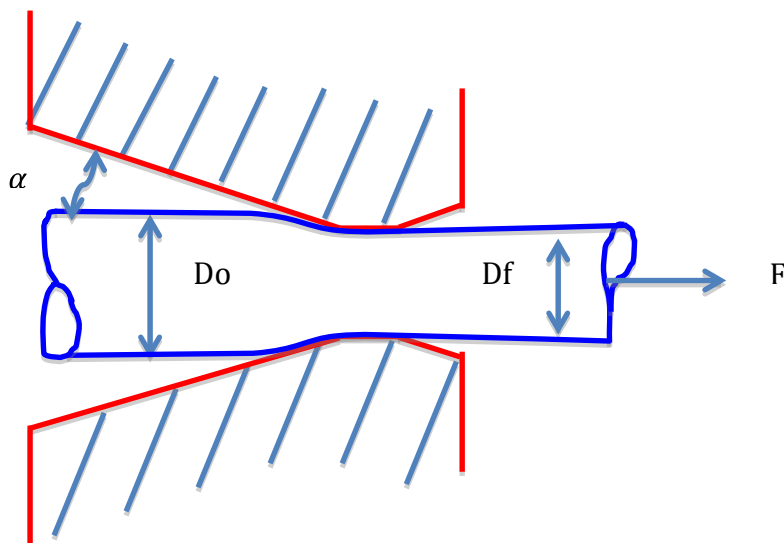


# Bar, Wire and tube drawing

## 1. Analysis of wire drawing:

### 1.1 Analysis of wire drawing

Consider the drawing of a wire through a conical shaped draw die, as shown below, schematically:



**Fig. 1.1.1: Schematic of wire drawing process**

Some of the important terms associated with wire drawing are to be understood first. They are:

Area reduction  $r$  is defined as  $(A_o - A_f)/A_o$  -----2.1

The drawing ratio  $R$  is defined as  $A_o/A_f = 1/(1-r)$  -----2.2

The important parameters which affect the wire drawing force are the drawing ratio, die angle, material flow stress, friction etc.

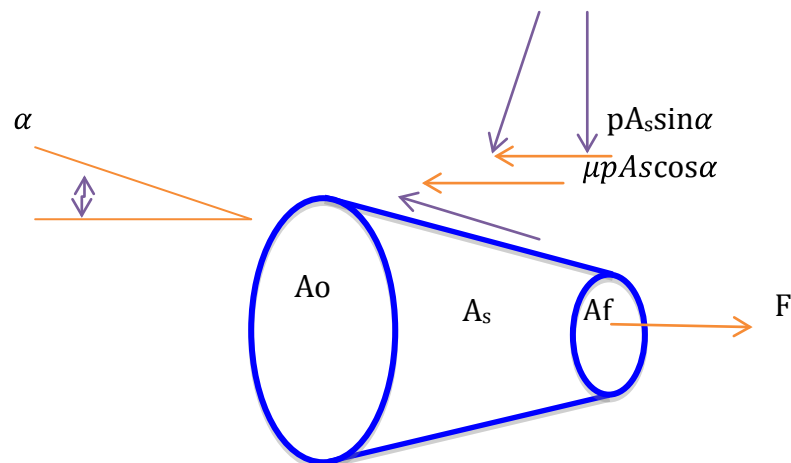
Approximate expression for drawing force can be written based on plastic work or strain energy. Ignoring friction and redundant work we can write the draw pressure as:

$$\text{Draw pressure } p = \bar{Y} \ln\left(\frac{A_o}{A_f}\right) = \bar{Y} \ln R = \bar{Y} \ln\left(\frac{1}{1-r}\right) \text{ -----2.3}$$

The draw pressure is dependent on draw ratio R. As draw ratio is increased the draw pressure increases.

$\bar{Y}$  is the average flow stress of the material.

### 1.2 Analysis of wire drawing with friction:



**Fig. 1.2.1: Stresses acting on elemental section of the drawn wire during drawing**

Consider a conical element of the workpiece inside the die. The surface area of the element is taken to be  $A_s$ . Let  $\alpha$  be the semi-cone angle of the die.  $A_o$  is the cross sectional area of the work piece at entry of the die.  $A_f$  is the exit cross-section area.

