Development of Mathematical Models for Analysis of a Vehicle Crash

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Abstract: Nowadays, each newly produced car must conform to the appropriate safety standards and norms. The most direct way to observe how a car behaves during a collision and to assess its crashworthiness is to perform a crash test. Because of the fact that vehicle crash tests are complex and complicated experiments it is advisable to establish their mathematical models. This paper contains an overview of the kinematic and dynamic relationships of a vehicle in a collision. There is also presented basic mathematical model representing a collision together with its analysis. The main part of this paper is devoted to methods of establishing parameters of the vehicle crash model and to real crash data investigation i.e. – creation of a Kelvin model for a real experiment, its analysis and validation. After model's parameters extraction a quick assessment of an occupant crash severity is done.

Key-Words: Modeling, vehicle crash, Kelvin model, data processing, ridedown criterion.

1 Introduction

The main objective of this paper is to establish a mathematical model of a vehicle collision. The purpose of this task is to simulate how the crash looks like – i.e. what are the main parameters describing the collision – without performing any real test. Real world experiments are difficult to realize – there are needed appropriate facilities, measuring devices, data acquisition process, qualified staff and of course – a car. Those factors make the test complicated, time – consuming and expensive enterprise. Therefore it is justified to propose a mathematical model of a collision and analyze it instead of a real experiment to approximate its results. One of the significant advantages of such a modeling is that the equations of motion of the models can be solved explicitly with closed – form solutions. This allows us to predict the behavior of the real car without performing complicated crash tests.
In our main interest it is to analyze in details a Kelvin model. Having knowledge concerning one such a system we are able to extend the model e.g. to a couple of Kelvin elements in order to obtain a more accurate response (we can represent car elements and connections between them exactly by multiple spring-mass-damper models). When it comes to modeling the vehicle crash we can distinguish two main approaches. The first one utilizes CAE (Computer Aided Engineering) software including FEA (Finite Element Analysis) while the second one bases on the analytical method presented in this paper. Many researches have been done so far in both of those areas.
[1] to [3] provide a brief overview of different types of vehicle collisions.
Yang et al. [4] presented a feasibility study of using numerical optimization methods to design structural components for crash. The presented procedure required several software, which included parametric modeling (Pro/ENGINEER), automatic mesh generation (PDA PATRAN3), nonlinear finite element analysis (RADIOSS), and optimization programs. Both single and multiple objective formulations were used for numerical optimization, which resulted in better designs. It was found that crash optimization was feasible but costly and that finite element mesh quality was essential for successful crash analysis and optimization.
Mahmood et al. [6] have described in detail a procedure for rapid simulation and design of the frame of an automotive structure. They developed a simplified program, called V-CRUSH, for rapid simulation of the structure. The program used special collapsible 3-D thinwall beam elements and was used to design full front-end frame for a light truck. The frame was divided into several substructures that were designed and tested. Experiments were also performed on the structures. Correlation between the experimental and simulation results was very good. Similar approach has been used in [5] – a try to simulate a vehicle collision with means of

3D Computer Aided Design (CAD) software.
Huang et al. [7] described Ford's Energy Management System that used CRUSH (Crash Reconstruction Using Static History) lumped mass modelling capability. In that system, the energy absorbing (EA) structural components were represented by nonlinear springs. Force-displacement characteristics of the EAs were obtained through static crush tests. Those were input directly to the program. Dynamic environment of the crash event was treated by the velocity sensitivity factors (dynamic amplification factors). Using the system, barrier loads and passenger compartment loads were calculated and compared to the test results in a frontal crash. Above brief overview of the literature has been done according to Kim et al. [8].
Because of the fact that a crash pulse is a complex signal, it is justified to simplify it. One solution for this is covered in [9]. To perform analysis of the crash event one can use wavelet – based approximation of the crash pulse: accuracy of this method is very good. [10] to [13] talk over commonly used ways of describing a collision – e.g. investigation of tire marks or the crash energy approach.
Vehicle crash investigation is an area of up-to-date technologies application. [14] to [16] discuss usefulness of such developments as neural networks or fuzzy logic in the field of modeling of crash events. It is extremely important to assess what factors have an influence on the crash severity for an occupant. As in the case of a vehicle crash simulation, also here we can distinguish two main ways of examining the occupant behavior during an impact. [17] focuses on finding the relationship between the car's damage and occupant injuries. On the other hand, [18] employs FEM software to closely study the crash severity of particular body parts.
In this work (which bases on [19]) we provide basic information concerning kinematic and dynamic relationships in a vehicle collision together with methods of crash pulses approximation. Furthermore we cover the spring – mass – damper modeling of the vehicle crash. We start with an overview of a Kelvin model – an element in which a mass is attached to a spring and a damper which are connected in parallel. Subsequently we give information about factors which determine crash severity for an occupant during collision. The largest part of this work is devoted to answer the following question – how to establish a model from real crash data? After presenting two methods for solution of this problem we proceed to analysis measurements from real collision.

2 Vehicle collision simulation – Kelvin model

A Kelvin model is shown in Fig. 1. It contains a mass together with spring and damper which are connected in parallel.
This model can be utilized to simulate the vehicle-to-vehicle (VTV) collision, vehicle-to-barrier collision (VTB) as well as for component impact modeling.
In majority of cases the response of the system is underdamped therefore we focus on this type of behavior.

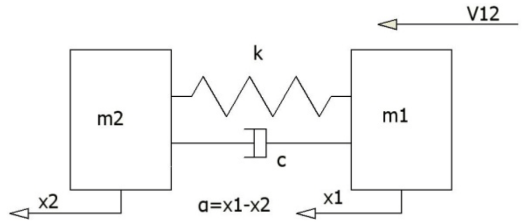

Fig. 1: Kelvin model.

