Design of power screws

Instructional Objectives

At the end of this lesson, the students should have the knowledge of

- Stresses in power screw.
- Design procedure of a power screw.

6.2.1 Stresses in power screws

Design of a power screw must be based on the stresses developed in the constituent parts. A power screw is subjected to an axial load and a turning moment. The following stresses would be developed due to the loading:

a) Compressive stress is developed in a power screw due to axial load. Depending on the slenderness ratio it may be necessary to analyze for buckling. The compressive stress $\sigma_c$ is given by $\sigma_c = \frac{P}{\pi d_c^2}$ where $d_c$ is the core diameter and if slenderness ratio $\lambda$ is more than 100 or so buckling criterion must be used. $\lambda$ is defined as $\lambda = \frac{L}{k}$ where $I = Ak^2$ and $L$ is the length of the screw. Buckling analysis yields a critical load $P_c$ and if both ends are assumed to be hinged critical load is given by $P_c = \pi^2 \frac{EI}{L^2}$. In general the equation may be written as $P_c = n\pi^2 \frac{EI}{L^2}$ where $n$ is a constant that depends on end conditions.

b) Torsional shear stress is developed in the screw due to the turning moment and this is given by $\tau = \frac{16T}{\pi d_c^3}$ where $T$ is the torque applied.
c) Bending stresses are developed in the screw thread and this is illustrated in figure-6.2.1.1. The bending moment \( M = \frac{Fh}{2} \) and the bending stress on a single thread is given by \( \sigma_b = \frac{My}{I} \). Here \( y = \frac{t}{2}, I = \frac{\pi d^3}{12} \) and \( F \) is the load on a single thread. Figure-6.2.1.2 shows a developed thread and figure-6.2.1.3 shows a nut and screw assembly. This gives the bending stress at the thread root to be \( \sigma_b = \frac{3Fh}{\pi d_m t^2} \). This is clearly the most probable place for failure.

Assuming that the load is equally shared by the nut threads

d) Bearing stress \( \sigma_{br} \) at the threads is given by \( \sigma_{br} = \frac{F'/n'}{\pi d_m h} \)

e) Again on similar assumption shear stress \( \tau \) at the root diameter is given by

\[ \tau = \frac{F'/n'}{\pi d_c t} \]

Here \( n' \) is the number of threads in the nut. Since the screw is subjected to torsional shear stress in addition to direct or transverse stress combined effect of bending, torsion and tension or compression should be considered in the design criterion.

6.2.2 Design procedure of a Screw Jack

A typical screw jack is shown in figure-6.2.2.1. It is probably more informative to consider the design of a jack for a given load and lift. We consider a reasonable value of the load to be 100KN and lifting height to be 500mm. The design will be considered in the following steps:
1. **Design of the screw**

A typical screw for this purpose is shown in figure-6.2.2.2.

Let us consider a mild steel screw for which the tensile and shear strengths may be taken to be approximately 448 MPa and 224 MPa respectively. Mild steel being a ductile material we may take the compressive yield strength to be also close to 448 MPa. Taking a very high factor of safety of 10 due to the nature of the application and considering the axial compression the core diameter of the screw \( d_c \) is given by

\[
\frac{100 \times 10^3}{\pi \left( \frac{448 \times 10^6}{10} \right)} = \frac{d_c}{\pi \left( \frac{448 \times 10^6}{10} \right)^{\frac{3}{2}}}
\]

which gives \( d_c \approx 54 \) mm.

From the chart of normal series square threads in table-6.1.1.1 the nearest standard nominal diameter of 70 mm is chosen, with pitch \( p = 10 \) mm.

Therefore, core diameter \( d_c = 60 \) mm, Major diameter \( d_{maj} = 70 \) mm, Mean diameter \( d_m = 65 \) mm, Nominal diameter \( d_n = 70 \) mm.

The torque required to raise the load is given by

\[
T = Fd_m \left( \frac{l + \mu \pi d_m}{\pi d_m - \mu l} \right)
\]

Where \( l = np, n \) being the number of starts. Here we have a single start screw and hence \( l = p = 10 \) mm, \( d_m = 65 \) mm, \( F = 100 \times 10^3 N \)

Taking a safe value of \( \mu \) for this purpose to be 0.26 and substituting the values we get

\[ T = 1027 \text{ Nm}. \]

**Check for combined stress**

The screw is subjected to a direct compressive stress \( \sigma_c \) and a torsional shear stress \( \tau \). The stresses are given by

\[
\sigma_c = \frac{4F}{\pi d_c^2} = \frac{4 \times 100 \times 10^3}{\pi (0.06)^2} = 35.3 \text{ MPa}
\]
The principal stress can be given by
\[ \sigma_{1,2} = \frac{35.3}{2} \pm \sqrt{\left(\frac{35.3}{2}\right)^2 + (24.22)^2} = 47.6 \text{MPa and } -12.31 \text{MPa} \]
and maximum shear stress \( \tau_{\text{max}} = 29.96 \text{ MPa} \).

The factor of safety in compression = \( \frac{448}{12.31} = 36.4 \) and in shear = \( \frac{224}{29.96} = 7.48 \). Therefore the screw dimensions are safe. Check for buckling and thread stress are also necessary. However this can be done after designing the nut whose height and number of threads in contact is needed to determine the free length of the screw.

