D_1 = \frac{\left[\begin{array}{l} E_0\{1-\alpha(1-(x/a))(1-(y/a)^2)\}l_0^3\{1+ \\ \frac{(3\beta x/a)}{1!}+\frac{(3\beta x/a)^2}{2!}+\frac{(3\beta x/a)^3}{3!}+\frac{(3\beta x/a)^4}{4!}+\frac{(3\beta x/a)^5}{5!}\} \end{array}\right]}{12(1-\nu^2)} \tag{3.7}$$

IV. SOLUTION OF EQUATION OF MOTION

To find a solution of equation of motion, Rayleigh Ritz technique is applied. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(S-K) = 0 \tag{4.1}$$

The expression for strain energy S and kinetic energy K are:

$$S = \frac{1}{2}\int_0^a\int_0^a D_1[(W_{,xx})^2 + (W_{,yy})^2 + 2\nu W_{,xx}W_{,yy} + 2(1-\nu)(W_{,xx})^2]dydx \tag{4.2}$$

$$K = \frac{1}{2}\rho p^2\int_0^a\int_0^a lW^2 dydx \tag{4.3}$$

Also the plate is assumed as clamped at all four sided, so the boundary conditions for square plate are

$$\left.\begin{array}{ll} W = W_{,x} = 0 & \text{at} \quad x = 0, a \\ W = W_{,y} = 0 & \text{at} \quad y = 0, a \end{array}\right\} \tag{4.4}$$

To satisfy equation (4.4), the corresponding two term deflection function is taken as

$$W = [(x/a)(y/a)(1-(x/a))(1-(y/a))]^2(A_1 + A_2(x/a)(y/a)(1-(x/a))(1-(y/a))] \tag{4.5}$$

Assuming the non-dimensional variable as

$$X = x/a, Y = y/a, \overline{W} = W/a, \bar{l} = l/a \tag{4.6}$$

and using equation (3.2), (3.3), (3.6) & (4.4) in equation (4.2) and (4.3), one obtains

$$S = L\int_0^a\int_0^a [1-\alpha(1-(X))(1-(Y)^2)][1+\frac{(3\beta X)}{1!}+\frac{(3\beta X)^2}{2!}+\frac{(3\beta X)^3}{3!}+\frac{(3\beta X)^4}{4!}+\frac{(3\beta X)^5}{5!}] \; D_1[(W_{,xx})^2+(W_{,yy})^2+2\nu W_{,xx}W_{,yy}+2(1-\nu)(W_{,xx})^2]dydx \tag{4.(4.7)}$$

$$K = \frac{1}{2}\rho_0 p^2 \bar{l}_0 a^5\int_0^a\int_0^a \rho_0(1+\alpha_1 X^2)\left[\begin{array}{l} 1+\frac{(\beta X)}{1!}+\frac{(\beta X)^2}{2!}+\frac{(\beta X)^3}{3!} \\ +\frac{(\beta X)^4}{4!}+\frac{(\beta X)^5}{5!} \end{array}\right]\overline{W}^2 dydx \tag{4.8}$$

where, $L = E_0 l_0^3 a^3/12(1-\nu^2)$

On using equations (4.7) & (4.8) in equation (4.1), one get

$$(S^* - \lambda^2 K^*) = 0 \tag{4.9}$$

where,

$$S^* = \int_0^a\int_0^a \left[\begin{array}{l} 1-\alpha(1-(X))(1-(Y)^2)\left(\begin{array}{l} 1+\frac{(3\beta X)}{1!}+\frac{(3\beta X)^2}{2!}+\frac{(3\beta X)^3}{3!} \\ +\frac{(3\beta X)^4}{4!}+\frac{(3\beta X)^5}{5!} \end{array}\right) \\ D_1[(W_{,xx})^2+(W_{,yy})^2+2\nu W_{,xx}W_{,yy}+2(1-\nu)(W_{,xx})^2 \end{array}\right] dydx$$

$$K^* = \int_0^a\int_0^a \rho_0(1+\alpha_1 X^2)\left[1+\frac{(\beta X)}{1!}+\frac{(\beta X)^2}{2!}+\frac{(\beta X)^3}{3!}+\frac{(\beta X)^4}{4!}+\frac{(\beta X)^5}{5!}\right]\overline{W}^2 dydx$$

and $\lambda^2 = 12\rho_0 p^2 (1-\nu^2) / E_0 l_0^2$

Here λ is a frequency parameter.

Equation (4.9) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W from equation (4.5). These two constants are to be determined as follows

$$\partial(S^* - \lambda^2 K^*) / \partial A_n, \qquad n = 1, 2 \qquad (4.10)$$

On simplifying (4.10), one gets

$$b_{n1}A_1 + b_{n2}A_2 = 0, \qquad n = 1, 2 \qquad (4.11)$$

where, $b_{n1}, b_{n2} (n = 1, 2)$ involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (4.11) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \qquad (4.12)$$

From equation (4.12), one can obtain a bi-quadratic equation in λ from which different values of λ can be calculated for different values of taper constant, non-homogeneity constant and thermal gradient.