2.1 Underdamped system (1>ζ>0)

Equation of motion (EOM):

$$\ddot{\alpha}+2\zeta\omega_e\dot{\alpha}+\omega_e^2\alpha=0 \qquad (1)$$

where $\zeta=\dfrac{c}{2m\omega_e}$ and $\omega_e=\sqrt{\dfrac{k}{m}}$

Transient responses of the underdamped system are:

$$\alpha(t)=\frac{v_0e^{-\zeta\omega_e t}}{\sqrt{1-\zeta^2}\,\omega_e}\sin(\sqrt{1-\zeta^2}\,\omega_e t) \qquad (2)$$

displacement (dynamic crush)

$$\dot{\alpha}(t)=v_0e^{-\zeta\omega_e t}[\cos(\sqrt{1-\zeta^2}\,\omega_e t)-\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,\omega_e t)]$$

(*3*) velocity

$$\ddot{\alpha}(t)=v_0\omega_e e^{-\zeta\omega_e t}[-2\zeta\cos(\sqrt{1-\zeta^2}\,\omega_e t)+\frac{2\zeta^2-1}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,\omega_e t)] \qquad (4)$$

deceleration.

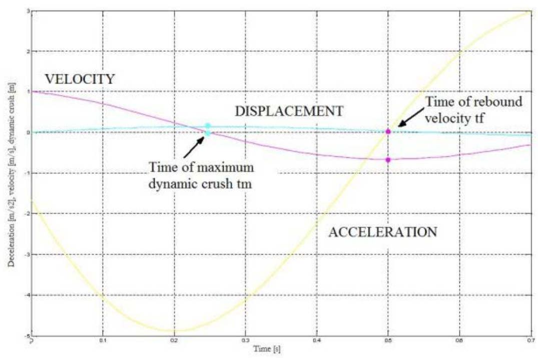

Fig. 2: Relationships between t_m, t_f and deceleration, velocity, displacement.

We see that above closed-form results are complex. To obtain the responses of the Kelvin model we use Matlab Simulink software.
In the analysis of the crash pulse (deceleration) alongside with velocity and displacement graphs we are able to observe specific relationships between them and between two timings: t_m – time of dynamic crush and t_f – time of rebound (or time of separation velocity). Those

dependences are shown in Fig. 2. The values on the graph below are just for presenting the principle – they do not come from any of the experiments.

At t_m the corresponding velocity is zero and the dynamic crush reaches its maximum value. At t_f the corresponding deceleration is zero and velocity reaches its maximum value. Please note that t_f is twice as long as t_m (in Fig. 2 t_f=0.5s and t_m=0.25s).

2.2 Coefficient of restitution (COR)

In the impact of the dynamic system the coefficient of restitution (COR) is defined as the ratio of relative separation velocity to the relative approach velocity. During the deformation phase, the relative approach velocity decreases from its initial value to zero due to the action of the deformation impulse, as shown in Fig. 3.

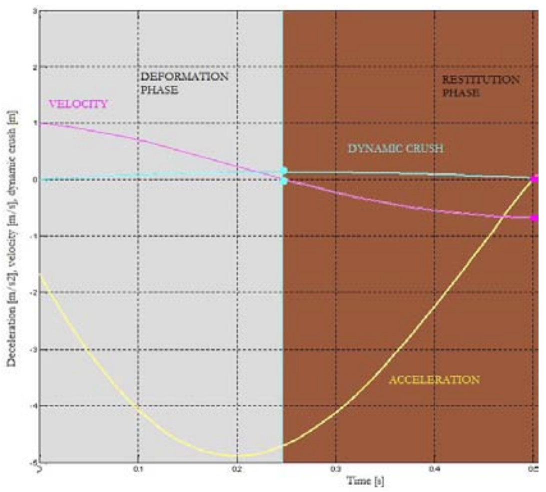

Fig. 3: Deformation and restitution phase during a crash.

At the time when the relative approach velocity is zero, the maximum dynamic crush occurs. The relative velocity in the rebound phase then increases negatively up to the final separation (or rebound) velocity, at which time the two masses separate from each other (or a vehicle rebounds from the barrier). At the separation time, there is no more restitution impulse acting on the masses, therefore, the relative acceleration at the separation time is zero [20]. To derive the relationship between the coefficient of restitution and damping factor of the system we use (4).

At the time of separation ($t=t_r=t_f$) the relative deceleration $\ddot{\alpha}=0$

Therefore from (4):

$$2\zeta\cos(\sqrt{1-\zeta^2}\omega_e t)+\frac{2\zeta^2-1}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\omega_e t)=0$$

We rewrite it in the following form:

$$\tan(\sqrt{1-\zeta^2}\omega_e t)=\frac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1}$$

from Pythagorean Theorem we get:

$$\cos(\sqrt{1-\zeta^2}\omega_e t)=2\zeta^2-1 \quad (5)$$

COR=relative separation velocity/relative approach velocity = $\dfrac{\dot{\alpha}(t)}{v_0}$

$$COR=e^{-\zeta\omega_e t}[\cos(\sqrt{1-\zeta^2}\omega_e t)-\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\omega_e t)] \quad (6)$$

There are three special cases:

1. No damping in the system ζ=0, then COR = 1.
2. Critically damped system ζ=1, then COR = 0.135.
3. Highly overdamped system ζ=∞, then COR = 0.

We simplify (6) by susbstituting:

$\sin(\sqrt{1-\zeta^2}\omega_e t)=2\zeta\sqrt{1-\zeta^2}$ so that we get:

$$COR=e^{-\left[\frac{\zeta}{\sqrt{1-\zeta^2}}\arccos(2\zeta^2-1)\right]} \quad (7)$$

Fig. 4 shows graphically relationship given by (7).

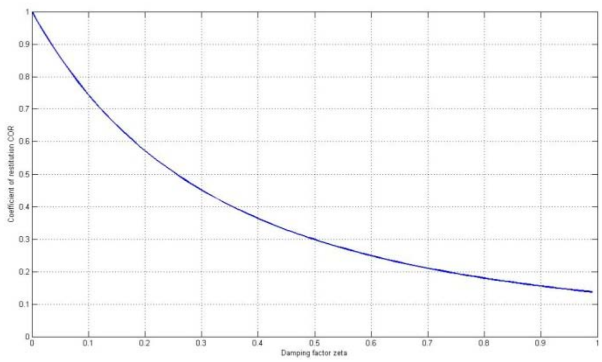

Fig. 4: Coefficient of restitution vs damping factor.

