Vibration Measurement Techniques: Data Acquisition

Introduction:

There is a considerable variety of extent and methods used to acquire and log vibration and other diagnostic monitored machinery parameters. The methods and corresponding products available to accomplish data acquisition tasks constitute a constantly changing field that parallels the rapid and perpetual advancements in PCs and workstations. Data acquisition systems are used to acquire, store and analyze vibration data received from sensors. Sensing Systems utilizes state of the art computer based data acquisition systems. The equipment is optimized to sample at rates commensurate with the highest frequency expected during testing. Data may be acquired and submitted in different formats for further review and analysis by our customers. Filtering may be performed during acquisition or digitally following data acquisition. Data analysis such as Fourier Transforms may be performed following data acquisition. In general, vibration sensor serves to detect mechanical vibrations and convert it into electrical voltage proportionally. And then conditioned to fit the requirements requested by the data acquisition system, so that the voltage may be amplified, filtered, or others. The vibration sensor module can be divided into two parts, an accelerometer sensor circuit and signal conditioning circuit.

The first item in any noise or vibration measuring system is the transducer. As its name implies, this instrument converts a signal in one physical form into another, that is, a transducer converts a sound pressure signal or a vibration signal into an electrical signal. Normally, the electrical signal obtained from a transducer is not suitable for direct analysis or readout, and a signal conditioner is then used to amplify, attenuate, or transform the signal using analog-to digital (A/D) conversion. It is optional at this stage to include a data storage item in the system before the signal is passed to a signal processor as shown in Fig. 9.9.

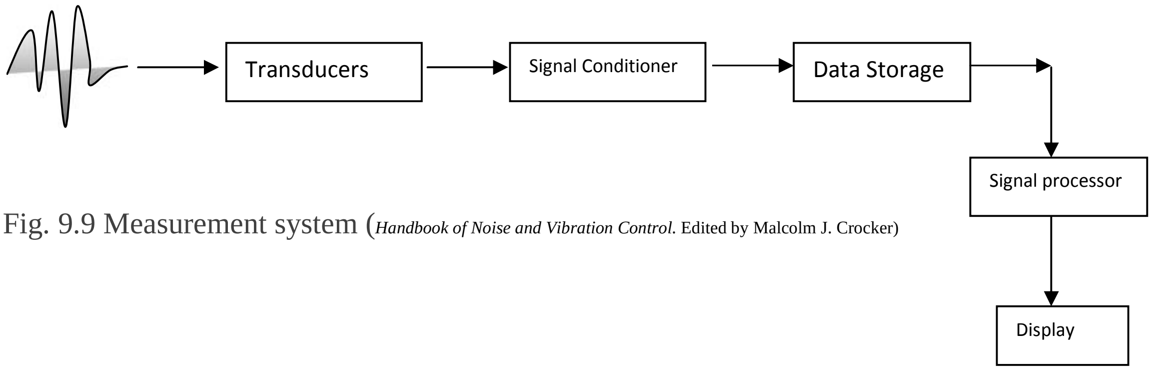


Fig. 9.9 Measurement system (*Handbook of Noise and Vibration Control.* Edited by Malcolm J. Crocker)

Sound and vibration signals produced by transducers are not normally in a suitable form for the study of noise and vibration problems. Frequency analysis is the most common approach used in the solution of such problems. Vibration analysis first begins with acquiring an accurate time-varying signal from an industry standard vibration transducer, such as an accelerometer. The raw analog signal is typically brought into a portable, digital instrument that processes it for a variety of user functions. Depending on user requirements for analysis and the native units of the raw signal, it can either be processed directly or routed to mathematical integrators for conversion to other units of vibration measurement. Depending on the frequency of interest, the signal may be conditioned through a series of high-pass and low-pass filters. Depending on the desired result, the signal may be sampled multiple times and averaged. If time waveform analysis is desired in the digital instrument, it is necessary to decide the number of samples and the sample rate. The time period to be viewed is the sample period times the number of samples. Most portable instruments also incorporate FFT (Fast Fourier Transform) processing as the method for taking the overall time-varying input sample and splitting it into its individual frequency components. Vibration analysis starts with a time-varying, real world signal from a transducer or sensor. From the input of this signal to a vibration measurement instrument, a variety of options are possible to analyze the signal. It is the intent of this paper to focus on the internal signal processing path, and how it relates to the ultimate root-cause analysis of the original vibration problem.

