Principles of Passive Vibration Control: Design of absorber

INTRODUCTION

The term 'vibration absorber' is used for passive devices attached to the vibrating structure. Such devices are made up of masses, springs and dampers. From a control point of view, vibration absorbers can be considered as passive controllers. (The term 'passive' is used loosely here, meaning that the controller can be constructed using masses, springs and dampers). For such controllers stability is not an issue, since the closed loop system is also a passive one. Vibration absorbers are devices attached to flexible structures in order to minimize the vibration amplitudes at a specified set of points. Design of vibration absorbers has a long history. First vibration absorber proposed by Frahm in 1909 [Den Hartog, 1956] consists of a second mass-spring device attached to the main device, also modeled as a mass-spring system, which prevents it from vibrating at the frequency of the sinusoidal forcing acting on the main device. If the absorber is tuned so that its natural frequency coincides with the frequency of the external forcing, the steady state vibration amplitude of the main device becomes zero.

From a control perspective, the absorber acts like a controller that has an internal model of the disturbance, which therefore cancels the effect of the disturbance. The vibrating systems generally consist of a mass, linked to the structure by a spring and a damper. The vibration absorbers are set up so that the mass can vibrate at a specific frequency, close to that of the structure as shown in Fig. 7.9. Thus, when the structure starts to move under an excitation, the mass of the vibration absorber move as well, this creates a new system with a different behaviour. It is then impossible to reach the uncomfortable accelerations reached with the structure alone.

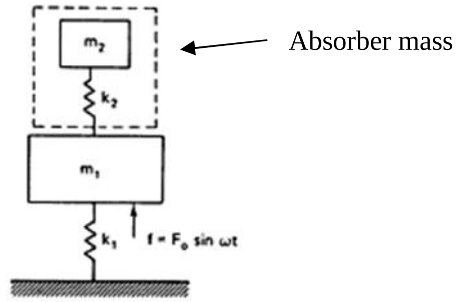


Fig. 7.9 Vibration absorber

A sinusoidal force $f = F_0 \sin\mu t$ acts on an undamped main mass-spring system (without the absorber mass attached). When the forcing frequency equals the natural frequency of the main mass the response is infinite. This is called resonance, and it can cause severe problems for vibrating systems. Mass Addition will reduce the effect (system response) of a constant excitation force. Mass Addition applies Newton's 2nd Law, which implies that if the mass of a system is increased while the force input remains constant, acceleration (vibration response) will decrease. When an absorbing mass-spring system is attached to the main mass and the resonance of the absorber is tuned to match that of the main mass, the motion of the main mass is reduced to zero at its resonance frequency. Thus, the energy of the main mass is apparently "absorbed" by the tuned dynamic absorber. It is interesting to note that the motion of the absorber is finite at this resonance frequency. This is because the system has changed from a 1-DOF system to a 2-DOF system and now has two resonance frequencies, neither of which equals the original resonance frequency of the main mass (and also the absorber).

This approach to vibration control is especially useful for equipment that has inherent high vibrations or transient (impacting) forces, such as diesel engines, hammer mills, positive displacement pumps, etc. Low natural frequency isolation requires a large deflection isolator such as a soft spring. However, the use of soft springs to control vibration can lead to rocking motions which are unacceptable. Hence, an inertia block mounted on the proper isolators can be effectively used to limit the motion and provide the needed isolation. Inertia blocks are also useful in applications where a system composed of a number of pieces of equipment must be continuously supported. An example of such equipment is a system employing calibrated optics. Thus, inertia blocks are important because they lower the center of gravity and thus offer an added degree of stability; they increase the mass and thus decrease vibration amplitudes and minimize rocking; they minimize alignment errors because of the inherent stiffness of the base; and they act as a noise barrier between the floor on which they are mounted and the equipment that is mounted on them. One must always keep in mind, however, that to be effective, inertia blocks must be mounted on isolators.