2.3 Separation time t_f

The contact duration of the two masses includes both contact times in deformation and restitution phases. When the relative acceleration becomes zero and relative separation velocity reaches its maximum recoverable value, separation of the two masses occurs.

From (5):

$\arccos(2\zeta^2-1)=\sqrt{1-\zeta^2}\omega_e t_f$ and

$$t_f=\frac{\arccos(2\zeta^2-1)}{\sqrt{1-\zeta^2}\omega_e} \quad (8)$$

separation time.

For ζ=0 (no damping in the model) we have a special case when:

$t_f=\dfrac{\arccos(-1)}{\omega_e}=\dfrac{\pi}{2\pi f}=\dfrac{1}{2f}$ therefore $t_f = 2t_m$.

3 Vehicle collision simulation – Maxwell model

A Maxwell model (shown in Fig. 5) consists of a spring and a damper connected in series. This two-parameter model is suitable for component modeling (creep and relaxation) as well as for localized vehicle impact modeling. According to the various damping characteristics, the Maxwell model gives us a wide range of timing when the maximum dynamic crush occurs. It is even possible to extend the timing at dynamic crush to infinity – then the deflection approaches an asymptote and the damping coefficient for which this occurs is termed as transition damping coefficient. The Maxwell model is suitable for soft impacts modeling such as localized pole and offset collisions where the timing at dynamic crush is fairly long. Since an experiment considered by us is a vehicle to pole collision – we present basic principles of modeling this type of crash by using an arrangement in which a spring and a damper are connected in series to a mass. However, in the analysis introduced in this work we will focus more on previously mentioned Kelvin model because of its simplicity and effectiveness.

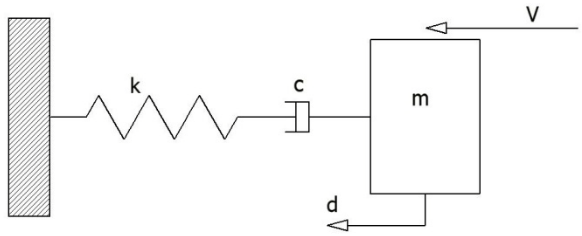

Fig. 5: Maxwell model.

3.1 Equations of motion for spring and damper

We are able to derive the equations of motion separately for the damper deflection d_c and total deflection d (as well as for spring deflection d_k).

EOM:

$$f\,(force) = f_k = f_c = kd_k = c\dot{d}_c$$

$$d\,(deflection) = d_k + d_c$$

$$\dot{d} = \frac{k}{c}d_k = \frac{k}{c}(d - d_c) \qquad (11)$$

$$\ddot{d} = -\frac{f}{m} = -\frac{c\dot{d}_c}{m} \qquad (12)$$

It is important that since deflections of spring and damper are additive in the Maxwell model, so are the deflection rates as shown in (13).

$$d = d_k + d_c \quad \text{and} \quad \dot{d} = \dot{d}_k + \dot{d}_c \qquad (13)$$

This statement allows us to obtain the overall response of the system by adding particular responses of the spring and damper.

3.2 Zero mass method

In this section, to determine equations of motion of a mass in the Maxwell model, a small mass m’ is placed between the spring and the damper as shown in Fig. 6. After deriving sets of differential equations for mass m and m’ we simply set mass m’ to zero and as a result we obtain a third order differential equation which describes the behavior of the system.

d – absolute displacement of mass m
d’ – absolute displacement of mass m’

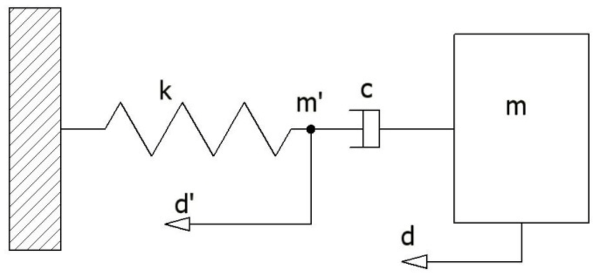

Fig. 6: Zero – mass approach.

EOM:

$$m\ddot{d} = -c(\dot{d} - \dot{d}') \quad \text{and} \quad m'\ddot{d}' = c(\dot{d} - \dot{d}') - kd'$$

now we differentiate both equations with respect to time and set m’ = 0 so that we obtain

$$m\dddot{d} = -c(\ddot{d} - \ddot{d}') \quad \text{and} \quad 0 = c(\ddot{d} - \ddot{d}') - k\dot{d}'$$

we sum both sides of the equations and transform them so that we obtain

$$\dot{d}' = \frac{-m}{k}\dddot{d}$$

and finally:

$$\dddot{d} + \frac{k}{c}\ddot{d} + \frac{k}{m}\dot{d} = 0 \qquad (14)$$

Characteristic equation: $s[s^2 + \frac{k}{c}s + \frac{k}{m}] = 0$ (15)

4 Bases of occupant – vehicle modeling

In this section we present basic notions and terms needed to asses the crash severity for an occupant. As the crash pulse approximation we use an ESW (Equivalent Square Wave). Fig. 7 shows an unbelted occupant in a vehicle during a collision.

v_0 – initial vehicle rigid barrier impact velocity
v^* – occupant to interior surface contact velocity
δ – occupant free travel space
c – vehicle dynamic crush at time t
t^* – time when occupant contacts restraint

t_m – time of dynamic crush

EOM for vehicle:

$$\ddot{x}_v = -\frac{F}{M} = -ESW \tag{16}$$

$$\dot{x}_v = v_0 - ESWt \tag{17}$$

$$x_v = v_0 t - \frac{1}{2} ESWt^2 \tag{18}$$

EOM for occupant:

$$\ddot{x}_o = -ESW[1 - \cos(p) + \omega t^* \sin(p)] \tag{19}$$

$$\dot{x}_o = \dot{x}_v + \frac{ESW}{\omega}[\sin(p) + \omega t^* \cos(p)] \tag{20}$$

$$x_o = x_v + \delta + \frac{ESW}{\omega^2}[1 - \cos(p) + \omega t^* \sin(p)] \tag{21}$$

