Design Consideration in Material Selection: Design Sensitivity

INTRODUCTION:

Large damping in a structural material may be either desirable or undesirable, depending on the engineering application at hand. For example, damping is a desirable property to the designer concerned with limiting the peak stresses and extending the fatigue life of structural elements and machine parts subjected to near-resonant cyclic forces or to suddenly applied forces. It is a desirable property if noise reduction is of importance. On the other hand, damping is undesirable if internal heating is to be avoided. It also can be a source of dynamic instability of rotating shafts and of error in sensitive instruments. Resonant vibrations of large amplitude are encountered in a variety of modern devices, frequently causing rough and noisy operation and, in extreme cases, leading to seriously high repeated stresses. Various types of damping may be employed to minimize these resonant vibration amplitudes.

Design sensitivity analysis usually refers to the study of the effect of parameter changes on the result of an optimization procedure or an eigenvalue–eigenvector computation. In particular, if a design change causes a system parameter to change, the eigen solution can be computed without having to recalculate the entire eigenvalue / eigenvector set. This is also referred to as a reanalysis procedure and sometimes falls under the heading of structural modification. This section develops the equations for discussing the sensitivity of natural frequencies and mode shapes for conservative systems. The motivation for studying such methods comes from examining the large-order dynamical systems often used in current vibration technology. Making changes in large systems is part of the design process. However, large amounts of computer time are required to find the solution of the redesigned system. It makes sense, then, to develop efficient methods to update existing solutions when small design changes are made in order to avoid a complete reanalysis. In addition, this approach can provide insight into the design process.

Two general types of units are used to specify the damping properties of structural materials: (1) the energy dissipated per cycle in a structural element or test specimen and (2) the ratio of this energy to a reference strain energy or elastic energy.

Absolute damping energy units are:

D_T = total damping energy dissipated by entire specimen or structural element per cycle of vibration, N.m/cycle

D_{Avg} = average damping energy, determined by dividing total damping energy D_T by volume V_0 of specimen or structural element which is dissipating energy, $N.m/m^3$/cycle

D = specific damping energy, work dissipated per unit volume and per cycle at a point in the specimen, $N.m/m^3$/cycle

Of these absolute damping energy units, the total energy D_T usually is of greatest interest to the engineer. The average damping energy D_{Avg} depends upon the shape of the specimen or structural element and upon the nature of the stress distribution in it, even though the specimens are made of the same material and have been subjected to the same stress distribution at the same temperature and frequency. Thus, quoted values of the average damping energy in the technical literature should be viewed with some reserve. The specific damping energy D is the most fundamental of the three absolute units of damping since it depends only on the material in question and not on the shape, stress distribution, or volume of the vibrating element. However, most of the methods discussed previously for measuring damping properties yield data on total damping energy D_T rather than on specific damping energy D. Therefore, the development of the relationships between these quantities assumes importance. If the specific damping energy is integrated throughout the stressed volume,

$$D_T = \int_0^{V_0} D\,dV \qquad (6.11)$$

This is a triple integral; dV = dx dy dz and D is regarded as a function of the space coordinates x, y, z. If there is only one nonzero stress component, the specific damping energy D may be considered a function of the stress level σ. Then

$$D_T = \int_0^{\sigma_d} D \frac{dV}{d\sigma} d\sigma \qquad (6.12)$$

In this integration, V is the volume of material whose stress level is less than σ. The integration is a single integral, and σ_d is the peak stress. The integrands may be put in dimensionless form by introducing D_d, the specific damping energy associated with the peak stress level reached anywhere in the specimen during the vibration (i.e., the value of D corresponding to $\sigma = \sigma_d$). Then

$$D_T = D_d V_0 \alpha \qquad (6.13)$$

$$\alpha = \int_0^1 \left(\frac{D}{D_d}\right) \frac{d\left(\frac{V}{V_0}\right)}{d\left(\frac{\sigma}{\sigma_d}\right)} d\left(\frac{\sigma}{\sigma_d}\right) \qquad (6.14)$$

The average damping energy is

$$D_a = \frac{D_0}{V_0} = D_d \alpha \qquad (6.15)$$

The relationship between the damping energies D_T, D_{Avg}, and D depends upon the dimensionless damping energy integral α.The integrand of α may be separated into two parts: (1) a damping function D/D_d which is a property of the material and (2) a volume-stress function $d\left(\frac{V}{V_0}\right)/d\left(\frac{\sigma}{\sigma_d}\right)$ which depends on the shape of the part and the stress distribution.