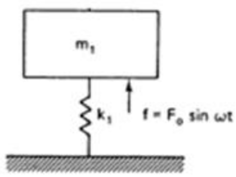
$$\boldsymbol{n}_{1}\ddot{\boldsymbol{x}}_{1} + \boldsymbol{k}_{1}\boldsymbol{x}_{1} + \boldsymbol{k}_{2}(\boldsymbol{x}_{1} - \boldsymbol{x}_{2}) = \boldsymbol{f}$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0 \qquad (7.12)$$

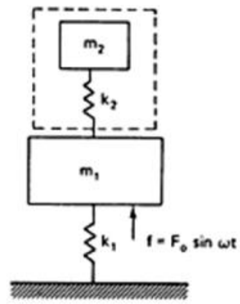
If no damping is present in the system as shown, the response of the 2-DOF system is infinite at these new frequencies. While this may not be a problem when the machine is running at its natural frequency, an infinite response can cause problems during startup and shutdown. A finite amount of damping for both masses will prevent the motion of either mass from becoming infinite at either of the new resonance frequencies. However if damping is present in either mass-spring element, the response of the main mass will no longer be zero at the target frequency.

Typically, the mass of the system is increased at the equipment foundation. Therefore, to successfully apply this method for vibration control, machines must be firmly connected to the foundation. From a machine design perspective, foundations that include a well designed sole plate, epoxy grouted to a concrete base, will help to achieve vibration control and maintenance free equipment operation. One rule of thumb states that the weight of the foundation should be 5X the machine weight. Note that other vibration control techniques which include adding mass to change a system natural frequency, fn, and/or the use of large “inertia block” foundations, are not considered Mass Addition, but rather Tuning and Isolation (rearrangement of force inputs).

Vibration absorber is applied to the machine whose operation frequency meets its resonance frequency as shown in Fig.7.10. Vibration absorber is often used with machines run at constant speed or systems with const. excited freq., because the combined system has narrow operating bandwidth as shown in Fig. 7.11.



(a) SDOF



(b) 2-DOF

Fig. 7.10 Mass addition as vibration absorber

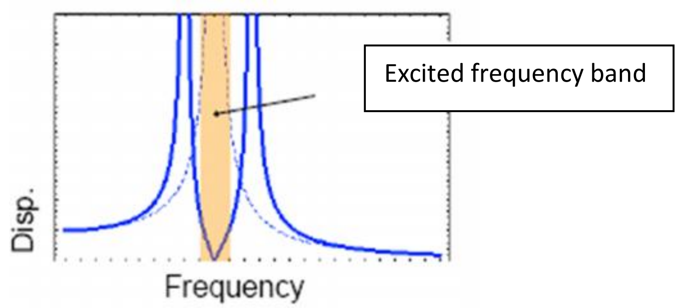


Fig. 7.11 Frequency Band of vibration excitation

When mass is added as vibration absorber, system acts as a two degree of freedom and the equation of motion in state space form is as;

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_1 \\ -k_1 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \sin\omega t \\ 0 \end{Bmatrix} \tag{7.13}$$

When the system is excited due to harmonic excitation, the input function is as;

$x_1(t) = X_1 \sin \omega t$
$x_2(t) = X_2 \sin \omega t$

$$\begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_1 \\ -k_1 & k_2 - m_2\omega^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \sin \omega t = \begin{Bmatrix} F_0 \sin \omega t \\ 0 \end{Bmatrix} \quad (7.14)$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} X \begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_1 \\ -k_1 & k_2 - m_2\omega^2 \end{bmatrix}^{-1} \quad (7.15)$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \frac{1}{\delta} \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} X \begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_1 \\ -k_1 & k_2 - m_2\omega^2 \end{bmatrix}^{-1} \quad (7.16)$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \frac{1}{\delta} \begin{bmatrix} (k_1 + k_2 - m_1\omega^2)\, F_0 \\ k_1 F_0 \end{bmatrix} \quad (7.17)$$

Where, $\delta = (k_1 + k_2 - m\omega^2)\,(k_2 - m_2\,\omega^2) - k_2^2$

$$X_1 = \frac{(k_1 + k_2 - m\omega^2)\, F_0}{\delta} \quad (7.18)$$

$$X_2 = \frac{k_2\, F_0}{\delta}$$

Here, m_2 and k_2 can be chosen such that X= 0, so

$$\omega^2 = \frac{k_2}{m_2}$$

Motion of absorber mass:

$$x_a(t) = -\frac{F_0}{k_a}\sin\omega t \quad ; \qquad X_a = -\frac{F_0}{k_a}$$

Force acting on the absorber mass:

$$k_a x_a = k_a(-F_0 / k_a) = -F_0$$

Force provided by m_2 = Disturbance force; Hence, zero net force acting on the primary mass.

$$X = \frac{(k_a - m_a\omega^2)F_0}{(k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2} \tag{7.19}$$

$\omega_p = \sqrt{\frac{k}{m}}$; Natural frequency of primary system without absorber