$$\ddot{x}_o|_{max} = -ESW[1 + \sqrt{1 + (\omega t^*)^2}] \tag{22}$$

where $p = \omega(t - t*)$ for $t \geq t*$ and $\omega = \sqrt{\frac{k}{m_{occupant}}}$

$$t* = \sqrt{\frac{2\delta}{ESW}} \tag{23}$$

restraint contact time

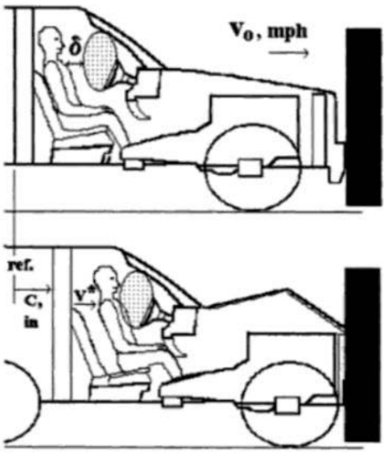

Fig. 7: Occupant during collision [20].

4.1 Ridedown criterion

Occupant relative contact velocity is less than the initial barrier impact speed if and only if the free travel space is less than the dynamic crush. Therefore we may write:

for $\delta < c$ then $v^* < v_0$ and $\frac{v*}{v_0} = \sqrt{\frac{\delta}{c}}$ as well as $\frac{t*}{t_m} = \sqrt{\frac{\delta}{c}}$.

In order for ridedown to exist, the conctact velocity v^* must be smaller than the barrier impact velocity v_0. The motion of the occupant can then be slowed down by the interior surface or restraint system during the deformation phase. The physical constraint needed to achieve this ridedown is having the interior space or restraint slack smaller than the vehicle dynamic crush. However, in the fixed barrier crashes, the dynamic crush of a truck is frequently smaller than that of cars. In order to minimize the occupant impact severity, the restraint slack is frequently reduced by using the pretensioner for both lap belt buckle and the shoulder belt retractor [20].

4.2 Dynamic Amplification Factor (DAF)

Let us define dynamic amplification factor as the ratio of maximum occupant chest deceleration to the ESW:

$$DAF = \frac{\ddot{x}_o|_{max}}{ESW} = \frac{-ESW[1 + \sqrt{1 + (\omega t^*)^2}]}{ESW}$$

$$DAF = 1 + \sqrt{1 + (\omega t^*)^2} = 1 + \sqrt{1 + (2\pi f t^*)^2} \tag{24}$$

where $\omega = 2\pi f$ and f is restraint natural frequency

This coefficient is convenient in analysis of the occupant behavior because the effect of the model parameters on the occupant response can be described exactly by the dynamic amplification factor.

4.3 Prediction of occupant deceleration using ridedown criterion and DAF

Since $DAF = \gamma = 1 + \sqrt{1 + (2\pi f t^*)^2}$ and we approximate the crash pulse by ESW we can write that the occupant chest deceleration is given by:

$a_0 = ESW \cdot DAF = ESW[1 + \sqrt{1 + (2\pi f t^*)^2}]$ where $v^* = ESW \cdot t^*$

therefore

$$a_0 = ESW + \sqrt{ESW^2 + (2\pi f v^*)^2} \tag{25}$$

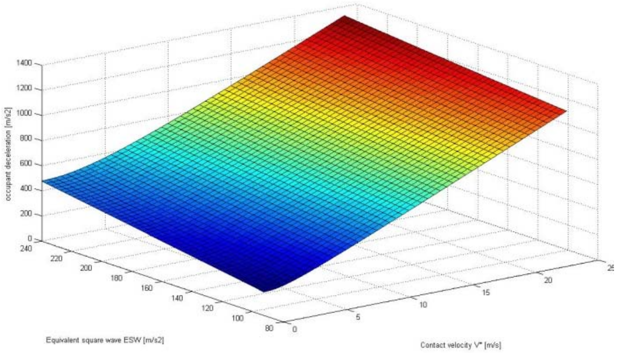

Fig. 8: Occupant deceleration vs ESW and contact velocity.

According to (25) we are able to predict the occupant deceleration by knowing the crash pulse, initial barrier impact velocity, restraint slack, maximum dynamic crush and restraint natural frequency. In Fig. 8 there is shown a graph based on (25). For the restraint natural frequency we have assumed a typical value of 7 Hz. This graph is useful because knowing what is an allowable value of a_0

(e.g. from national standards) we are able to determine for what values of interior contact velocity and ESW we keep a_0 in such an allowable occupant deceleration range. And this is helpful in car design since interior contact velocity depends on the restraint slack. Therefore we know what should be δ to stay in the limits given by v^* and ESW from a_0 condition.
As it has been mentioned above it is a common practice to install in trucks pretensioners . This is because of the fact that the ESW of a truck is higher that that of a car. Therefore if we want to decrease the occupant deceleration we need to decrease the restraint slack and that is justified by the DAF relationship.

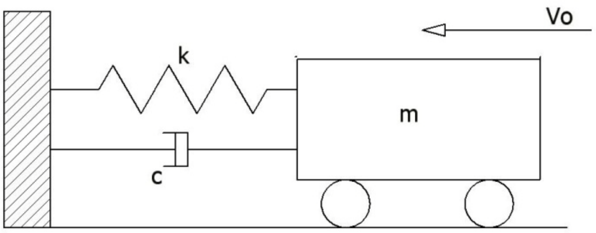

Fig. 9: VTB collision – Kelvin model.