Several approaches are available for performing a sensitivity analysis. The one presented here is based on parameterizing the eigenvalue problem. Consider a conservative n-degree-of-freedom system defined by

$$M(\alpha)\ddot{q}(t) + K(\alpha)q(t) = 0 \qquad (6.16)$$

where the dependence of the coefficient matrices on the design parameter α is indicated. The parameter α is considered to represent a change in the matrix M and/or the matrix K. The related eigenvalue problem is

$$M^{-1}(\alpha)K(\alpha)u_i(\alpha) = \lambda_i(\alpha)u_i(\alpha) \qquad (6.17)$$

Here, the eigenvalue $\lambda_i(\alpha)$ and the eigenvector $u_i(\alpha)$ will also depend on the parameter α.

The mathematical dependence is discussed in detail by Whitesell (1980). It is assumed that the dependence is such that M, K, $\lambda_i(\alpha)$ and $u_i(\alpha)$ are all twice differentiable with respect to the parameter α .

Proceeding, if $u_i(\alpha)$ is normalized with respect to the mass matrix, differentiation of Equation with respect to the parameter α yields

$$\underset{\alpha}{—}(\lambda_i) = u_i^T\left[\frac{d}{d\alpha}(K) - \lambda_i\frac{d}{d\alpha}(M)\right]u_i \qquad (6.18)$$

Here, the dependence of α has been suppressed for notational convenience. The second derivative of λ_i can also be calculated as

$$\frac{d^2}{d\alpha^2}(\lambda_i) = 2u_i'^T\left[\frac{d}{d\alpha}(K) - \lambda_i\frac{d}{d\alpha}(M)\right]u_i' + u_i^T\left[\frac{d^2}{d\alpha^2}(K) - \frac{d}{d\alpha}(\lambda_i)\frac{d}{d\alpha}(M) - \lambda_i\frac{d^2}{d\alpha^2}(M)\right]u_i \qquad (6.19)$$

The notation u' denotes the derivative of the eigenvector with respect to u. The expression for the second derivative of λ_i requires the existence and computation of the derivative of the corresponding eigenvector. For the special case where M is a constant, and with some manipulation (Whitesell, 1980), the eigenvector derivative can be calculated from the related problem for the eigenvector v_i from the formula

$$\underset{\alpha}{—}(V_i) = \sum_{k=1}^{n} C_k(i,\alpha)V_k \qquad (6.20)$$

where the vectors V_k are related to u_k by the mass transformation $V_k = M_1/2u_k$

The coefficients $C_k(i,\alpha)$ in this expansion are given by

$$C_k(i,\alpha) = \begin{cases} 0 & i=k \\ \vdots & \\ \frac{1}{\lambda_i - \lambda_k} u_k^T \frac{dA}{d\alpha} u_i & i \neq k \end{cases} \qquad (6.21)$$

Where the matrix A is the symmetric matrix $M^{-1/2}KM^{-1/2}$ depending on α. Above equations yield the sensitivity of the eigenvalues and eigenvectors of a conservative system to changes in the stiffness matrix. More general and computationally efficient methods for computing these sensitivities are available in the literature. Adhikari and Friswell (2001) give formulae for damped systems and reference to additional methods.

Another method for design of damping is taken as a laminated metal material, which offers an effective method to increase the inherent level of damping in sheet-metal components. To assist the product designer considering the use of laminated metal material in place of traditional sheet metal, various practical modeling techniques are available that can be used both as a damping prediction and design optimization tool. Optimization of the laminate construction, as with all constrained layer type treatments, is a function of other parameters in addition to the actual properties of the viscoelastic material. This complexity offers more design flexibility as the thickness and type of the damping core as well as the constraining layers can be altered to optimize effectiveness of the laminated metal product. Two specific approaches are available to help assist in the selection and design of viscoelastic-based damping treatments.

Simplified RKU approach:

One approach is to simplify a real world component down to an equivalent 3-layer beam or plate system. This was first suggested by Ross, Kerwin, Ungar, and the RKU method uses a fourth order differential equation for a uniform beam with the sandwich construction of the 3-layer laminate system represented as an equivalent complex stiffness. The equation for the flexural rigidity, EI, of this system has been reported in many technical references, and is therefore not duplicated here. The most common assumption made when using this method is that the mode shapes of the theoretical structure are sinusoidal in nature, therefore implying a simply-supported boundary

condition. When using this approach with other boundary conditions, which may be necessary in working with actual structures, approximations must be made in the results depending on the mode shape in question. The RKU method is better suited as a damping indicator as opposed to a precise damping predictor when applied to complex, real world structures. The goal is to use this simplified method to develop design trends that will lead to the selection of a damping material, constraining layers, and thickness which yield optimized damping performance.

