FRACTAL CHARACTERIZATION AND COMPARISON OF CHAOS IMPACT OF EXCITED DUFFING OSCILLATOR PARAMETERS

SALAU, T.A.O.
Department of Mechanical Engineering, University of Ibadan
Ibadan, Nigeria
tao.salau@mail.ui.edu.ng

AJIDE, O.O.
Department of Mechanical Engineering, University of Ibadan
Ibadan, Nigeria
ooe.ajide@mail.ui.edu.ng

Abstract:

This study adopted two popular Runge-Kutta algorithms to investigate the chaotic driven impact of two important parameters of excited Duffing oscillator using fractal disk dimension characterization and comparison. The constant time step simulation of 'family' of Duffing oscillator beyond unsteady periods was employed to create scatter phase plots at every end of excitation period and over one thousand consecutive excitation periods. The resulting distorted Poincare images were quantified using fractal disk dimension obtained by use of optimum disk count method. However, the comparison of dimension distribution was made on one hundred equal intervals between limits for two distinct cases. The spectrum of estimated fractal disk dimension is noisy. Its variation is $1.312 \le D \le 1.681$ and $1.126 \le D \le 1.358$ respectively for damp and excitation amplitude based cases using fourth order Runge-Kutta algorithms as simulation tool. However the variation of dimension is $1.277 \le D \le 1.688$ and $1.140 \le D \le 1.384$ for corresponding case using fifth order Runge-Kutta algorithms. Furthermore, both cases enjoyed unimodal disk dimension distribution for all simulation algorithms. The approximated modal dimension is 1.52 and 1.23 in favour of damp and excitation amplitude value while the average modal relative frequency is 8.0%.The study results therefore supported damp parameter properly tune as easier agent of impacting chaotic behaviour in Duffing oscillator compares with its excitation amplitude counterpart other simulation conditions remains same.

Keywords*:* Excited Duffing oscillator, Runge-Kutta algorithms and Fractal disk dimension

1. Introduction

Extensive literature study has affirmed the fact that parameters alteration has a very high potential in influencing the dynamics of engineering systems. The impact of high-speed machine tool parameters on the contouring accuracy of a system has been investigated (Richard et al, 2004).Ricardo et al (2005) stated in their paper that parameter variations are very important in the modelling of nonlinear systems. It is reported in the paper that parameters choice often lead to interesting dynamics such as complex periodicity and chaos. The analytical formulation of contouring error in the instance of a straight line, circle and corner crossing was derived using a simplified axis drive model including the main servo parameters and dominating mechanical mode. Christopher (1999) studied the parameter space boundary for escape and chaos in the Duffing Twin-Well oscillator. The construction of a physical, nonlinear air-tract oscillator with ultrasound position detection systems allowed the author to observe a wide range of oscillatory behaviours including chaotic dynamics. It is inferred from his study that the range of space parameter or conditions has a great impact on the dynamics of Duffing Twin-Well Oscillator. An interesting analysis of a nonlinear oscillator system with one degree of freedom such as Vander Pol self – excitation term and parametric excitation of the Mathieu type was carried out by Jerzy (2001). It was found in the author's paper that a small external force causes qualitative and quantitative changes in the main parametric resonance. It is reported that the external harmonic force changes the system from chaos to regular motion. Abdelhak and Mohamed (2009) investigated the effect of fast harmonic excitation on frequency-locking in a Vander Pol-Mathieu Duffing oscillator. The outcome of their work implies that fast harmonic excitation parameter can significantly influence the nonlinear characteristics of spring behaviour as well as the entrainment region. Bogdan et al (2004) investigated how chaotic dynamics can be exploited as a tool for detecting parameter variations in Aeroelastic systems. The authors demonstrated that the sensitivity of chaotic behaviour to parametric changes is an effective tool in detecting structural changes such as variations in the stiffness parameter of the mounting point of the upstream end of the system panel. Alexander (1993) examined the influence of parameters variation on the dynamic system. It was shown in the paper that chaos appearing with a result of quasiperiodic motion may be easily suppressed by weak parametric perturbation of the system. Insook (1994) investigated how the dynamics of chaotic oscillator can be explored in the musical industry. The author was able to develop an interface which was extensively used for exploring parameter regions for pre-compositional activity and for sending control signals to both analog and simulated versions of the oscillator in real-time performance. This has no doubt serves as a clue for enriching concert performance. Syed et al (2010) showed the influence of variation in parameters as tools for generation and enhancement of chaos in erbium-doped fiber-ring lasers. It is reported in this paper that the degree of chaos determines the level of security in chaotic optical communication systems. In order to have an insight to the chaotic signatures of this system, certain parameters (such as the width and height of individual pulses, relationship of their time periods, gain quenching, shape, formation of bunches, and bumps of the chaotic waveforms) need to be analyzed. It is deduced from the study that the individual and cumulative behaviour of all parameters in influencing optical chaos provides a reliable platform in designing secure communication systems. The influence of parameters variation on the phase portrait in the mixing model has been studied (Lonescu and Cosrescu, 2008). The authors observed that the challenge of flow kinematics is far from complete solving. The mathematical methods developed in the field of mixing theory have established a significant relation between turbulence and chaos. Earlier study of the authors on the 3D non-periodic mixing models exhibited a quite complicated dynamic behaviour. Findings of the authors as stated in their paper showed that parameter variations has great influence on the length and surface deformations of the system models. As part of the efforts of the scientists and engineers to ensure practical realization of the model-based emergency forecasting in the pulse system, the effect of parameter changes on this system has been examined (Yury and Anna, 2003). The fractal diagram developed established a one to one correspondence between the parameter and phase subspaces by line up stages. This implies that parameter variations have a great influence on the system's model. The results of their study have provided a reliable platform for forecasting in the pulse system. In the effort of Dimitrios (2011) to develop a model for predicting population dynamics of the flour beetle (Tribolium Freemani), the influence of parameter variations was identified as key to population dynamics. Dimitrios found that for certain parameter manipulations, the model for insect species predicts chaotic behaviour with strong statistical confidence.