5 Obtaining parameters of the Kelvin model from tests

Fig. 9 presents a Kelvin model of a vehicle-to-barrier impact.
k – spring stiffnes
c – damping coefficient
m – mass of the vehicle
v_0 – barrier initial impact velocity

5.1 Method 1 - analytical

To obtain structural parameters k and c first we need to determine two other parameters: ζ – damping factor and f – structure natural frequency. Before we do that let us first remind the centroid time concept.
Centroid time – it is a time at the geometric center of area of the crash pulse from time zero to the time of dynamic crush. We define it as follows:

$$t_c = \frac{C}{v_0} \qquad (26)$$

Referring to the Section 2 we may write:

$$\omega_e = \sqrt{\frac{k}{m}} \quad \text{and} \quad \zeta = \frac{c}{2m\omega_e}.$$

We define normalized centroid time and angular position at dynamic crush as:

$$\tau_c = t_c\omega_e = \left(\frac{\alpha_m}{v_0}\right)\omega_e = e^{-\zeta\tau_m}$$

$$\tau_m = t_m\omega_e = \frac{1}{\sqrt{1-\zeta^2}}\arctan\frac{\sqrt{1-\zeta^2}}{\zeta}$$

where α_m is the maximum dynamic crush.

After transforming above two equations we get:

$$\frac{\tau_c}{\tau_m} = \frac{t_c}{t_m} = \frac{\sqrt{1-\zeta^2}}{\arctan\frac{\sqrt{1-\zeta^2}}{\zeta}} e^{\left[\frac{-\zeta}{\sqrt{1-\zeta^2}}\arctan\frac{\sqrt{1-\zeta^2}}{\zeta}\right]} \qquad (27)$$

relative centroid location.

$\tau_m = t_m(2\pi f)$ so

$$ft_m = \frac{1}{2\pi\sqrt{1-\zeta^2}}\arctan\frac{\sqrt{1-\zeta^2}}{\zeta} \qquad (28)$$

(27) is illustrated in Fig. 10. Once we find the relative centroid location by determining t_c and t_m we can get damping factor ζ from the equation or from the plot.

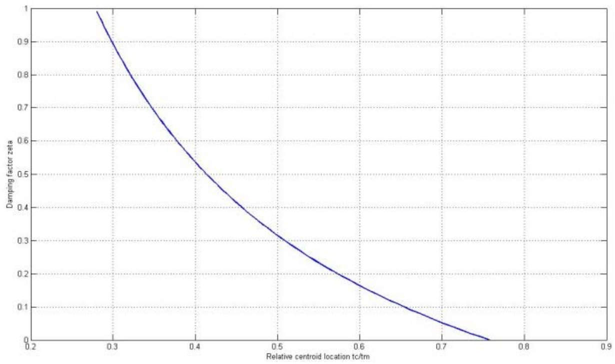

Fig. 10: Damping factor vs relative centroid location.

(28) is illustrated in Fig. 11. After deriving damping factor ζ and knowing time of dynamic crush t_m we can obtain the value of structure natural frequency from the plot or from the calculation.

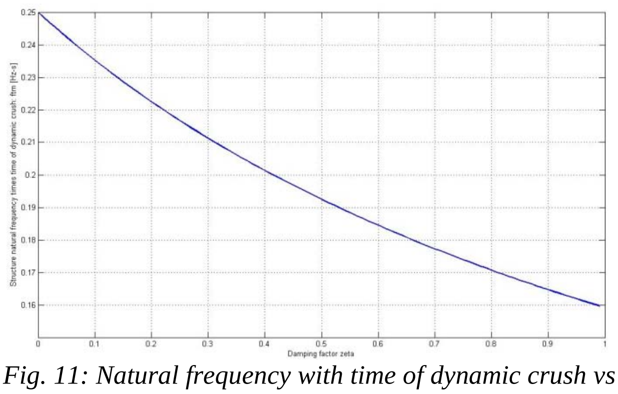

Fig. 11: Natural frequency with time of dynamic crush vs damping factor.

Having already values of ζ and f we determine structural parameters of the model: k and c:

Since $\omega_e = 2\pi f = \sqrt{\frac{k}{m}}$ and $\zeta = \frac{c}{2m\omega_e}$

$$k = 4\pi^2 f^2 m \qquad (29)$$

$$c = 4\pi f \zeta m \qquad (30)$$

In order to estimate parameters of the Kelvin model basing on the real crash pulse data we need just main information concerning the collision: time of dynamic crush t_m, initial impact velocity v_0, dynamic crush C and mass of the vehicle m. Taking into consideration the complexity of the collision phenomena it is a significant advantage – we can e.g. assess the stiffness and damping of a frontal structure of a car using simple data mentioned above.

5.2 Method 2 – Using Matlab Identification Toolbox

This Toolbox allows us to obtain the parameters of the system according to the input and output data. As an example we are going to use the Simulink model of the second order differential equation (second order oscillating element). The forcing factor is the external force over mass (acceleration) - initial conditions (velocity and displacement) are set to zero.

Data:
F = 300 N; k = 100 N/m; c = 5N-s/m; m = 3kg;
$v_0 = d_0 = 0$.

Equation of second order oscillating element is [21]

$$T^2 \frac{d^2 y(t)}{dt^2} + 2\zeta T \frac{dy(t)}{dt} + y(t) = Kx(t) \qquad (31)$$

where y(t) – output and x(t) – input.

By taking Laplace transform of (31) with zero initial conditions we get:

$$T^2 s^2 Y(s) + 2\zeta Ts Y(s) + Y(s) = KX(s) \qquad (32)$$

Therefore the transfer function of the system given by (32) is:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{K}{T^2 s^2 + 2\zeta Ts + 1} \qquad (33)$$

From the EOM of the Kelvin model we have:

$$mu(t) = m\frac{d^2 y}{dt^2} + c\frac{dy}{dt} + ky \qquad (34)$$

input u(t) is an acceleration.

By taking Laplace transform of (34) with zero initial conditions we obtain the following transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{m}{ms^2 + cs + k} \qquad (35)$$

(33) and (35) are describing the same model. Therefore they are equal to each other if and only if:

$$T = \sqrt{\frac{m}{k}} \quad \text{and} \quad K = \frac{m}{k} \quad \text{and} \quad \zeta = \frac{c}{2m\omega_e} \quad \text{and} \quad \omega_e = \sqrt{\frac{k}{m}}$$

.

With this knowledge we proceed to Identification Toolbox. We select the appropriate type of estimation – in our case – since we use Kelvin model - an underdamped system with two poles.

Parameters obtained from the estimation are shown in Fig. 12.