Modal Strain Energy: Another prediction method known as the Modal Strain Energy (MSE) approach utilizes a finite element analysis (FEA) representation of a structure as the basis for of modeling the damping effect. This method has been shown to be an accurate predictor of damping levels in structures comprising layers of elastic and viscoelastic elements. The MSE principle states that the ratio of composite system loss factor to the viscoelastic material loss factor for a given mode of vibration can be estimated from the ratio of elastic strain energy in the viscoelastic elements to the total strain energy in the model for a given mode. This is shown mathematically in the following equation: Typically, the MSE approach is used in conjunction with an undamped, normal modes analysis to compute the strain energy ratio. The strain energies are determined from the relative mode shapes. It is assumed that the viscoelastic properties are linear in terms of the dynamic strain rate.

$$\eta_S^m = \eta_{VEM} \frac{U_{VEM}^m}{U_{Total}^m} \qquad (6.22)$$

Where, η_s^m = System damping for nth mode of vibration

η_{VEM} = material damping for appropriate frequency and temperature

η_{VEM}^m = elastic strain energy stored in viscoelastic core

η_{total}^m = total strain energy for n^{th} mode shape

Example 2

Consider the system discussed previously in example 1. Here, take M =I, and K becomes

$$\bar{K} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} = K$$

The eigenvalues of the matrix are $\lambda_{1,2} = 2, 4$ and the normalized eigenvectors are u1= v1=(1/√2)[1 1]T and u2=v2=(1/√2)[−1 1]T. It is desired to compute the sensitivity of the natural frequencies and mode shapes of this system as a result of a parameter change in the stiffness of the spring attached to ground. To this end, suppose the new design results in a new stiffness matrix of

$$K(a) = \begin{bmatrix} 3+\alpha & -1 \\ 1 & 3 \end{bmatrix}$$

Then

$$\frac{d}{d\alpha}(M) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$\frac{d}{d\alpha}(K) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Following Equations (6.25) and (6.27), the derivatives of the eigen values and eigenvectors become

$$\frac{d\lambda_1}{d\alpha} = 0.5, \quad \frac{d\lambda_2}{d\alpha} = 0.5, \quad \frac{du_1}{d\alpha} = \frac{1}{4\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \frac{du_2}{d\alpha} = \frac{1}{4\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

These quantities are an indication of the sensitivity of the eigen solution to changes in the matrix K.

To see this, substitute the preceding expressions into the expansions for $\lambda_i(\alpha)$ and $u_i(\alpha)$ are,

$$\lambda_1(\alpha) = 2 + 0.5\alpha$$

$$\lambda_2(\alpha) = 4 + 0.5\alpha$$

$$u_1(\alpha) = 0.707\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.177\alpha\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$u_2(\alpha) = 0.707\begin{bmatrix} -1 \\ 1 \end{bmatrix} - 0.177\alpha\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

This last set of expressions allows the eigenvalues and eigenvectors to be evaluated for any given parameter change α without having to resolve the eigenvalue problem. These formulae constitute an approximate reanalysis of the system.

It is interesting to note this sensitivity in terms of a percentage. Define the percentage change in λ_1

$$\frac{\lambda_1(\alpha) - \lambda_1}{\lambda_1} 100\% = \frac{(2 + 0.5\alpha) - 2}{2} 100\% = (25\%)\alpha$$

If the change in the system is small, say $\alpha = 01$, then the eigenvalue λ_1 changes by only 2.5%, and the eigenvalue λ_2 changes by 1.25%. On the other hand, the change in the elements of the eigenvector u_2 is 2.5%. Hence, in this case the eigenvector is more sensitive to parameter changes than the eigenvalue is.

By computing higher-order derivatives of λ_i and u_i, more terms of the expansion can be used, and greater accuracy in predicting the eigensolution of the new system results. By using the appropriate matrix computations, the subsequent evaluations of the eigenvalues and eigenvectors as the design is modified can be carried out with substantially less computational effort (reportedly of the order of n2 multiplications). The sort of calculation provided by eigenvalue and eigenvector derivatives can provide an indication of how changes to an initial design will affect the response of the system. In the example, the shift in value of the first spring is translated into a percentage change in the eigenvalues and hence in the natural frequencies. If the design of the system is concerned with avoiding resonance, then knowing how the frequencies shift with stiffness is critical.

Source:
http://nptel.ac.in/courses/112107088/19