Fourier series and Transformation:

Fourier series decomposes periodic functions or periodic signals into the sum of a (possibly infinite) set of simple oscillating functions, namely sines and cosines (or complex exponentials). The study of Fourier series is a branch of Fourier analysis. Most machines emit periodic disturbances to the surroundings, either in the form of fluctuating forces, acting via the machine mounts, or in the form of sound. The reasons for these periodic disturbances can be, to name a few examples, the meshing of gear teeth, imbalances in rotating shafts, or periodic pressure fluctuations that arise in the cylinders of internal combustion engines due to the intake-exhaust cycles. To analyze the problem in the frequency domain, a method is needed to divide up a measured signal into its harmonic components, so that they can be individually analyzed. For periodic signals, it is possible to use a Fourier series expansion.

A Fourier series is an expansion of a periodic function $f(x)$ in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical.

a. Approximation of signals

As a first step in deriving a method to decompose a periodic signal into its harmonic components, we will study how to best approximate a signal *a (t)* with a signal *b(t).* We assume that the fitting of the two signals is to take place during the time interval 0 < t < T. To carry out the approximation in the simplest possible way, we multiply *b(t)* by a constant that is varied to adjust the approximation as well as possible,

$$a(t) \approx \quad b(t) \qquad (9.2)$$

The error in the our approximation then becomes

$$e(t) = a(t) - \quad b(t) \qquad (9.3)$$

Next, the averaged squared error is computed over the entire time interval 0 < *t* < *T*,

$$£ = \frac{1}{T}\int_0^T \left(a(t) - \quad b(t)\right)^2 dt \qquad (9.4)$$

We minimize £ with respect to   by differentiating, and setting the resulting derivative equal to zero,

$$\frac{d£}{d} = \frac{1}{T}\int_0^T 2(\quad b^2(t) - a(t)b(t))dt = 0 \qquad (9.5)$$

from which

$$= \int_0^T a(t)b(t)dt \Big/ \int_0^T b^2(t)dt \qquad (9.6)$$

We now introduce the useful concept of *orthogonality*. If the signals *a*(*t*) and *b*(*t*) are orthogonal, there is no connection between the two signals, and   equals zero. The orthogonality condition for the two signals *a*(*t*) and *b*(*t*) on the time interval 0 < *t* < *T* is therefore

$$\int_0^T a(t)b(t)dt = 0 \qquad (9.7)$$

The energy of a signal is proportional to the time integral of the signal squared, thus,

$$E_a = \sim \int_0^T a^2(t)dt \qquad (9.8)$$

in which α is a proportionality constant. Given a signal composed of two parts, *v(t)=a(t)+b(t)*, its energy becomes

$$E_v = \sim \int_0^T \left(a(t)+b(t)\right)^2 dt = \sim \int_0^T a^2(t)dt + \sim \int_0^T b^2(t)dt + \sim \int_0^T 2a(t)b(t)dt \qquad (9.9)$$

If the signals are orthogonal, we get

$$E_v = \sim \int_0^T a^2(t)dt + \sim \int_0^T b^2(t)dt = E_a + E_b \qquad (9.10)$$

Thus, the energy of the combined signal is the sum of the individual energies. If the signals are not orthogonal, previous equation must be used.

