DESIGN SPECIFICATIONS

The actual design of a mechanism starts and ends with a list of performance objectives or criteria. These qualitative criteria are eventually stated in terms of quantitative design specifications. Sample specifications form the topic of this section. Three performance criteria are considered in this section: speed of response, relative stability, and resonance. The speed of response addresses the length of time required before steady state is reached.

In classical control this is measured in terms of rise time, settling time, and bandwidth. In vibration analysis, speed of response is measured in terms of a decay rate or logarithmic decrement. Speed of response essentially indicates the length of time for which a structure or machine experiences transient vibrations. Hence, it is the time elapsed before the steady state response dominates. If just a single output is of concern, then the definitions of these quantities for multiple-degree-of-freedom systems are similar to those for the single-degree-of-freedom systems. For instance, for an n-degree-of-freedom system with position vector

$$q = [q_1(t)\ q_2(t) \ldots \ldots \ldots \ldots \ldots .\, q_n(t)]^T \qquad (6.23)$$

If one force is applied, say at position m1, and one displacement is of concern, say $q_1(t)$, then specifications for the speed of response of $q_1(t)$ can be defined as follows. The settling time is the time required for the response $q_1(t)$ to remain within ±α percent of the steady state value of $q_1(t)$. Here, α is usually 2, 3, or 5. The rise time is the time required for the response $q_n(t)$ to go from 10 to 90% of its steady state value. The log decrement discussed in Equation can be used as a measure of the decay rate of the system. All these specifications pertain to the transient response of a single-input, single-output (SISO) configuration.

On the other hand, if interest is in the total response of the system, i.e., the vector q, then the response bounds yield a method of quantifying the decay rate for the system. In particular, the constant β, called a decay rate, may be specified such that

$$\|q(t)\| < Me^{-\beta t} \qquad (6.24)$$

is satisfied for all t>0. This can also be specified in terms of the time constant defined by the time, t, required for βt =1. Thus, the time constant is t =1/ β.

Example 3

Consider the system. The response norm of the position is the first component of the vector x(t) so that $q(t) = (1 - e^{-t})e^{-t}$ and its norm is $|(e^{-t} - e^{-2t})| < |e^{-t}| = e^{-t}$. Hence β =1, and the decay rate is also 1.

Some situations may demand that the relative stability of a system be quantified. In particular, requiring that a system be designed to be stable or asymptotically stable may not be enough. This is especially true if some of the parameters in the system may change over a period of time or change owing to manufacturing tolerances or if the system is under active control. Often the concept of a stability margin is used to quantify relative stability.

In this several systems are illustrated that can become unstable as one or more parameters in the system change. For systems in which a single parameter can be used to characterize the stability behavior of the system, the stability margin, denoted by sm, of the system can be defined as the ratio of the maximum stable value of the parameter to the actual value for a given design onfiguration. The following example illustrates this concept.

Example 4

Consider the system defined with γ = 1, c1 = 6, and c2 = 2 and calculate the stability margin of the system as the parameter changes. Here, η is being considered as a design parameter. As the design parameter η increases, the system approaches an unstable state. Suppose the operating value of η, denoted by ηop, is 0.1. Then, the stiffness matrix becomes semi-definite for η =1 and indefinite for $\eta > 1$, and the maximum stable value of η is $\eta_{max}=1$. Hence, the stability margin is

$$sm = \frac{\eta_{max}}{\eta_{op}} = \frac{1}{0.1} = 10$$

If the design of the structure is such that $\eta_{op} = 0.5$, then sm = 2. Thus, all other factors being equal, the design with $\eta_{op} = 0.1$ is 'more stable' than the same design with $\eta_{op} = 0.5$, because $\eta_{op} = 0.1$ has a larger stability margin.

The resonance properties, or modal properties, of a system are obvious design criteria in the sense that in most circumstances resonance is to be avoided. The natural frequencies, mode shapes, and modal damping ratios are often specified in design work. Methods of designing a system to have particular modal properties have been discussed briefly in this chapter in terms of passive and active control.

(d) MODEL REDUCTION

A difficulty with many design and control methods is that they work best for systems with a small number of degrees of freedom. Unfortunately, many interesting problems have a large number of degrees of freedom. One approach to this dilemma is to reduce the size of the original model by essentially removing those parts of the model that affect its dynamic response of interest the least. This process is called model reduction, or reduced-order modeling.

