Constant Acceleration: The physics of sailing

Sailing gives examples of physics: Newton's laws, vector subtraction, Archimedes' principle and others. This support page from Physclips asks

- How can a boat sail upwind?
- How can boats sail faster than the wind?
- Why are eighteen foot skiffs always sailing upwind?

We introduce the physics of sailing to answer these and some other questions. But first:

A puzzle.

A river runs straight from West to East at 10 knots. A 10 mile race is held: the boats sail downstream, from West to East. The first heat is held in the morning, when there is no wind. The second heat is held in the afternoon, when there is a 10 knot wind from the West. In which heat are the faster times recorded?

(Answer below.)

Sailing downwind (parallel to the wind, like the boat at left) is easy to understand: the wind blows into the sails and pushes against them. The wind is faster than the boat so the air is decelerated by the sails. The sails push backwards against the wind, so the wind pushes forward on the sails. But for a boat with normal sails, the catch is that, downwind, you can only ever sail more slowly than the
wind, even with a spinnaker. Which is comfortable, but not the most interesting sailing.

You know this force: In a strong wind, it is easier to walk, run or bicycle with the wind pushing on your back. Usually, the wind pushes you in the direction it is going.

**Sailing directly upwind** (exactly anti-parallel to the wind, like the boat at right) is also easy to understand: it's impossible (with sails*). You just sit there with your sails flapping. This is also not interesting sailing.

* A boat with a wind turbine driving a propellar could go directly upwind.

But boats can sail at say 40° to the wind and, by tacking (alternate lines on either side of the wind direction) they can go where they like. So let's think about....

**An experiment.** Here is what my left hand looks like as I bicycle, signalling a left turn. If my hand is flat and horizontal, I just feel the drag force of the wind acting backwards. But if I tilt my hand up a little at the front, I feel lift force as well: the force on my hand is both upwards and backwards. The arrows show the wind speed relative to me. To get past my hand, the wind is deflected down, and this pushes my hand up (as well as back).

In this diagram, the quantities force and velocity have arrows, because they have a magnitude as well as a direction. Try this link for an Introduction to vectors.
Sailing close to the wind uses the shape of the sails to generate lift. To flow around the sails, the wind has to deviate in direction, as shown by the arrows for initial velocity $v_i$ and final velocity $v_f$, which are given with respect to the boat. The change of velocity $dv$ is in the direction shown. The acceleration $a_a$ of the air is $dv/dt$, so the force $F_a$ that sails exert on the air is in the same direction. (Newton's first and second laws: $F = ma$.) The force $F_w$ that the wind exerts on the sails is in the opposite direction. (There is also a Bernoulli effect, which contributes in a secondary way.)

*Note that nowhere in this argument did we need to say that the wind was faster than the boat.*

Now this force is mainly sideways on the boat, and it gets more and more sideways as you get closer to the wind. However, part of the force is forward: the direction we want to go. So...

**Why doesn't the boat drift sideways?** Well it does a little, but when it does, the keel, a large nearly flat area under the boat, has to push a lot of water sideways. The water resists this, and exerts the sideways force $F_k$ on the keel.
This cancels the sideways component of $F_w$. As to the forwards component: it accelerates the boat until the drag force $F_d$ holding it back is big enough so that

$$F_w = -(F_k + F_d).$$

So a boat can sail close to the wind: typically 45°, although many high performance boats go closer than that.

A little digression: the sideways components of wind and water on the boat make the boat heel (tilt) away from the wind, as is shown in the diagram below. These two horizontal components have equal size but opposite direction: as forces they cancel, but they make a torque tending to rotate the boat clockwise. This is cancelled by another pair of forces. The buoyancy and the weight are also equal and opposite, and they make a torque in the opposite direction. As the boat heels to starboard, the lead on the bottom of the keel, which has a substantial fraction of the weight, moves to port and exerts an anticlockwise torque. These two torques cancel.
So now back to our question:

**How can boats sail faster than the wind?** Lots of boats can---especially the eighteen footer skiffs on Sydney Harbour. Ask a sailor how, and he'll say "These boats are so fast that they make their own wind", which is actually true. Ask a physicist, and she'll say that it's just a question of vectors and relative velocities.

Downwind (diagram at left) is easy. If the wind is 10 kt, and the boat makes 6 kt in the same direction, then the crew feels a wind of 4 kt coming over the stern of the boat. The true wind $\mathbf{v}_w$ equals the speed of the boat $\mathbf{v}_b$ plus the relative wind $\mathbf{v}_r$. The equation $\mathbf{v}_w = \mathbf{v}_b + \mathbf{v}_r$ tells us the problem: as the boat speed approaches the wind speed, the relative wind drops towards zero and so there is no force on the sail. So you can't go faster than the wind. When the wind is at an angle, we have to add the arrows representing these velocities (vector addition). Upwind (right), exactly the same equation holds: $\mathbf{v}_w = \mathbf{v}_b + \mathbf{v}_r$.

The faster that the boat goes, the greater the relative wind, the more force there is on the sails, so the greater the force dragging the boat forwards. So the boat accelerates until the drag from the water balances the forward component of the force from the sails.
Why are eighteen footers always sailing upwind?

In a fast boat, there's no point going straight downwind: you can never go faster than the wind. So you travel at an angle. But if your boat is fast enough, then the relative wind always seems to be coming mainly from ahead of you, as these arrows show. So the eighteen footers never set ordinary spinnakers: they have asymmetrical sails that they can set even when they are travelling at small angles to the apparent wind.