The dearth of literature which accounts for impact of parameter variations on the chaos dynamics of excited Duffing oscillator is a strong motivation for the research that is being reported in this paper. The research question of whether damp parameter or excitation amplitude is the key agent for impacting chaotic behaviour in Duffing oscillator demanded for an answer in order to further explore this dynamic system. The objective of this study was to characterize and compare chaos impact of excited Duffing Oscillator parameters.

2. Methodology

2.1. *Harmonically excited duffing's oscillator*

This study investigated normalized governing equation of harmonically excited Duffing system given by equation (1) with reference to Moon (1987), Dowell (1988) and Narayanan and Jayaraman (1989b).

$$\ddot{x} + \gamma\dot{x} - \frac{x}{2}(1 - x^2) = P_o Sin(\omega t) \tag{1}$$

In equation (1) x, $\dot{x}$ and $\ddot{x}$ represents respectively displacement, velocity and acceleration of the oscillator about a set datum. The damp coefficient is γ. Amplitude strength of harmonic excitation, frequency and time are respectively P_o, ω and t. According to literature combination of γ = 0.168, P_o = 0.21, and ω = 1 .0 or γ = 0.0168, P_o = 0.09 and ω = 1.0 parameters leads to chaotic behaviour. However, the present study focuses on fourth and fifth orders Runge-Kutta simulations of two distinct cases: fixed excitation frequency and amplitude coupled with constant step incremental of damp ($0.0168 \le \gamma \le 0.1680$) and fixed excitation frequency and damp coupled with constant step incremental of excitation amplitude ($0.09 \le P_o \le 0.21$) over large number of excitation period. The scatter phase plots of simulation results of '**family**' of excited Duffing oscillators are captured and characterised periodically using fractal disk dimension obtained by optimum disk count method. The relative distribution of the fractal disk dimensions were obtained and compared over one hundred sub-intervals for the cases and simulation algorithms.

In Salau and Ajide (2012) observation scale (X), optimum disk counted and the fractal disk dimension (D) are related by power law given by equation (2) for constant of proportionality (C). The slope of line of best fit to log-log plot of X versus Y gives estimate of the fractal disk dimension (D).

$$Y = CX^D \tag{2}$$

2.2. *Parameters of cases investigated*

A constant time step (Δt = 0.01), initial conditions (1, 0), random number generating seed value (9876), ten (10) observation scales and five (5) iterates are common to all investigated cases. The unsteady and steady solutions spanned the first twenty (20) and one thousand (10000) simulation periods of harmonic excitation respectively.

Case-I: Two thousand of damp value at constant spacing in the range

$(0.0168 \le \gamma \le 0.1680)$, P_o = 0.21, and ω = 1.

Case-II: Two thousand of excitation amplitude value at constant spacing in the range

$(0.09 \le P_o \le 0.21)$, γ = 0.168, and ω = 1.