### 2. Design of the nut
A suitable material for the nut, as shown in figure- 6.2.2.3, is phosphor bronze which is a Cu-Zn alloy with small percentage of Pb and the yield stresses may be taken as
Yield stress in tension \( \sigma_{y,\text{t}} = 125 \text{MPa} \)
Yield stress in compression \( \sigma_{y,\text{c}} = 150 \text{MPa} \)
Yield stress in shear \( \sigma_{y,\text{s}} = 105 \text{MPa} \)
Safe bearing pressure \( P_b = 15 \text{MPa} \).
Considering that the load is shared equally by all threads bearing failure may be avoided if
\[ F = \frac{\pi}{4} \left( d_{\text{maj}}^2 - d_c^2 \right) P_b n' \]
where \( n' \) is the number of threads in contact. Substituting values in the above equation we have \( n' = 6.52 \). Let \( n' = 8 \). Therefore \( H = n' p = 8X10 = 80 \text{mm} \).
The nut threads are also subjected to crushing and shear. Considering crushing failure we have
\[ F = n' \frac{\pi}{4} \left( d_{maj}^2 - d_c^2 \right) \sigma_c \]

This gives \( \sigma_c = 12.24 \text{ MPa} \) which is adequately safe since \( \sigma_{cy} = 150 \text{ MPa} \) and therefore crushing is not expected. To avoid shearing of the threads on the nut we may write \( F = \pi d_{maj} t' \tau \) where \( t \) is the thread thickness which for the square thread is \( \frac{d}{2} \) ie 5. This gives \( \tau = 11.37 \text{ MPa} \) and since \( \tau_y = 105 \text{MPa} \) shear failure of teeth is not expected. Due to the screw loading the nut needs to be checked for tension also and we may write

\[ CF = \frac{\pi}{4} (D_1^2 - d_c^2) \sigma_{ty} \]

A correlation factor \( C \) for the load is used to account for the twisting moment. With \( C=1.3 \) and on substitution of values in the equation \( D_1 \) works out to be 70mm. But \( D_1 \) needs to be larger than \( d_{maj} \) and we take \( D_1 = 100 \text{mm} \).

We may also consider crushing of the collar of the nut and to avoid this we may write

\[ F = \frac{\pi}{4} (D_2^2 - d_c^2) \sigma_{cy} \]

Substituting values we have \( D_2 = 110 \text{mm} \). To allow for the collar margin we take \( D_2 = 120 \text{mm} \). Considering shearing of the nut collar \( \pi D_1 a \tau_y = F \).

Substituting values we have \( a = 4 \text{mm} \) Let \( a = 15 \text{mm} \)

3. **Buckling of the Screw.**

Length \( L \) of the screw = Lifting height + H.

This gives \( L = 500 + 80 = 580 \text{ mm} \)

With the nominal screw diameter of 70mm ,

\[ l = \frac{\pi(0.07)^4}{64} = 1.178 \times 10^{-6} \]

and \( K = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.178 \times 10^{-6}}{\frac{\pi (0.07)^2}{4}}} = 0.0175 \text{ mm} \).

The slenderness ration \( \lambda = \frac{L}{K} = \frac{0.58}{0.0175} = 33 \)
This value of slenderness ratio is small ($< 40$) and the screw may be treated as a short column. No buckling of the screw is therefore expected.

4. Tommy bar

A typical tommy bar for the purpose is shown in figure-6.2.2.4.a.

Total torsional moment without the collar friction is calculated in section 6.2.2.1 and $T = 1027$ Nm. The collar friction in this case (see figure-6.2.2.1) occurs at the interface $I$. However in order to avoid rotation of the load when the screw rotates a loose fitting of the cup is maintained.

Length $l$ of the tommy bar = $l_1 + D_3$ and we may write the torque $T$ as

$$T = F_1 l$$

Where $F_1$ is the maximum force applied at the tommy bar end and this may be taken as approximately 400 N. This gives $l = \frac{1027}{400} = 2.56$ m. This length of the tommy bar is too large and one alternative is to place the tommy bar centrally and apply force at both the ends. This alternative design of the tommy bar is also shown in figure-6.2.2.4.b The bar is subjected to a bending moment and its maximum value may be taken as 1027 Nm. This means to avoid bending we may write

$$\frac{\pi}{32} d_1^3 \sigma_{ty} = 1027$$

where $d_1$ is the tommy bar diameter as shown in figure-6.2.2.4.b If we choose a M.S bar of $\sigma_{ty} = 448$ MPa the tommy bar diameter $d_1$ works out to be $d_1 = 0.0285$ m.

Let $d_1 = 30$ mm and we choose $d_2 = 40$ mm

5. Other dimensions

$D_3 = (1.5$ to $1.7) \times d$ Let $D_3 = 112$ mm

$D_4 = \frac{D_3}{2} = 56$ mm

Let $L_1 = 100$ mm and $t_4 = 10$ mm
\( t_1 = 0.25 \, d_n \approx 18 \text{ mm} \), \( D_5 = 2.25 \, D_2 = 270 \text{ mm} \), \( D_6 = 1.75 \, D_5 = 473 \text{ mm} \), \( t_3 = \frac{t_1}{2} = 9 \text{ mm} \).

6.2.1.1 \textit{F-} Loading and bending stresses in screw threads

6.2.1.2 \textit{F-} Dimensions of a developed thread
6.2.1.3 F- A screw and nut assembly
6.2.2.1F - A typical screw jack
6.2.2.3F - A phosphor bronze nut for the screw jack

6.2.2.2F - The screw with the provision for tommy bar attachment
6.2.2.4.a F - A typical tommy bar with a holding end.

6.2.2.4.b F - A typical centrally located tommy bar
6.2.3 Summary of this Lesson

In this lesson firstly the stresses developed in a power screw are discussed. Design procedure of a screw jack is then considered and the components such as the screw, the nut and the tommy bar are designed for strength. Finally the assembled screw jack along with the components are shown in the dimensioned figures.

6.2.4 Reference for Module-6


Source:
http://nptel.ac.in/courses/Webcourse-contents/IIT%20Kharagpur/Machine%20design1/pdf/module-6%20lesson-2.pdf