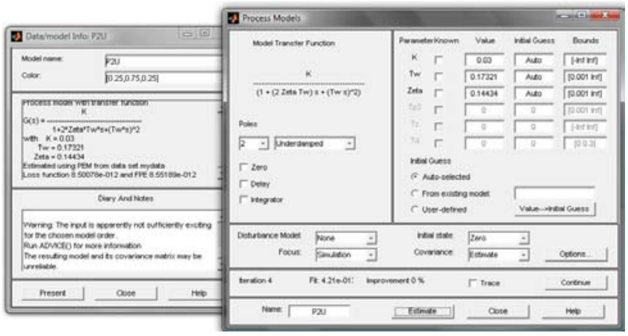

Fig. 12: Identification Toolbox – results.

After obtaining values which are describing the estimated model we check what are the values of T, K and ζ for our reference model – and we compare them with those ones from the estimated model.

For k = 100 N/m, c = 5N-s/m, m = 3kg we have:

$$K = \frac{m}{k} = \frac{3}{100} = 0.03 m/N$$

ID toolbox: K = 0.03m/N

$$T = \sqrt{\frac{m}{k}} = \sqrt{\frac{3}{100}} = 0.17321 s$$

ID toolbox: T = 0.17321s

$$\zeta = \frac{c}{2m\omega_e} = \frac{5}{2 \cdot 3 \cdot \sqrt{\frac{100}{3}}} = 0.14434$$

ID toolbox: ζ = 0.14434.

The results of approximation are perfect. Time constant T, damping coefficient ζ and gain K for both models – reference and our estimated are the same. It means that we can use Identification Toolbox to precisely determine what are the coefficients of the Kelvin model when we are given an input and an output to this system and the initial conditions are set to zero.

6 Investigation of real crash data

6.1 Experiment procedure [22]

In the experiment conducted by UiA [22] the test vehicle, a standard Ford Fiesta 1.1L 1987 model was subjected to a central impact with a vertical, rigid cylinder at the initial impact velocity v_0 = 35km/h. Mass of the vehicle (together with the measuring equipment and dummy) was 873kg. Scheme of the experiment is shown in Fig. 13.

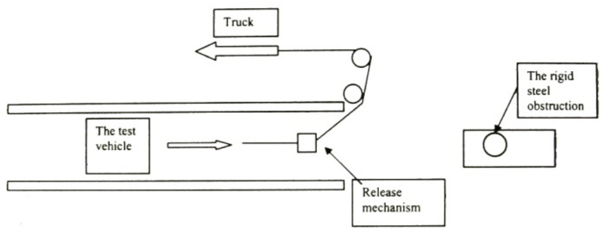

Fig. 13: Scheme of the test collision [22].

The acceleration field was 100m long, consisted of two anchored parallel pipelines with distance such as to give a clearance of 5mm to the front wheel tires. The force to accelerate the test vehicle was generated using a truck and a tackle. The release mechanism was placed 2m before the end of the pipeline. From the end of pipelines to the test item the distance was about 6.5m.

Vehicle accelerations in three directions (longitudinal, lateral and vertical) together with the yaw rate at the center of gravity were measured. The impact speed of the test vehicle was confirmed. Using normal-speed and high-speed video cameras, the behavior of the obstruction and the test vehicle during the collision was recorded.

6.2 Data processing

Since we are given accelerations in 3 directions (longitudinal – x, lateral – y, vertical – z) we are able to propose 3 different Kelvin models for every direction. Because of the fact that we are mostly interested in what happens in the direction in which a car hits the obstacle, we are going to analyze x – direction (longitudinal). Also, the data used here is not filtered – that task will be covered by us in our further work.

To approximate the crash pulse we use Identification Toolbox with Gaussian approximation as it is shown in Fig. 14.

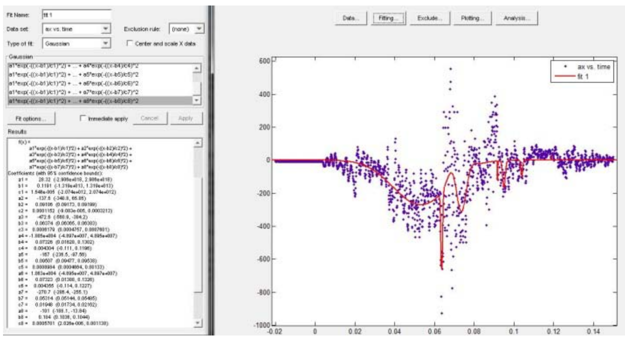

Fig. 14: Curve Fitting Toolbox – preparation of measured data.

To obtain the velocity curve we integrate the approximated pulse (using Matlab Curve Fitting Toolbox) – the result is shown in Fig. 15.

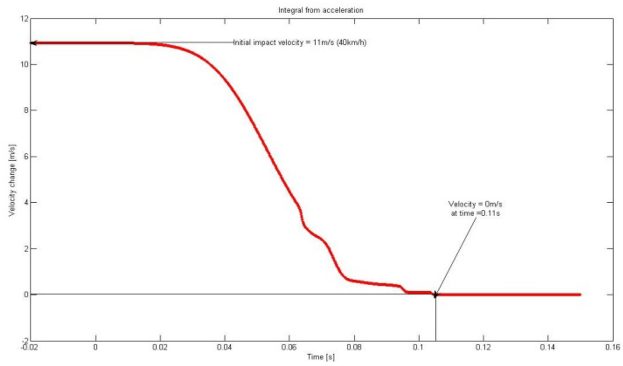

Fig. 15: Velocity obtained from measured acceleration.

We see in Fig. 15 that the initial velocity is not equal to 35km/h as it was stated in the experiment's description but is about 5km/h higher. This discrepancy is a result of using raw data – without filtering. From this plot we read the value of time of dynamic crush t_m = 0.11s.

To get the displacement graph we proceed in the manner described above – we approximate and integrate the velocity curve from Fig. 15. The plot of displacement is shown in Fig. 16.