$\omega_a = \sqrt{\frac{k_a}{m_a}}$; Natural frequency of absorber as it is attached to primary system

Normalize parameters

$\mu = \frac{m_a}{m}$

$\beta = \frac{\omega_a}{\omega_p}$

$r = \frac{\omega}{\omega_a}$

Normalize displacement of the primary mass as;

$$\left|\frac{Xk}{F_0}\right| = \left|\frac{1 - r^2}{(1 + \mu\beta^2 - r^2)(1 - r^2) - \mu\beta^2}\right| \tag{7.20}$$

- Damping can reduce the resonance amplitude of the system,
- Amplitude at operating point increase with increasing damping
- Select ω which will be tuned to zero amplitude

- Relation between *k2* and *m2* is obtained from $\omega^2 = ka/ma$
- Select *ma* and *ka*(consider restrictions: force, motion of absorber mass)

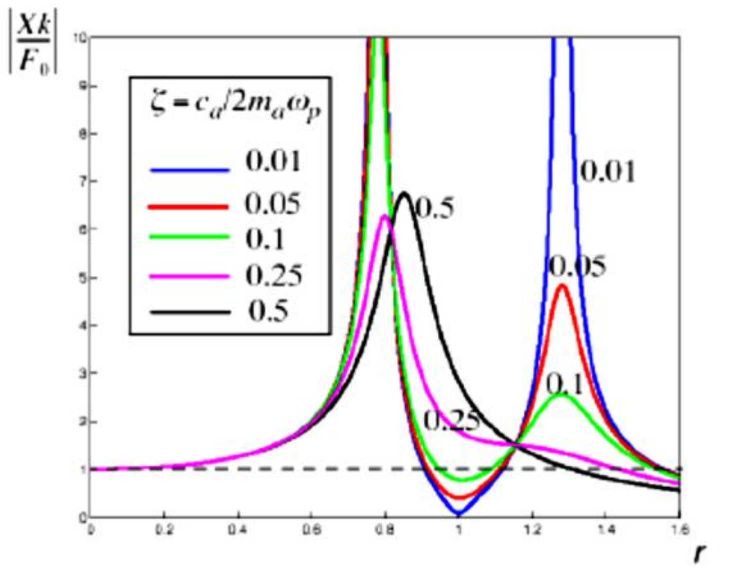


Fig. 7.12 Variation of amplitude with damping ratio

The 2-DOF system has two natural frequencies, corresponding to the two natural modes of vibration for the system. In the lower frequency mode both masses move in the same direction, in-phase with each other. In the higher frequency mode the two masses move in opposite direction, 180° out of phase with each other. The animation below shows the motion of the 2-DOF system at normalized forcing frequencies of f_{left}**=0.67** (in-phase mode), f_{middle}**=1** (undamped classical tuned dynamic absorber), and f_{right}**=1.3** (opposite-phase mode). The arrows in the movie represent the magnitude and phase of the force applied to the main mass. Often in the design of systems, damping is introduced to achieve a reduced level of vibrations, or to perform vibration suppression. Consider a symmetric system of the form

$$M\ddot{x} + D\dot{x} + kx = 0$$

where M, D, and K are the usual symmetric, positive definite mass, damping, and stiffness matrices, to be adjusted so that the modal damping ratios, ζ_i, have desired values. This in turn provides insight into how to adjust or design the individual elements m_i, C_i and k_i such that the desired damping ratios are achieved.

Often in the design of a mechanical part, the damping in the structure is specified in terms of either a value for the loss factor or a percentage of critical damping, i.e., the damping ratio. This is mainly true because these are easily understood concepts for a single degree- of-freedom model of a system. However, in many cases, of course, the behavior of a given structure may not be satisfactorily modeled by a single modal parameter. Hence, the question of how to interpret the damping ratio for a multiple-degree-of-freedom system such as the symmetric positive definite system of Equation arises.

An n-degree-of-freedom system has n damping ratios, ζ_i. These damping ratios are, in fact, for the normal mode case (i.e., under the assumption that $DM^{-1}K$ is symmetric). Recall that, if the equations of motion decouple, then each mode has a damping ratio ζ_i defined by;

$$\zeta_i = \frac{\lambda_i(D)}{2\omega_i} \quad (7.21)$$

where ω_i is the ith undamped natural frequency of the system and $\lambda_i(D)$ denotes the ith eigenvalue of matrix D.