V. RESULTS AND DISCUSSION

Calculation of the frequency parameter is done for a non-homogeneous clamped square plate for various values of thermal gradient, taper constant and non-homogeneity constant for the following parameters:

l_0 =0.01m ρ_0 = 2.80 × 10^3 kg/m^3 & ν = 0.345 (For visco elastic material Duralium).

Table 1 contains the numeric values of frequency parameter of a square plate for various values of thermal gradient. This shows that as the value of thermal gradient increases then the frequency parameter is decreases continuously for the different combination of taper constant and non-homogeneity constant i.e.

(β =0.2= α_1 , β =0.4=α_1 & β =0.8=α_1).

It is also noticed that as the combination of taper constant and non-homogeneity constant increases then the frequency parameter also increases. Variations of frequency are also shown in figure 1 for all the three combinations.

Table 1: Frequency vs Thermal gradient

α	$\alpha_1 = \beta = 0.2$		$\alpha_1 = \beta = 0.4$		$\alpha_1 = \beta = 0.8$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.0	38.96	152.05	42.71	166.04	52.85	203.80
0.2	37.74	147.28	41.49	161.33	51.58	199.13
0.4	36.47	142.35	40.23	156.48	50.27	194.34
0.6	35.16	137.24	38.92	151.48	48.92	189.44
0.8	33.79	131.94	37.56	146.31	47.51	184.42
1.0	32.35	126.42	36.14	140.95	46.04	179.25

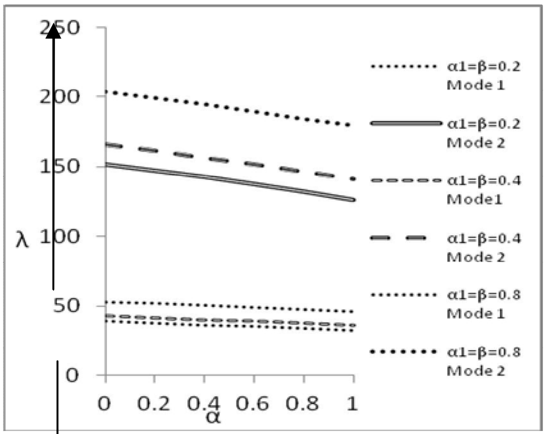

Figure1: Frequency vs Thermal Gradient

Table 2 shows the values of frequency parameter for various values of non-homogeneity constant of square plate. It is clearly seen that as the value of non-homogeneity constant increases then the frequency parameter decreases continuously for the different combination of taper constant and thermal gradient i.e. (α = β = 0.2, α = β = 0.4 & α = β = 0.8). Also, it can be seen that as combination of taper constant and thermal gradient increases, frequency parameter also increases. Variations of frequency are shown in figure 2 for all three combinations.

Table 2: Frequency vs Non-homogeneity Constant

α_1	$\alpha = \beta = 0.2$		$\alpha = \beta = 0.4$		$\alpha = \beta = 0.8$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.0	38.77	151.72	42.43	166.07	52.71	207.74
0.2	37.74	147.28	41.29	161.06	51.25	201.08
0.4	36.79	143.21	40.23	156.48	49.91	195.03
0.6	35.90	139.46	39.25	152.28	48.66	189.50
0.8	35.08	135.99	38.34	148.39	47.51	184.42
1.0	34.31	132.77	37.49	144.79	46.43	179.73

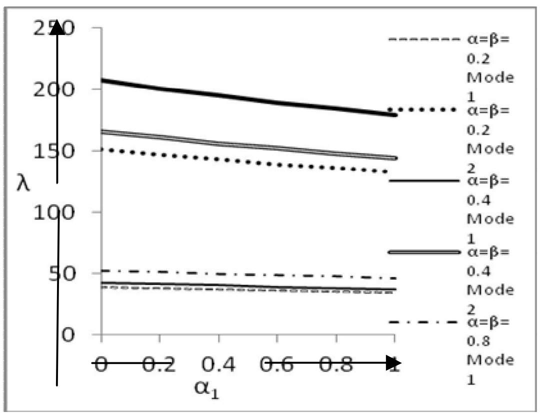

Figure 2: Frequency vs Non-Homogeneity

Table 3 contains the values of frequency parameter of a square plate for various values of taper constant for fixed non-homogeneity constant (α_1 =0.2) for the different values of thermal gradient.

It can be noted that the frequency parameter increases continuously (with increased values of taper constant) for both the modes of vibration for three values of thermal gradient. Also, table 3 shows that frequency decreases continuously for both the modes of vibration as thermal gradient increases from 0.0 to 0.8. Variations of are shown in figure 3.