Possibly, the approximation of the signal *a*(*t*) given by above equation in combination with first equation gives an inadequate fit. To further reduce the error, we incorporate another signal *c(t)* with a proportionality constant ¥ , and write the approximation as

$$a(t) \approx \quad b(t) + \yen c(t) \qquad (9.11)$$

The error in that case becomes

$$e(t) = a(t) - \quad b(t) - \yen\, c(t) \qquad (9.12)$$

and the mean squared error is

$$\pounds = \frac{1}{T}\int_0^T \left(a(t) - \quad b(t) - \yen\, c(t)\right)^2 dt \qquad (9.13)$$

First minimize £ with respect to ,

$$\frac{\partial £}{\partial} = \frac{1}{T}\int_0^T 2(\quad b^2(t) + ¥\, b(t)c(t) - a(t)b(t))dt = 0 \qquad (9.14)$$

If we can now choose *b(t)* and *c(t)* to be mutually orthogonal, the second term in above equation is eliminated, and gives

$$= \int_0^T a(t)b(t)dt \Big/ \int_0^T b^2(t)dt \qquad (9.15)$$

i.e., the same expression as in equation 5. If we minimize the error with respect to ¥ we obtain

$$\frac{\partial £}{\partial ¥} = \frac{1}{T}\int_0^T 2(¥c^2(t) + \quad b(t)c(t) - a(t)c(t))dt = 0 \qquad (9.16)$$

If *b(t)* and *c(t)* are orthogonal, we can solve for ¥ in the same way as before,

$$¥ = \int_0^T a(t)c(t)dt \Big/ \int_0^T c^2(t)dt \qquad (9.17)$$

By comparison to equations, the result obtained is the same as what would have followed from the approximation $a(t) = ¥ \quad c(t)$. To improve upon that, we therefore only need to add another orthogonal signal and minimize the mean squared error versus $a(t)$, independently of the other signals included in the approximation.

Examples of functions that are orthogonal are $\sin(n\mu_0 t)$ and $\cos(n\mu_0 t)$. They are orthogonal for all integer values of *n* over the time interval $T = 2\pi / \mu_0$. In the next part, the advantage of that orthogonality property to decompose a periodic signal into sine and cosine components can be taken. That gives the desired decomposition of the signal into its frequency components, given by $f_n = n / T$.

Fourier series decomposition

Assume that a signal *a*(*t*) that is periodic, with period *T*, and which we wish to approximate with the help of sine and cosine functions,

$$a(t) = {}_0 + \sum_{n=1}^{\infty} {}_n \cos(n\mu_o t) + \sum_{n=1}^{\infty} \text{¥}_n \sin(n\mu_o t) \qquad (9.18)$$

That is called a *Fourier series decomposition* of the signal *a*(*t*). The coefficients ${}_n$ and ¥_n can be calculated separately, and given by equation,

$${}_0 = \int_{-T/2}^{T/2} a(t) 1 dt \Bigg/ \int_{-T/2}^{T/2} 1^2 \, dt = \frac{1}{T} \int_{-T/2}^{T/2} a(t) dt$$

(9.19)

$${}_n = \int_{-T/2}^{T/2} a(t) \cos(n\mu_0 t) dt \Bigg/ \int_{-T/2}^{T/2} \cos^2(n\mu_0 t) dt = \frac{2}{T} \int_{-T/2}^{T/2} a(t) \cos(n\mu_0 t) dt, \quad n = 1,2,3,\ldots$$

$$\text{¥}_n = \int_{-T/2}^{T/2} a(t) \sin(n\mu_0 t) dt \Bigg/ \int_{-T/2}^{T/2} \sin^2(n\mu_0 t) dt = \frac{2}{T} \int_{-T/2}^{T/2} a(t) \sin(n\mu_0 t) dt, \quad n = 1,2,3,\ldots$$

The interval of integration in all above equations is -*T/2* to *T/2*, but could just as well have been 0 to *T*. The coefficient ${}_0$ represents the signal's time average. It can also be shown that corresponding sine and cosine terms can be combined into a single cosine term with a phase angle ¨_n.

$$a(t) = \quad {}_0 + \sum_{n=1}^{\infty} ¢_n \cos(n µ_0 t - ¨_n) \qquad (9.22)$$

where $¢_n = \sqrt{\ {}^2_n + ¥^2_n}$ $\qquad ¨_n = \arctan(¥_n / \ _n) + m®$, $\quad m = 0, 1, 2, \ldots,$

where $¢_1$ gives the signal's amplitude for the first tone, or fundamental tone,

$¢_2$ gives the signal's amplitude for the second tone, or the first overtone,

$¢_3$ gives the signal's amplitude for the third tone, or second overtone.