3. Results and Discussions

Figures 1 and 2 refer. These are sample of scatter phase plots in thousand simulated and characterised using fractal disk dimension for cases I and II in this study. Similarly table 1 is a sample of optimum variation of disks counted for ten (10) different observations scale and five (5) iterations. The slope of line of best fit in figure 3 is an expression of the fractal quantification of space filling ability of the images given in figures 1 and 2. Therefore, it can be argued that figure 1 fill space more than figure 2 with an estimated fractal disk dimension value of 1.3937 and 1.3585 respectively. These estimated fractal dimensions are supported by appropriate entries of sample table 2 out of a thousand estimated in this study. The corresponding spectrum and relative distributions are given comparatively in figures 4 to 7.

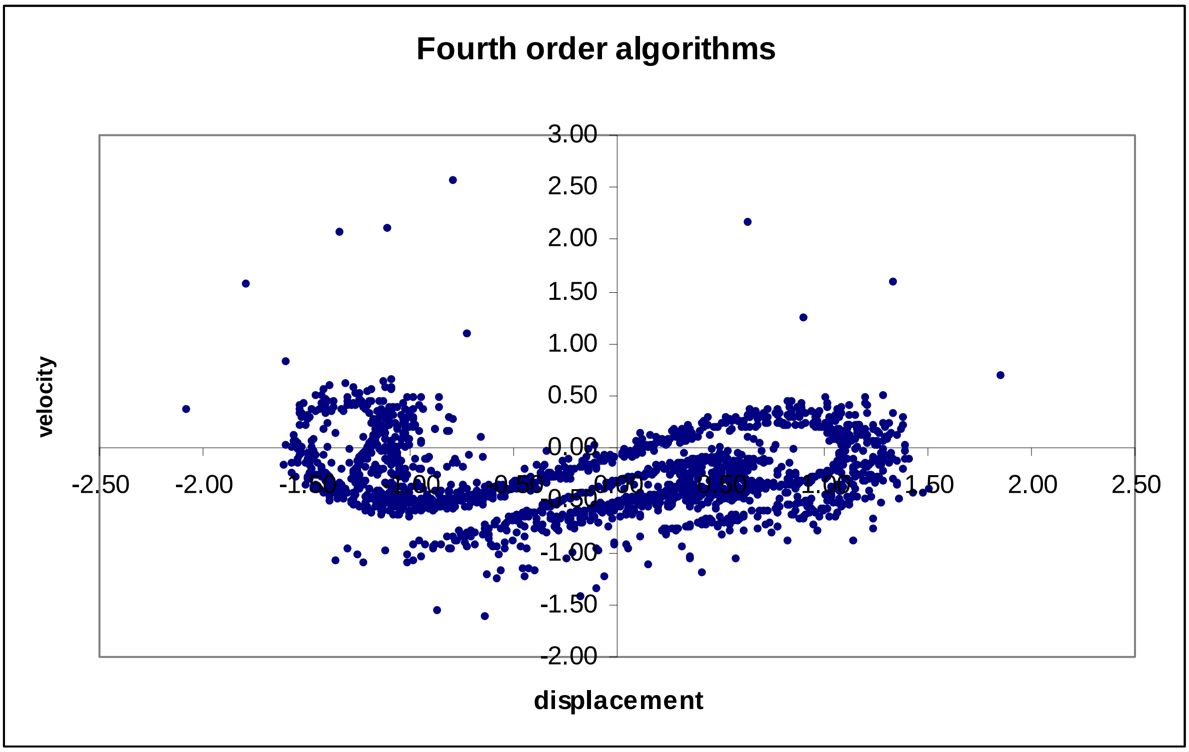

Figure 1: Scatter phase plots for Case-I at the end of twenty first (21) simulation periods (period of excitation).