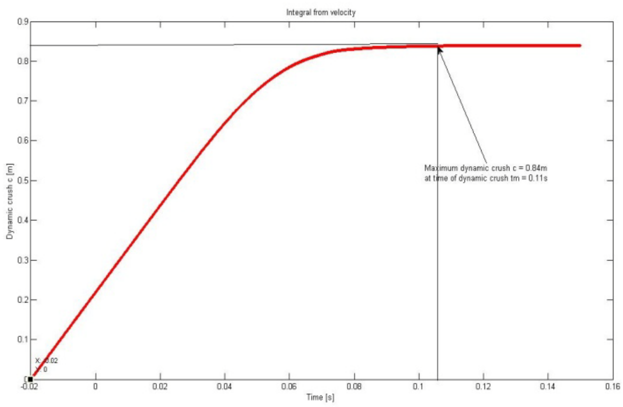

Fig. 16: Displacement obtained from measured acceleration.

From the plot we determine maximum dynamic crush C = 0.84m at time of dynamic crush = 0.11s.

6.3 Comparison between model and real data according to method 1

Knowing values of v_0 = 11m/s, t_m = 0.11s, C = 0.84m and m = 873kg from the real test, using method described in Section 4.1 we determine parameters: t_c, t_c/t_m, ζ, f, k, c:

$$t_c = \frac{C}{v_0} = \frac{0.84m}{11m/s} = 0.076s$$

$$\frac{t_c}{t_m} = \frac{0.076s}{0.11s} = 0.69$$ and furthermore from Fig. 10:

$\zeta = 0.05$ and from Fig. 11: ft_m = 0.24Hz-s

$$ft_m = 0.24Hz - s \quad \text{so} \quad f = \frac{0.24Hz - s}{t_m} = \frac{0.24Hz - s}{0.11s} = 2.2Hz$$

We calculate parameters of the Kelvin model:

$$k = 4\pi^2 f^2 m = 4\pi^2 (2.2Hz)^2 \cdot 873kg = 166809N/m$$

spring stiffness

$$c = 4\pi f \zeta m = 4\pi \cdot 2.2Hz \cdot 0.05 \cdot 873kg = 1207N - s/m$$

damping coefficient

Having parameters of the Kelvin model we investigate its response using the Simulink diagram with the initial velocity v_0 = 11m/s. The result is shown in Fig. 17.

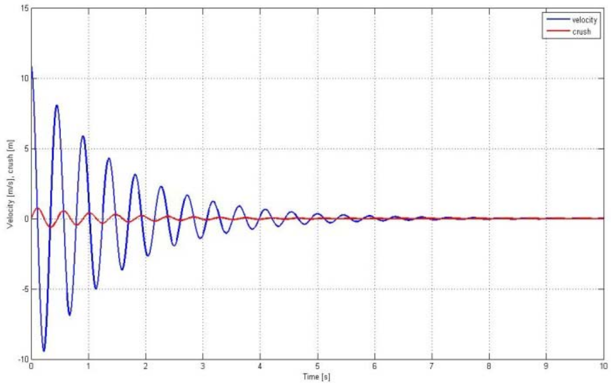

Fig. 17: Velocity and displacement vs time of the Kelvin model with estimated parameters.

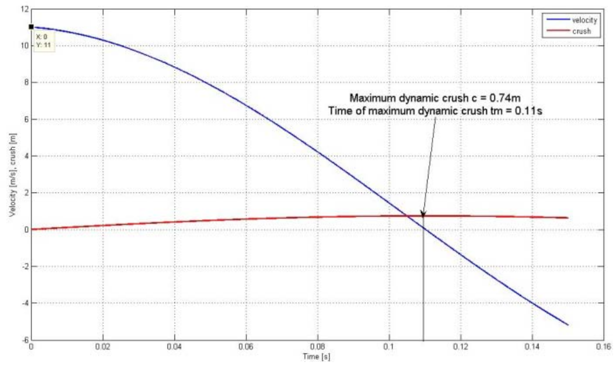

Fig. 18: Velocity and displacement vs time of the Kelvin model with estimated parameters in the crash interval

That is the response of the mass for 10 seconds. It is a typical one for the second order oscillating element – also Kelvin model.

In Fig. 18 you see the response in time interval used in the test data analysis (a magnified part of above plot).

Although the approximation of the velocity curve is not quite exact – we do not see e.g. a rebound, still the accuracy of approximation is very good. Time of dynamic crush t_m obtained from the model is exactly the same as in experiment: t_m = 0.11s and maximum dynamic crush C = 0.74m is about 12% less than that from the real test.

6.4 Estimation of the maximum chest deceleration of an occupant

Knowing initial impact velocity v_0 = 11m/s, maximum dynamic crush C = 0.84m, time when it occurs t_m = 0.11s and the distance between an occupant and a vehicle (restraint slack) δ = 0.6m we calculate:

$$ESW = 0.5\frac{v_0^2}{C} = 0.5\frac{11^2}{0.84} = 72m/s^2$$

$DAF = \frac{a_0}{ESW} = 1 + \sqrt{1 + (2\pi f t^*)^2}$, where $t* = \sqrt{\frac{\delta}{c}} t_m$ is the time when occupant contacts restraint, f is restraint natural frequency (we assume following [20] another typical value of f = 6Hz) and a_0 is the maximum occupant chest deceleration.

$$DAF = 1 + \sqrt{1 + (2\pi \cdot 6Hz \cdot \sqrt{\frac{0.6m}{0.84m}} 0.11s)^2} = 4.64$$

Maximum occupant chest deceleration:

$$a_0 = DAF \cdot ESW = 4.64 \cdot 72m/s^2 = 334m/s^2 = 34g$$

7 Conclusions

We have managed to prepare the crash data for analysis and extract the mathematical model from it. Challenges here were to choose an appropriate test data approximation and time interval in which we want to investigate the collision. Having this done we can determine maximum crush of a car, when it occurs, how the velocity changes and what are the changes in acceleration of a car during a crash. What is more – we have also estimated the maximum occupant deceleration – that is one of the main tasks in the area of crashworthiness study.