In order to be able to denote each frequency component as a complex, rotating vector, which, results in simpler computations and a more compact symbolic expression, a complex Fourier series can be defined,

$$a(t) = \sum_{n=-\infty}^{\infty} ¢_n e^{in µ_0 t} \qquad (9.23)$$

where the complex coefficient $¢_n$ can be determined from

$$¢_n = \int_{-T/2}^{T/2} a(t) e^{-in µ_0 t}\, dt \Bigg/ \int_{-T/2}^{T/2} 1^2\, dt = \frac{1}{T} \int_{-T/2}^{T/2} a(t) e^{-in µ_0 t}\, dt \qquad (9.24)$$

In equation, there are components with "negative frequencies". That is because of a real quantity can be described with the aid of two oppositely rotating complex vectors, each of which is the complex conjugate of the other.

A Fourier series converges to the function $\bar{f}$ (equal to the original function at points of continuity or to the average of the two limits at points of discontinuity)

$$\bar{f} \equiv \begin{cases} \frac{1}{2}\left[\lim_{x \to x_0^-} f(x) + \lim_{x \to x_0^+} f(x)\right] & \text{for } -\pi < x_0 < \pi \\ \frac{1}{2}\left[\lim_{x \to -\pi^+} f(x) + \lim_{x \to \pi_-} f(x)\right] & \text{for } x_0 = -\pi, \pi \end{cases} \qquad (9.25)$$

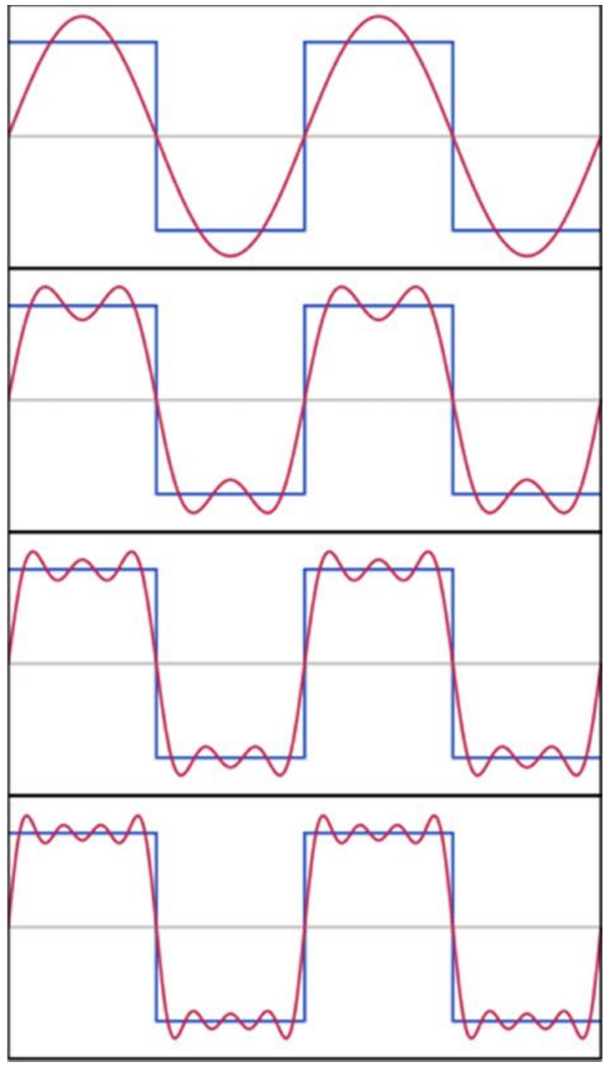
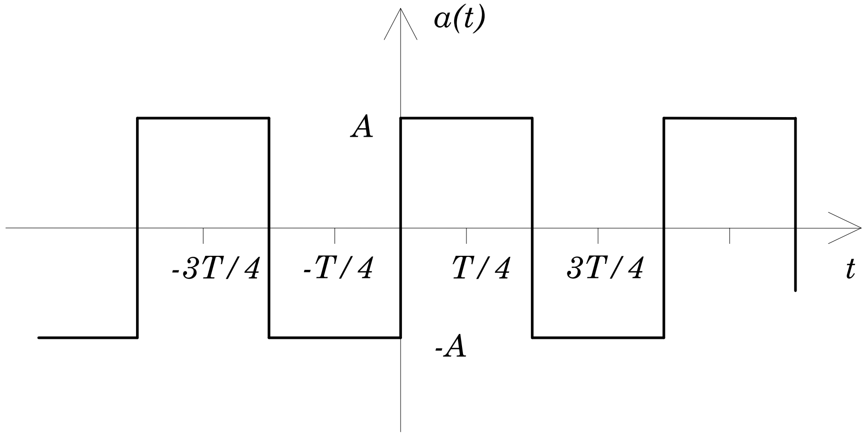