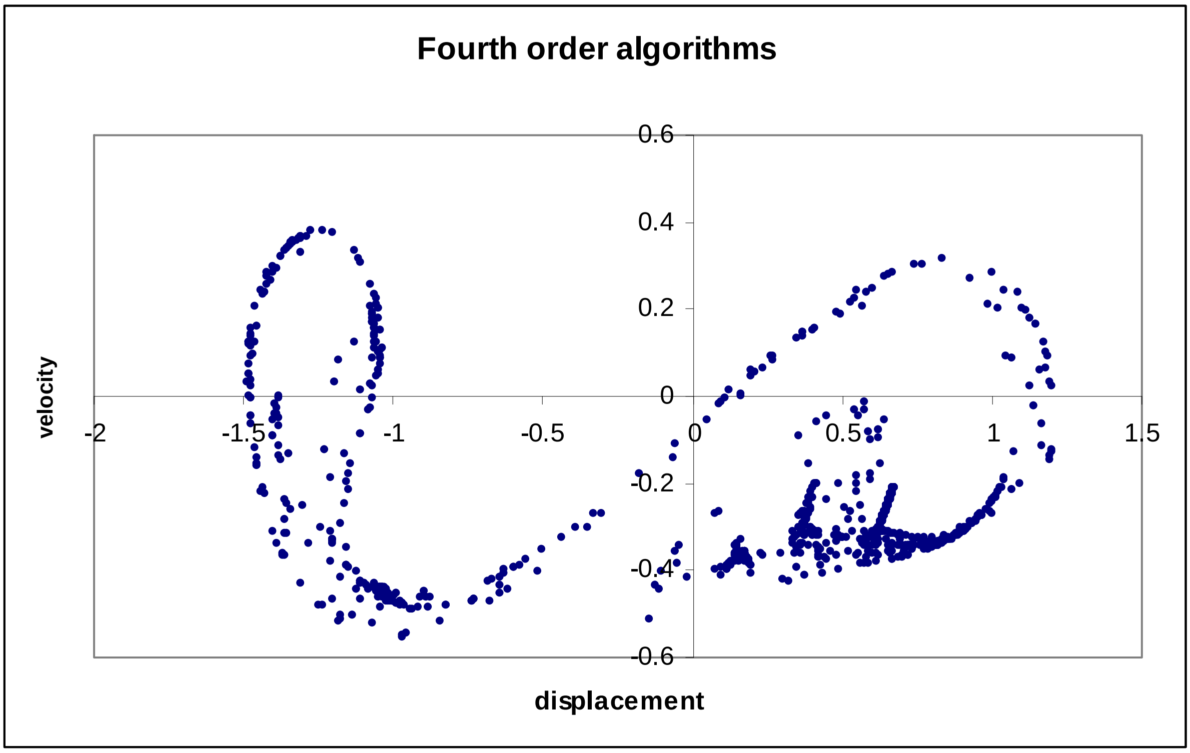

Figure 2: Scatter phase plots for Case-II at the end of twenty first (21) simulation periods (period of excitation).

Table 1: Variation of optimum counted disks with increasing observation scale number for cases I and II at the end twenty first (21) simulation periods (period of excitation).

Observation scales (**SC**)	Disks counted		Natural Logarithms		
	Case-I (**DCD**)	Case-II (**DCP**)	SC	DCD	DCP
1	3	2	0.00	1.10	0.69
2	7	3	0.69	1.95	1.10
3	14	7	1.10	2.64	1.95
4	24	11	1.39	3.18	2.40
5	28	15	1.61	3.33	2.71
6	34	19	1.79	3.53	2.94
7	45	25	1.95	3.81	3.22
8	52	28	2.08	3.95	3.33
9	63	32	2.20	4.14	3.47
10	73	36	2.30	4.29	3.58