Table 3: Frequency vs Taper Constant

β	$\alpha = 0.0, \alpha_1 = 0.2$		$\alpha = 0.2, \alpha_1 = 0.2$		$\alpha = 0.8, \alpha_1 = 0.2$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.0	35.05	136.88	32.59	127.26	29.92	116.85
0.2	38.96	152.05	36.47	142.35	33.79	131.94
0.4	43.84	170.89	41.29	161.06	38.55	150.60
0.6	49.82	194.09	47.18	184.07	44.34	173.48
0.8	57.03	222.16	54.24	211.87	51.25	201.08
1.0	65.47	255.27	62.49	244.64	59.30	233.54

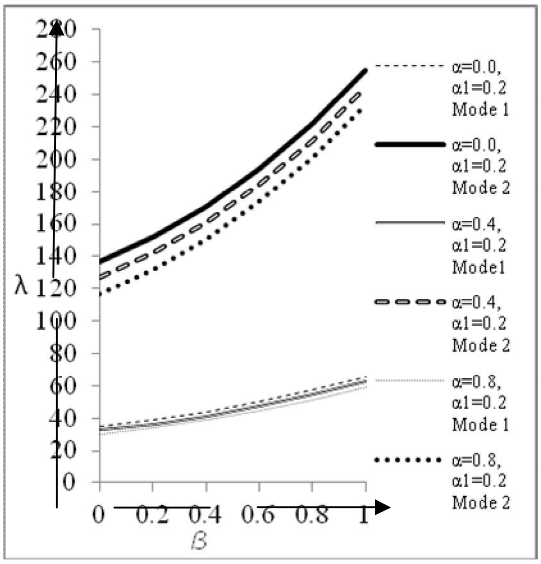

Figure 3: Frequency vs Taper constant

CONCLUSION

After comparing the results of present paper with [8], authors find a close agreement between them for the different values of corresponding parameters. Frequency for both the mode of vibration have slightly less in [8] than the present paper. With the analysis of tables and graphs, scientists & engineers can find the desired values of frequencies by the proper choice of different parameters. So, they are advised to analyze the findings of this paper in order to provide much better authentic structures and machines with more strength, durability & efficiency.

REFERENCES

[1] De, and D. Debnath, “Thermal effect on axi-symmetric vibrations of a circular plate of exponentially varying thickness and density,” International Journal of Mathematical Science & Engineering Applications, vol. 5, pp.325-334, 2011.

[2] A.K. Gupta, T. Johri, and R.P. Vats, “Thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness,” Proceedings of the World Congress on Engineering and Computer Science (San Francisco USA), pp.24-26, 2007.

[3] A.K. Gupta, and P. Sharma, “Thermal effect on vibration of non-homogeneous trapezoidal plate of linearly varying thickness” International Journal of Applied Mathematics and Mechanics, Vol. 7, pp.1-17, 2011.

[4] A.K. Gupta, and S. Sharma, “Study the Effect of Thermal Gradient on Transverse Vibration of Non Homogeneous Orthotropic Trapezoidal Plate of Parabolically Varying Thickness,” Applied Mathematics, Vol. 2, pp. 1-10, 2011.

[5] C. Imrak, and I. Gerdemeli, “The problem of isotropic rectangular plate with four clamped edges” Sadhna, Vol.32, pp.181-186, 2007.

[6] Y. Kumar, and R. Lal, “Vibrations of non homogeneous orthotropic rectangular plates with bilinear thickness variation resting on Winkler foundation,” Meccanica, Vol. 47, pp. 893-915, 2012.

[7] A. Khanna, N. Kaur, and A.K. Sharma, “Effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate,” Indian Journal of Science and Technology, Vol. 5, pp.3263-3267, 2012.

[8] A. Khanna, and N. Sharma, “An analytical approach on free vibrations of non-homogeneous tapered square plate with bi-linear temperature variations,” International journal of advanced technology and engineering research, Vol. 2, pp.16-20, 2012.

[9] A.W.Leissa, “Vibrations of plates,” NASA-SP, 160, 1969.

[10] M.J. Maurizi, and P.A Laura, “Vibrational analysis of Clamped, rectangular plates of generalized orthotropy,” Journal of sound and vibration, Vol. 26, pp.299-305, 1973.

[11] K. Nagaya, “Vibrations and dynamic response of visco-elastic plates on non-periodic elastic supports” Journal of engineering for industry, Vol.99, pp.404-409 1977.

[12] R.P. Singh, and SK Jain, “Free asymmetric transverse vibration of polar orthotropic annular sector plate with thickness varying parabolically in radial direction,” Sadhna, Vol. 29, pp.415-428, 2004.

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