Fig.9.10 First four series approximation for square wave

Example 0-1

Consider a rectangular wave, as in the figure below [1].



From equation

$$_0 = \frac{1}{T}\int_{-T/2}^{0} -A\,dt + \frac{1}{T}\int_{0}^{T/2} A\,dt = -\frac{A}{T}\left[t\right]_{-T/2}^{0} + \frac{A}{T}\left[t\right]_{0}^{T/2} = -\frac{A}{2} + \frac{A}{2} = 0$$

i.e., the time-averaged value is, as expected, equal to 0. Equation yields

$$_n = \frac{2}{T}\int_{-T/2}^{0} -A\cos(n\mu_0 t)\,dt + \frac{2}{T}\int_{0}^{T/2} A\cos(n\mu_0 t)\,dt =$$

$$= -\frac{2A}{T}\left[\frac{\sin(n\mu_0 t)}{n\mu_0}\right]_{-T/2}^{0} + \frac{2A}{T}\left[\frac{\sin(n\mu_0 t)}{n\mu_0}\right]_{0}^{T/2} = -\frac{A}{n\text{®}}\sin(n\text{®}) + \frac{A}{n\text{®}}\sin(n\text{®}) =$$

and equation gives

$$\text{¥}_n = \frac{2}{T}\int_{-T/2}^{0} -A\sin(n\mu_0 t)\,dt + \frac{2}{T}\int_{0}^{T/2} A\sin(n\mu_0 t)\,dt =$$

$$= \frac{2A}{T}\left[\frac{\cos(n\mu_0 t)}{n\mu_0}\right]_{-T/2}^{0} + \frac{2A}{T}\left[-\frac{\cos(n\mu_0 t)}{n\mu_0}\right]_{0}^{T/2} = \frac{2A}{n\text{®}}(1-\cos(n\text{®})) = \begin{cases} \frac{4A}{n\text{®}} & n = 1,3 \\ 0 & n = 0,2 \end{cases}$$

Thus, the Fourier series can be expressed as

$$a(t) = \frac{4A}{\text{®}}\left[\sin(\mu_0 t) + \frac{1}{3}\sin(3\mu_0 t) + \frac{1}{5}\sin(5\mu_0 t) + \ldots\right].$$

It only consists of the odd sine components. That only sine components appear in the Fourier series is because the rectangular wave is an odd function; odd functions can be decomposed into sine components, since the sine function itself is odd. By definition, a function is odd if it has the property that $a(-x) = -a(x)$, and even if it has the property that $a(-x)=a(x)$. If the rectangular wave is shifted $T/4$ to the left, making it symmetric about $t=0$, it becomes an even function instead, and be built up exclusively of cosine functions.

Source:
http://nptel.ac.in/courses/112107088/38