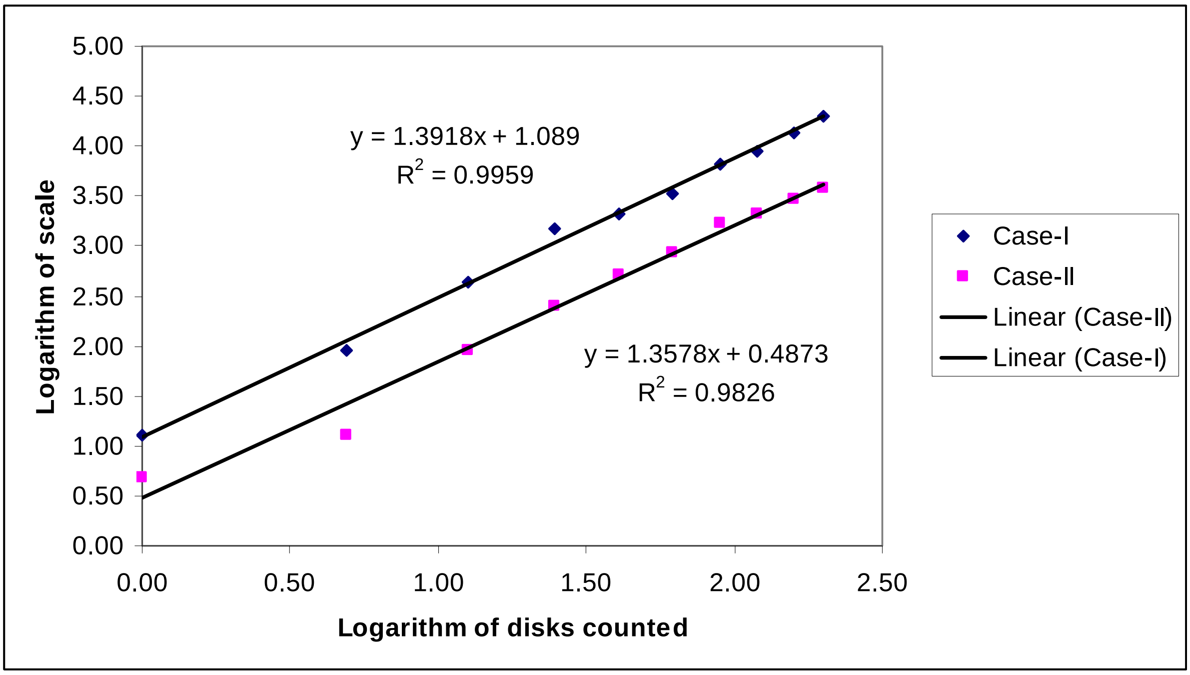

Figure 3: Log-Log plot of observation scale and disks counted for cases I and II.

Table 2: The variations of estimated fractal disk dimension with increasing periods of simulation.

Estimation time point in excitation periods	Estimated fractal disk dimensions			
	Fourth order algorithms		Fifth order algorithms	
	Case-I	**Case-II**	**Case-I**	**Case-II**
21	1.394	1.358	1.323	1.373
22	1.429	1.331	1.400	1.315
23	1.350	1.327	1.469	1.299
24	1.314	1.313	1.442	1.303
25	1.410	1.300	1.427	1.365
26	1.449	1.333	1.337	1.323
27	1.416	1.307	1.482	1.301
28	1.345	1.299	1.379	1.288
29	1.364	1.343	1.463	1.322
30	1.410	1.316	1.501	1.334
31	1.405	1.293	1.347	1.305
32	1.508	1.302	1.614	1.299
33	1.385	1.330	1.387	1.310
34	1.583	1.290	1.585	1.313
35	1.450	1.315	1.551	1.292
36	1.423	1.332	1.390	1.272
37	1.434	1.308	1.560	1.313
38	1.455	1.281	1.475	1.322
39	1.603	1.313	1.432	1.305
40	1.415	1.297	1.406	1.310

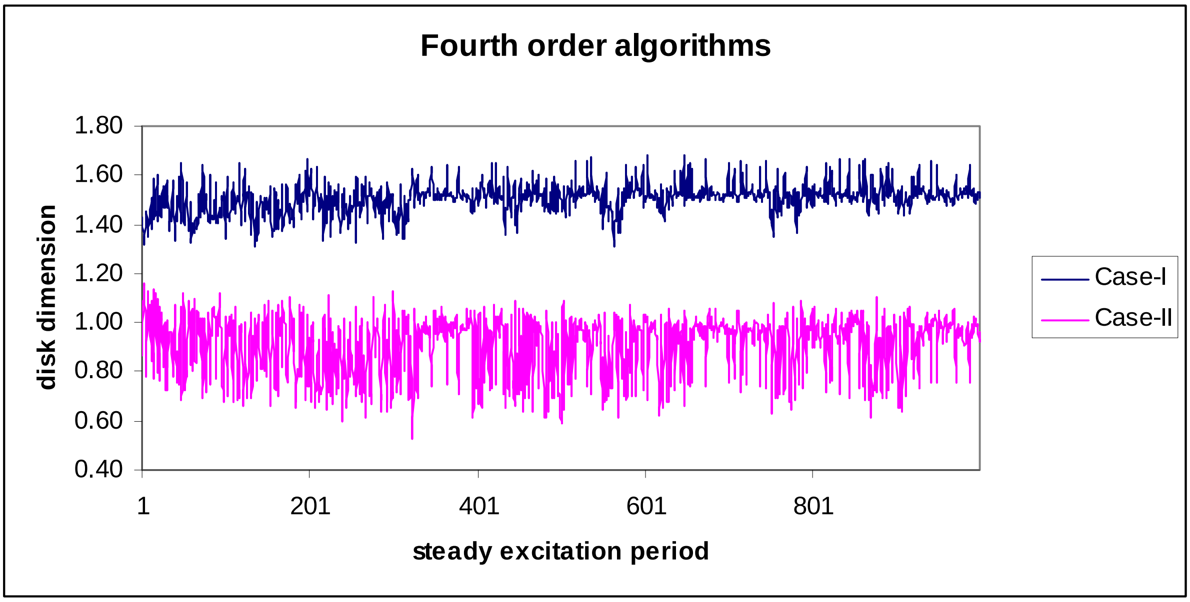

Figure 4: Spectrum of estimated fractal disk dimension for a fourth order algorithms simulations