Quite often the mass matrix of a system may be singular or nearly singular owing to some elements being much smaller than others. In fact, in the case of finite element modeling the mass matrix may contain zeros along a portion of the diagonal (called an inconsistent mass matrix). Coordinates associated with zero, or relatively small mass, are likely candidates for being removed from the model. Another set of coordinates that are likely choices for removal from the model are those that do not respond when the structure is excited. Stated another way, some coordinates may have more significant responses than others. Consider the undamped forced vibration the

Equation **x(**t)=U**z(**t) and partition the mass and stiffness matrices according to significant displacements, denoted by $\mathbf{q}_1$, and insignificant displacements, denoted by $\mathbf{q}_2$. This yields

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Note that the coordinates have been rearranged so that those having the least significant displacements associated with them appear last in the partitioned displacement vector $q^T = [q_1^T \quad q_2^T]$

Next consider the potential energy of the system defined by the scalar

$$V_e = \frac{q^T K q}{2}$$

or, in partitioned form,

$$V_e = \frac{1}{2} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}^T \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Likewise, the kinetic energy of the system can be written as the scalar

$$T_e = \frac{1}{2} \dot{q}^T M \dot{q}$$

which becomes

$$T_e = \frac{1}{2} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^T \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

in partitioned form. Since each coordinate **q**i is acted upon by a force **f**i, the condition that there is no force in the direction of the insignificant coordinates, $\mathbf{q}_2$, requires that $\mathbf{f}_2 = \mathbf{0}$ and that $\frac{\partial V_e}{\partial q_2} = 0$ This yields

$$\frac{\partial}{\partial q_2} (q_1^T K_{11} q_1 + q_1^T K_{12} q_2 + q_2^T K_{21} q_1 + q_2^T K_{22} q_2) = 0$$

Solving the Equation yields a constraint relation between $\mathbf{q}_1$ and $\mathbf{q}_2$ which (since $K_{12} = K_{21}^T$ is as follows:

$$q_2 = -K_{22}^{-1} K_{21} q_1$$

This last expression suggests a coordinate transformation (which is not a similarity transformation) from the full coordinate system **q** to the reduced coordinate system $\mathbf{q}_1$. If the transformation matrix P is defined by

$$P = \begin{bmatrix} I & 0 \\ -K_{22}^{-1} & K_{21} \end{bmatrix}$$

then, if $\mathbf{q} = P\mathbf{q}_1$ is substituted into Equation (6.32) and this expression is premultiplied by P^T , a new reduced-order system of the form

$$P^T MP\ddot{q}_1 + P^T KPq_1 = P^T f_1$$

$$P^T MP = M_{11} - K_{21}^T K_{22}^{-1} M_{21} - M_{12} K_{22}^{-1} K_{21} + K_{21}^T K_{22}^{-1} M_{12} K_{22}^{-1} K_{21}$$

$$P^T KP = K_{11} - K_{12} K_{22}^{-1} K_{21}$$

These last expressions are commonly used to reduce the order of vibration problems in a consistent manner in the case where some of the coordinates (represented by **q**2) are thought to be inactive in the system response. This can greatly simplify design and analysis problems in some cases.

If some of the masses in the system are negligible or zero, then the preceding formulae can be used to reduce the order of the vibration problem by setting $M_{22} = 0$ in Equation .This is essentially the method referred to as mass condensation.

Example 5

Consider a four-degree-of-freedom system with the mass matrix

$$M = \frac{1}{420}\begin{bmatrix} 312 & 54 & 0 & -13 \\ 54 & 156 & 12 & -22 \\ 0 & 13 & 8 & -3 \\ -13 & -22 & -3 & 4 \end{bmatrix}$$

and the stiffness matrix

$$K = \begin{bmatrix} 24 & -12 & 0 & 6 \\ -12 & 12 & -6 & -6 \\ 0 & -6 & 2 & 4 \\ 6 & -6 & 4 & 4 \end{bmatrix}$$

Remove the effect of the last two coordinates. The submatrices of Equation are easily identified:

$$M_{11} = \frac{1}{420}\begin{bmatrix} 312 & 54 \\ 54 & 156 \end{bmatrix} \qquad M_{12} = \frac{1}{420}\begin{bmatrix} 0 & -13 \\ 13 & -22 \end{bmatrix} = M_{21}^T$$

$$M_{22} = \frac{1}{420}\begin{bmatrix} 8 & -3 \\ -3 & 4 \end{bmatrix} \qquad K_{22} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} \qquad K_{11} = \begin{bmatrix} 24 & -12 \\ -12 & 12 \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} 0 & 6 \\ -6 & -6 \end{bmatrix} = K_{21}^T \qquad P^{\mathrm{T}}MP = \begin{bmatrix} 1.021 & 0.198 \\ 0.198 & 0.236 \end{bmatrix}$$

$$P^{\mathrm{T}}KP = \begin{bmatrix} 9 & 3 \\ 3 & 3 \end{bmatrix}$$

These last two matrices form the resulting reduced-order model of the structure. It is interesting to compare the eigenvalues (frequencies squared) of the full-order system with those of the reduced-order system, remembering that the transformation P used to perform the reduction is not a similarity transformation and subsequently does not preserve eigenvalues. The eigenvalues of the reduced system and full-order systems are

λ_1^{rom}= 6.981, λ_2^{rom}=12.916

λ_1^{rom}=6.965, λ_2^{rom}=12.916

λ_4^{rom}=3.833E3, λ_3^{rom}=230.934

where the superscript 'rom' refers to the eigenvalues of the reduced-order model. Note that in this case the reduced-order model captures the nature of the first two eigenvalues very well. This is not always the case because the matrix P defined in Guyan reduction, unlike the matrix P from modal analysis, does not preserve the system eigenvalues.

Source:
http://nptel.ac.in/courses/112107088/20