To formalize this definition and to examine the nonnormal mode case ($DM^{-1}K \neq KM^{-1}D$), the damping ratio matrix, denoted by Z, is defined in terms of the critical damping matrix D_{cr} of Section 3.6. The damping ratio matrix is defined by

$$Z = D_{CR}^{-1/2} \breve{D} D_{CR}^{-1/2} \quad (7.22)$$

where $\breve{D}$ is the mass normalized damping matrix of the structure. Furthermore, define the matrix $Z^{!}$ to be the diagonal matrix of eigenvalues of matrix Z, i.e.,

$$Z^{!} = \mathrm{diag}[\lambda_i(Z)] = \mathrm{diag}[\zeta_i^*] \quad (7.23)$$

Here, the ζ_i^* are damping ratios in that, if $0< \zeta_i^* <1$, the system is underdamped. Note, of course, that, if $DM^{-1}K = KM^{-1}D$, then $Z = Z^{!}$.

On further consideration, it is also apparent that one can bring about a reflection by incorporating an element with a differing inertia from that of the medium. Since elements of that type are often idealized as rigid masses, they are referred to as *blocking masses*. Practical realizations of the concept of blocking masses are, for example, *seismic blocks* and *added masses* at compliant points; as shown in Fig. 5.

Considering, however, that the most common construction materials are relatively stiff, such as steel and concrete, it is often simpler to accomplish significant discontinuities in the medium properties by the compliant-element approach. For that reason, it is much more common to use compliant than stiff elements. Nevertheless, for structural reasons, there are some cases in which it is necessary to use stiff elements.

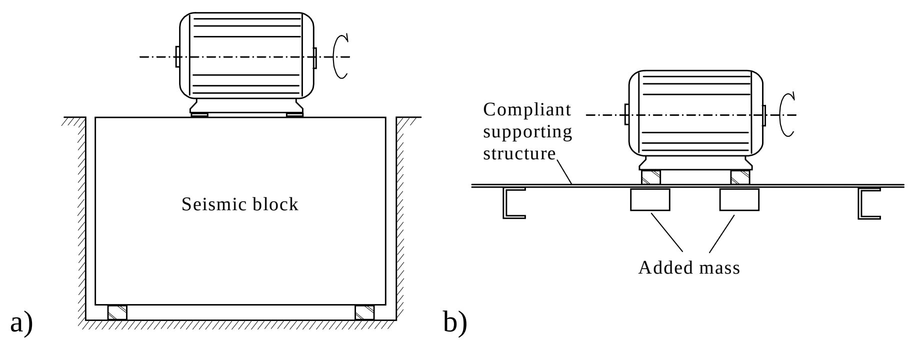


Figure 7.13 Blocking masses, preferably in combination with elastic elements, give very good vibration isolation and are frequently used in practice. a) Seismic block. b) Added mass at a compliant point. [1]

For a long time, machinery designers have mainly provided for vibration isolation using “trial and error” methods in combination with rough estimates obtained from very simple calculations. That approach tends to yield good results at lower frequencies, up to about 100 Hz, say. On the other hand, in the lion’s share of the audible frequency range, that approach provides little or no control over the actual isolation results obtained. In order to be able to achieve good vibration isolation by design, throughout the entire relevant frequency range, access to more advanced theoretical, as well as experimental, techniques is a necessity.

Design of vibration isolators

There are a number of rules of thumb that should be followed when designing vibration isolators. If these are adhered to, the results should be acceptable.

> (*i*) The isolator's (static) stiffness must be chosen so low that the highest *mounting resonance* falls far below the lowest interesting excitation frequency.
>
> (*ii*) The mounting positions on the foundation should be as stiff as possible.
>
> (*iii*) The points at which the machine is coupled to the isolators should also be as stiff as possible.

Rules (*ii*) and (*iii*) are normally not difficult to fulfill at low frequencies; at high frequencies, however, internal resonances make them problematic.

> (*iv*) The isolator should, if possible, be designed so that its first internal anti-resonance falls well above the highest excitation frequency of interest.

That rule is, in practice, very difficult to follow. If it cannot be followed, then one should ascertain by measurements or computations that at least the following alternative rules are fulfilled:

> (*v*) The isolator must be designed so that its internal resonances do not coincide with strong components of the excitation spectrum.
>
> (*vi*) The isolator must, furthermore, be designed so that its antiresonance frequencies do not coincide with the resonance frequencies of the foundation.

In addition to these rules, there are normally also a number of constraints related to geometric, strength, and stability concerns.

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