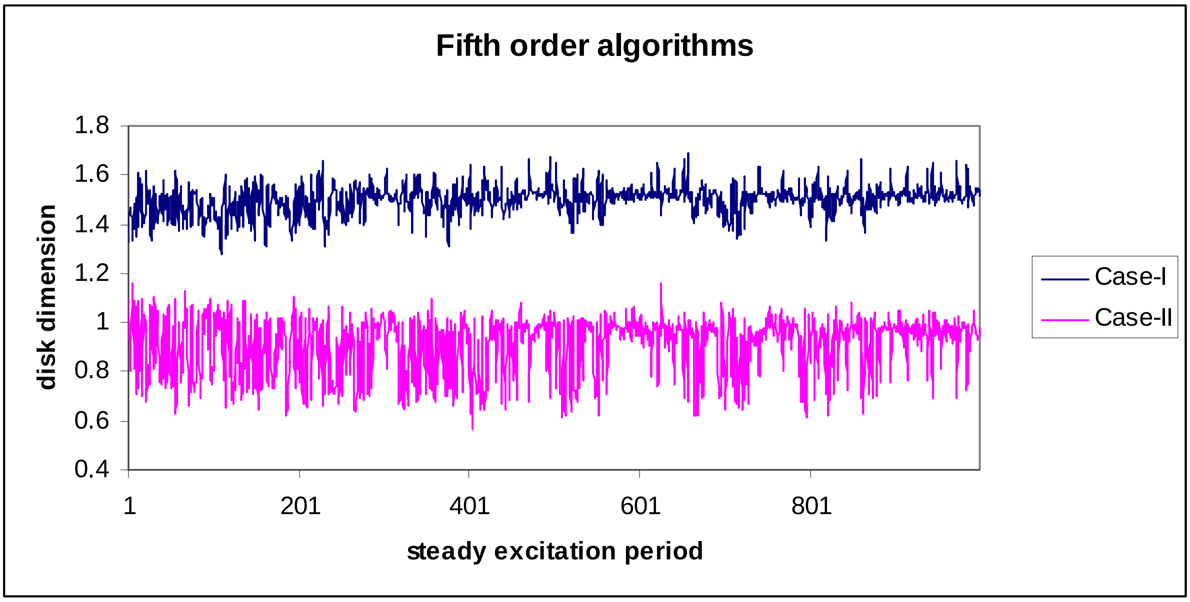

Figure 5: Spectrum of estimated fractal disk dimension for a fifth order algorithms simulations

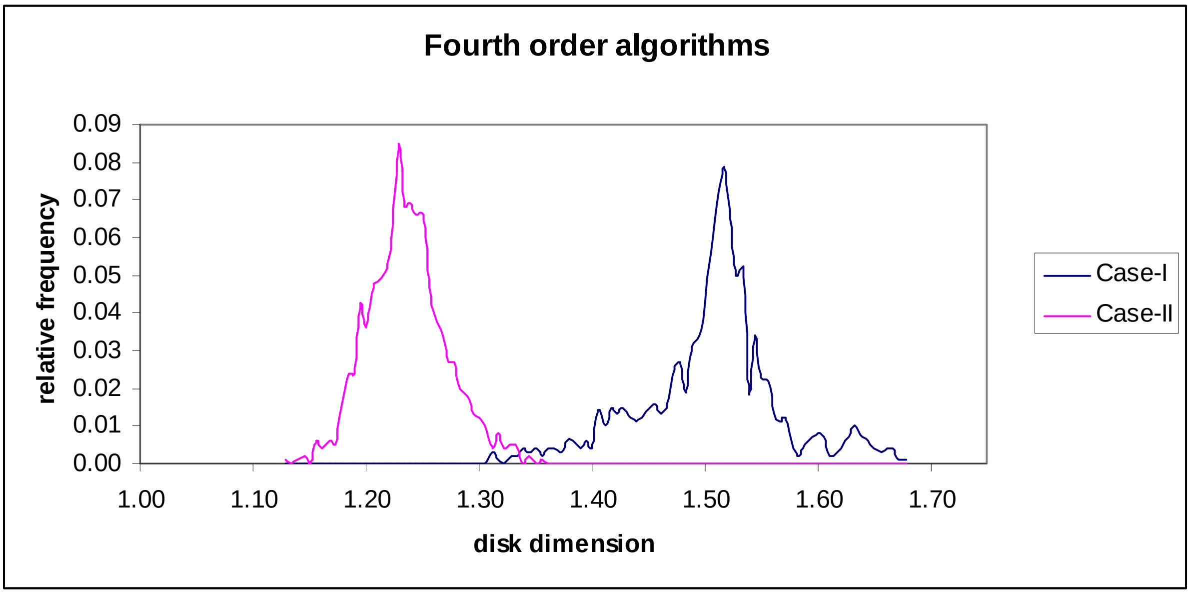

Figure 6: Relative distribution of estimated fractal disk dimension for a fourth order algorithms simulations

