Body Forces

Surface forces act on surfaces. As discussed in the previous section, these are the forces which arise when bodies are in contact and which give rise to stress distributions. Surface forces also arise *inside* materials, acting on internal surfaces, Fig. 3.2.1a, as will be discussed in the following section.

To complete the description of forces acting on real materials, one needs to deal with forces which arise even when bodies are not in contact; one can think of these forces as *acting at a distance*, for example the force of gravity. To describe these forces, one can define the **body force**, which acts on volume elements of material. Fig. 3.2.1b shows a sketch of a volume element subjected to a magnetic body force and a gravitational body force $\mathbf{F}_g$.

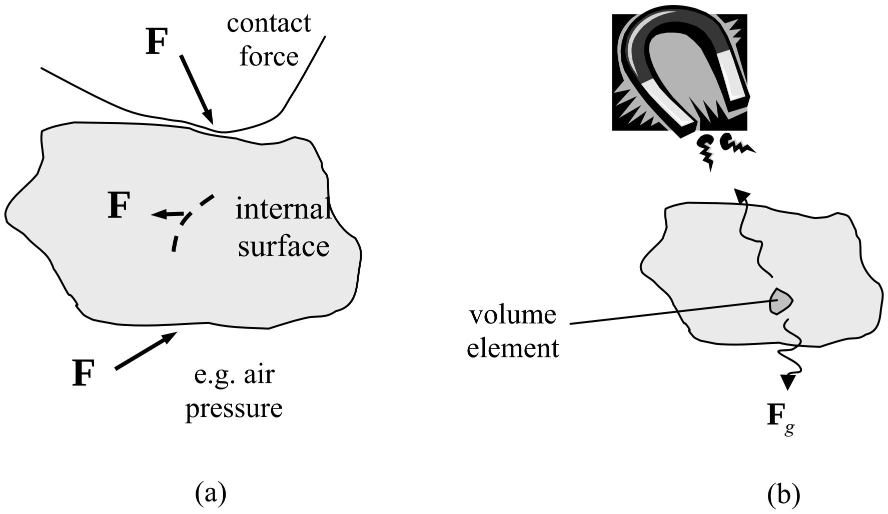

Figure 3.2.1: forces acting on a body; (a) surface forces acting on surfaces, (b) body forces acting on a material volume element

3.2.1 Weight

The most important body force is the force due to gravity, i.e. the weight force. In Chapter 2 there were examples involving the weight of components. In those cases it was simply stated that the weight could be taken to be a single force acting at the component centre (for example, Problem 3 in §2.2.3). This is true when the component is symmetrical, for example, in the shape of a circle or a square. However, it is not true in general for a component of arbitrary shape.

In what follows, the important case of a flat object of arbitrary shape will be examined.

The weight of a small volume element ΔV of material of density ρ is $dF = \rho g \Delta V$ and the total weight is

$$F = \int_V \rho g dV \qquad (3.2.1)$$

Consider the general two-dimensional case, Fig. 3.2.2, where material elements of area ΔA_i (and constant thickness *t*) are subjected to forces $\Delta F_i = t\rho g \Delta A_i$.

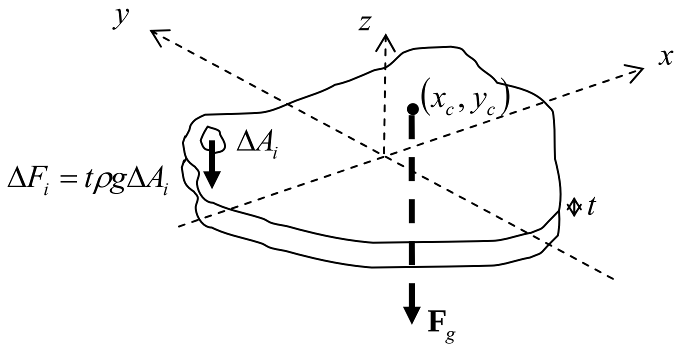

Figure 3.2.2: Resultant Weight on a body

The resultant, i.e. equivalent, weight force due to all elements, for a component with uniform density, is

$$F = \int dF = t\rho g \int dA = \rho g t A,$$

where *A* is the cross-sectional area.

The resultant moments about the *x* and *y* axes, which can be positioned anywhere in the body, are $M_x = t\rho g \int y dA$ and $M_y = t\rho g \int x dA$ respectively; the moment ΔM_x is shown in Fig. 3.2.3. The equivalent weight force is thus positioned at (x_c, y_c), Fig. 3.2.2, where

$$\boxed{x_c = \frac{\int x dA}{A}, \quad y_c = \frac{\int y dA}{A}} \qquad \textbf{Centroid of Area} \qquad (3.2.2)$$

The position (x_c, y_c) is called the **centroid of the area**. The quantities $\int x dA$, $\int y dA$, are called the **first moments of area** about, respectively, the *y* and *x* axes.

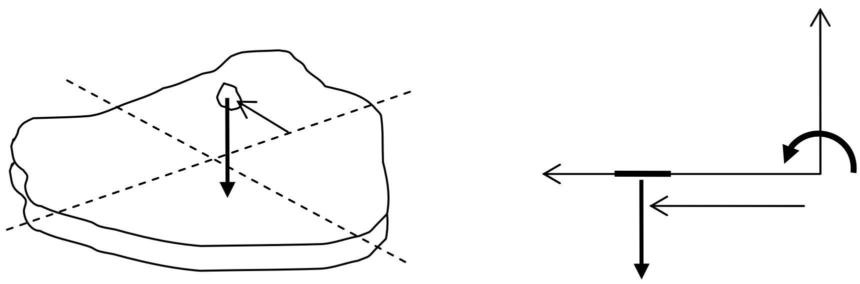

Figure 3.2.3: The moment M_x; (a) full view, (b) plane view

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