When it comes to the further work, we can extend our simple spring-mass-damper model to multiple Kelvin elements system. Then we will obtain more accurate results and – what is also important – for particular car components, not for a car as a one element. The other thing which could improve the results is using a Maxwell model (a mass together with a spring and damper connected in series) for a vehicle to rigid pole crash simulation. This system gives better approximation of offset impacts and localized pole collisions because it provides more accurate response for longer times of maximum dynamic crush. The last improvement is to filter the accelerometer measurements and to use more accurate type of curve approximation.

References

[1] M. Borovinsek, M. Vesenjak, M. Ulbin, Z. Ren: *Simulation of crash tests for high containment levels of road safety barriers*, EngineeringFailureAnalysis, Volume 14, Issue 8, December 2007, Pages 1711-1718, Elsevier Ltd.

[2] Rami Harb, Essam Radwan, Xuedong Yan, Mohamed Abdel-Aty: *Light truck vehicles (LTVs) contribution to rear-end collisions*, Accident Analysis & Prevention
Volume 39, Issue 5, September 2007, Pages 1026-1036, Elsevier Ltd.

[3] Adrian Soica, Simona Lache: *Theoretical and Experimental Approaches to Motor Vehicle: Pedestrian Collision*, 3rd WSEAS International Conference on Applied and Theoretcial Mechanics, Spain, December 14-16, 2007.

[4] R.J. Yang, L. Tseng, L. Nagy, J. Cheng, 1994: *Feasibility study of crash optimization*. In: B.J. Gilmore, D.A. Hoetzel, D. Dutta, H.A. Eschenauer: (eds.) Advances in design automation, DE-Vol. 69-2, pp. 549–556. ASME.

[5] Xiao-yun Zhang, Xian-long Jin, Wen-guo Qi, Yi-zhi Guo, 2008: *Vehicle crash accident reconstruction based on the analysis 3D deformation of the auto-body*, Advances in Engineering Software, Volume 39, Issue 6, June 2008, Pages 459-465, Elsevier Ltd.

[6] H.F. Mahmood, D.G. Wheatley, M.A. El-Bkaily, J.Y. Bakkar, 1993: On the front end design of automotive vehicle for crashworthiness. *Crashworthiness and occupant protection in transportation systems*, AMD-Vol. 169/BED-Vol. 25, pp. 311–318. New York: ASME.

[7] M. Huang, R. Chen, B. Margolin, 1995: Use of an advanced CRUSH model in estimating vehicle impact loading and energy absorption. In: *Crashworthiness and occupant protection in transportation systems*, AMD-Vol. 210/BED-Vol. 30, pp. 273–285. New York: ASME.

[8] C.H. Kim, A.R. Mijar, J.S. Arora, 2001: *Development of simplified models for design and optimization of automotive structures for crashworthiness*. In: Structural and Multidisciplinary Optimization, Vol. 22, No. 4, pp. 307-321. Springer Berlin / Heidelberg.

[9] H.R. Karimi, K.G. Robbersmyr: *Wavelet-Based Signal Analysis of a Vehicle Crash Test With a Fixed Safety Barrier*, WSEAS 4th European Computing Conference, Bucharest, Romania, April 20-22, 2010.

[10] Gašper Šušteršič, Igor Grabec, Ivan Prebil: *Statistical model of a vehicle-to-barrier* collision, International Journal of Impact Engineering, Volume 34, Issue 10, October 2007, Pages 1585-1593

[11] X.Y. Zhang, X.L. Jin and J. Shen, 2006: *Virtual reconstruction of two types of traffic accident by the tire marks*, Lecture Notes Comput Sci 4282, pp. 1128–1135

[12] Daniel Trusca, Adrian Soica, Bogdan Benea, Stelian Tarulescu: *Computer Simulation and Experimental Research of the Vehicle Impact*, WSEAS Transactions on Computers, Issue 1, Volume 8, January 2009, Pages 1185-1194.

[13] Dario Vangi: *Energy loss in vehicle to vehicle oblique impact*, International Journal of Impact Engineering, Volume 36, Issue 3, March 2009, Pages 512-521, Elsevier Ltd.

[14] T. Omar, A. Eskandarian, N. Bedewi: *Vehicle crash modelling using recurrent neural networks*, Mathematical and Computer Modelling, Volume 28, Issue 9, November 1998, Pages 31-42, Elsevier Ltd.

[15] Istvan A. Harmati, Andras Rovid, Laszlo Szeidl, Peter Varlaki*: Energy Distribution Modeling of Car Body Deformation Using LPV Representations and Fuzzy Reasoning*, WSEAS Transactions on Systems, Issue 1, Volume 7, January 2008, Pages 1228-1237.

[16] V. Giavotto, L. Puccinelli, M. Borri, A. Edelman, T. Heijer: *Vehicle dynamics and crash dynamics with minicomputer*, Computers & Structures, Volume 16, Issues 1-4, 1983, Pages 381-393, Elsevier Ltd.

[17] Carol Conroy, Gail T. Tominaga, Steve Erwin, Sharon Pacyna, Tom Velky, Frank Kennedy, Michael Sise, Raul Coimbra: *The influence of vehicle damage on injury severity of drivers in head-on motor vehicle crashes*, Accident Analysis & Prevention, Volume 40, Issue 4, July 2008, Pages 1589-1594, Elsevier Ltd.

[18] Yuqing Niu, Weixin Shen, James H. Stuhmiller: *Finite element models of rib as an inhomogeneous beam structure under high-speed impacts*, Medical Engineering & Physics, Volume 29, Issue 7, September 2007, Pages 788-798, Elsevier Ltd.

[19] W. Pawlus, J.E. Nielsen, H.R. Karimi, K.G. Robbersmyr: *Mathematical modeling and analysis of a vehicle crash*, 4th European Computing Conference, Bucharest, Romania, April 20-22, 2010.

[20] M. Huang, 2002: *Vehicle Crash Mechanics*, Boca Raton: CRC Press.

[21] J. Kowal, 2003: *Podstawy Automatyki, tom 1*, Kraków: Uczelniane Wydawnictwa Naukowo – Dydaktyczne AGH.

[22] K. G. Robbersmyr, 2004: *Project Report 43/2004, Calibration test of a standard Ford Fiesta 1.1L, model year 1987, according to NS – EN 12767*, Grimstad: Agder University College.