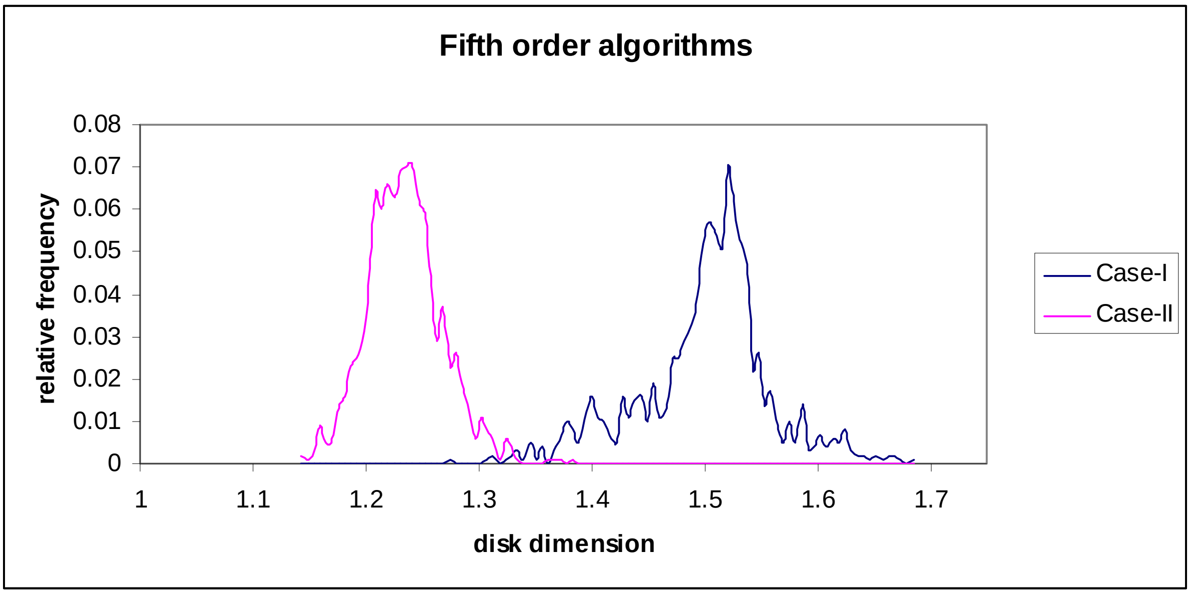

Figure 7: Relative distribution of estimated fractal disk dimension for a fifth order algorithms simulations

Figures 4 to 7 refer. The spectrums have noisy appearance for the cases and algorithms. The estimated fractal disk dimension variation is $1.312 \le D \le 1.681$ and $1.126 \le D \le 1.358$ respectively for case-I and II using fourth order Runge-Kutta algorithms as simulation tool over one thousand steady excitation periods. Similarly the recorded dimension variation is $1.277 \le D \le 1.688$ and $1.140 \le D \le 1.384$ respectively for case-I and II with fifth order Runge-Kutta algorithms as simulation tool. These ranges of disk dimensions suggest that the scatter phase plots vary from simple image to complex attractor. Both cases enjoyed unimodal disk dimension distribution for the two simulation algorithms investigated. The approximated modal dimension is 1.52 and 1.23 respectively for case-I and II while the average modal relative frequency is 8.0%. In view of these observed estimated dimension variation and modal dimension, it can be argued that damp parameter tune properly has higher potential of driven Duffing oscillator chaotically than its excitation amplitude counterpart provided other simulation conditions are same. It is to be noted that there is correlation between chaotic behaviour and fractal dimension, a higher dimension signifies higher unpredictability or chaos and vice-versa.

4. Conclusions

This study has quantified the chaotic driven impact of damp and excitation amplitude on Duffing oscillator using fractal dimension characterizing index. The constant step tuning of damp value and excitation amplitude leads to noisy spectrum and estimated dimension variation of $1.277 \le D \le 1.688$ and $1.126 \le D \le 1.384$ respectively. The tune of damp and excitation amplitude resulted in approximated modal dimension of 1.52 and 1.23 respectively with an average modal relative frequency being 8.0%. The foregoing dimensions supported damp parameter as easier agent of chaotic behaviour of Duffing oscillator compares with its excitation amplitude counterpart, other simulation conditions remains same.

Acknowledgement

We like to acknowledge Mr. BADA Ayokunle Abdul-Wasiu (**Matric. No: 123348**) whose effort on his B.Sc. final year (**Nov. 2009**) project work under the supervision of Dr. T.A.O. Salau of Mechanical Engineering University of Ibadan, Nigeria motivated the present study.

References

[1] Abdelhak F. and Mohamed B. (2009), Effect of Fast Harmonic Excitation on Frequency-Locking in a Vander Pol-Mathieu Duffing Oscillator. Communications in Nonlinear Science and Numerical Simulation, Elsevier, Volume 14, pp.244-253. www.elsevier.com/locate/cnsns

[2] Alexander Y.L.(1993),Dynamics Control of Chaotic Systems by Parametric Destochastization. Journal of Phy.A: Math. Gen.26 (1993), pp.4581-4594.

[3] Bogdan I.E., Liaosha S.T. and Michael P.P. (2004), Exploiting Chaotic Dynamics for Detecting Parametric Variations in Aeroelastic system. AIAA Journal, Volume 42,No.4

[4] Christopher I.R.(1999),The Parameter Space Boundary for Escape and Chaos in the Duffing Twin-Well Oscillator. A thesis presented in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in Physics at Massey University

[5] Dimitrios N. (2011), Predicting Induced Chaos in the Population Dynamics of an Insect Species. A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science of Imperial College London and Diploma of Imperial College London.

[6] Dowell E.H. (1988), Chaotic Oscillations in Mechanical Systems, Computational Mechanics, 3, 199-216.

[7] Francis C. Moon (1987), Chaotic Vibrations-An Introduction for Applied Scientists and Engineers, John Wiley & Sons, New York, ISBN 0-471-85685-1

[8] Insook C. (1994), Interactive Exploration of a Chaotic Oscillator for Generating Musical Signals in Real-time Concert Performance. Journal of the Franklin Institute, Elsevier Science Ltd., Volume 331B, No.6,pp. 785-818.

[9] Jerzy W. (2001), Synchronization Effects and Chaos in the Vander Pol-Mathieu Oscillator. Journal of Theoretical and Applied Mechanics, Volume 4, Issue 39, pp.861-884.

[10] Lonescu A. and Costescu M.(2008) ,The Inluence of Parameters on the Phaseportrait in the Mixing Model. International Journal of Computers, Communication and Control. 2008 Supplement, Volume 3,Issue 3,pp.333.

[11] Narayanan S. and Jayaraman K. (1989). Control of Chaotic Oscillations by Vibration Absorber. ASME Design Technical Conference, 12th Biennial Conference on Mechanical Vibration and Noise. DE 18.5, 391-394.

[12] Ricardo L.V.,Mário S.T. and Celso G. (2005), Estimation of Model Parameters in Nonlinear Engineering Systems: The Case of Suspension Bridge Oscillations. Proceedings of the workshop on Nonlinear Modelling and their Applications,4-6 May,2005-SP-Brasil.Edited by J M Balthazar RMLRF Brasil, EEN Macsu and BR Pontes

[13] Richard B., Pierre-Jean B. and Serge B. (2004),Influence of High Speed Machine Tool Control Parameters on the Contouring Accuracy. Application to Linear and Circular Interpolation. Journal of Intelligent and Robotic Systems, Volume 40, Issue 3, pp.321-342.

[14] Salau T.A.O. and Ajide O.O. (2012), Disk and Box Dimensions: Selected Case Studies of Fractals with IFS Codes, International Journal of Sciences and Technology (IJST), Volume 1, No. 5, pp.234-247.

[15] Syed Z.A., Muhammad K.I. and Muhammed Z. (2010), Effect of Parametric Variation on Generation and Enhancement of Chaos in Erbium-Doped Fiber Rings Lasers. Optical Engineering, Volume 49, Issue 10.

[16] Yury V.K. and Anna V.M.(2003), An Influence of Parameter variation on Model-Based Emergency Forecasting in the Pulse System. IEEE International Workshop on Intelligent Data Acquisition and Advanced Computing Systems. Technology and Applications, 8-10 September 2003, Lviv, Ukraine.