We can write the surface area of the element  $A_s$  as:

$$A_s = \frac{A_o - A_f}{\sin \alpha} \text{ -----2.4}$$

The forces acting on the elemental work piece are:

Force due to normal die pressure =  $pA_s \sin \alpha$

frictional force =  $\mu p A_s \cos \alpha$

Draw force =  $F$ ,

Making force balance on the element:

$$pA_s \sin \alpha + \mu p A_s \cos \alpha = F \text{ -----2.5}$$

Substituting for  $A_s$  from 4, we get:

$$p(A_o - A_f) + \mu p(A_o - A_f) \cot \alpha = F$$

$$F = (A_o - A_f) p [1 + \mu \cot \alpha] \text{ -----2.6}$$

We can eliminate p from the above equation by considering a frictionless drawing.

In the absence of friction:

$$\text{Draw force} = F = (A_o - A_f) p = \bar{Y}' \ln \left( \frac{A_o}{A_f} \right) A_o$$

$$\text{From this, we can write: } p = \frac{A_o}{A_o - A_f} \bar{Y}' \ln \left( \frac{A_o}{A_f} \right) \text{ -----2.7}$$

Substituting 7 in 6, we get:

$$F = A_o \bar{Y}' \ln \left( \frac{A_o}{A_f} \right) (1 + \mu \cot \alpha) \text{ -----2.8}$$

Or, the draw stress with friction can be written as:

$$p = \bar{Y}' \ln \left( \frac{A_o}{A_f} \right) (1 + \mu \cot \alpha) \text{ -----2.9}$$

As seen from the above equation, the draw stress depends on the die angle. Higher the die angle, higher the draw stress.

A simple equation proposed by Schey can also be used for the draw stress. It is given as:

$$\sigma_d = \bar{Y}' \left( 1 + \frac{\mu}{\tan \alpha} \right) \theta \ln \left( \frac{A_o}{A_f} \right) \text{ -----2.9A}$$

where  $\theta = 0.88 + 1.2 \frac{D}{L_c}$ , which accounts for redundant deformation. It is called inhomogeneity factor.

D is average diameter of the billet,  $L_c$  is contact length of the wire in the die.

$$L_c = \frac{D_o - D_f}{2 \sin \alpha} \quad \text{and} \quad D = \frac{D_o + D_f}{2}$$

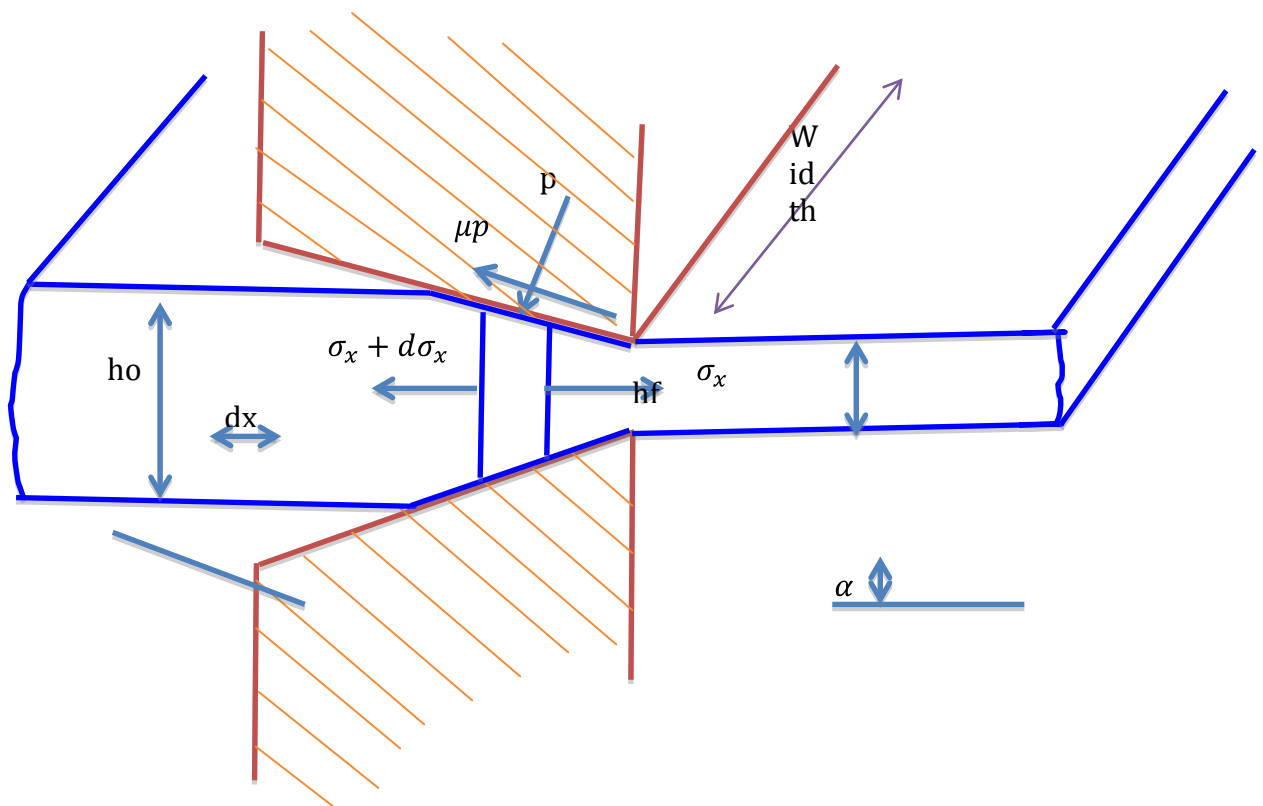
We can determine the draw force from 2.9A as:

$$F = A_f \sigma_d$$

### 1.3 Strip drawing – slab analysis:

Strip drawing is a process of drawing in which, metal of large thickness gets reduced in thickness and increase in length through a converging die.

Consider a rectangular strip of initial thickness  $h_0$  and uniform width. This strip is passed through a convergent die, so that its thickness gets reduced to  $h_f$ . The semi-die angle is taken to be  $\alpha$ .



**Fig. 1.3.1: Strip drawing – stresses acting**

In the analysis, we may assume plane strain compression of the strip, as the width of the strip does not change during the process.

Consider a strip of thickness  $dx$  within the die. Let the strip of initial thickness  $h+dh$  be reduced in thickness to  $h$  after the deformation. We can write the force balance on the elemental strip.

The slant area of the strip =  $dx/\cos\alpha$  (Width is taken as unity)

Resolving the forces along the direction of drawing and writing the force balance,

$$\sigma_x dh + h d\sigma_x + 2\mu p dx + 2p dx \tan\alpha = 0 \text{ -----2.10}$$

Dividing by dh on both sides,

$$\sigma_x + \frac{h d\sigma_x}{dh} + p(1 + \mu \cot\alpha) = 0 \text{ -----2.11}$$

Applying Tresca yield criterion, we have

$$\sigma_x + p = \bar{Y}' \text{ where } \bar{Y}' \text{ is plane strain yield strength and is } = 2Y/\sqrt{3} \text{ -----2.12}$$

$$p = \bar{Y}' - \sigma_x \text{ -----2.13}$$

Substituting in 2.11, and letting  $B = \mu \cot\alpha$  we get:

$$\frac{d\sigma_x}{B\sigma_x - \bar{Y}'(1+B)} = \frac{dh}{h} \text{ -----2.14}$$

Integrating and applying the boundary condition:

At  $h = h_0$ ,  $\sigma_x = 0$

We get:

$$\sigma_x = \bar{Y}' \frac{1+B}{B} \left(1 - \left[\frac{h_f}{h_0}\right]^B\right) \text{ -----2.15}$$

By applying the same procedure, we can derive a similar expression for the draw stress of wire drawing process.

The draw stress for wire drawing process is given by:

$$\sigma_x = \bar{Y}' \frac{1+B}{B} \left(1 - \left[\frac{d_f}{d_0}\right]^{2B}\right) \text{ -----2.16}$$

Example: A steel wire is drawn to 24% reduction from initial diameter of 10mm. The flow stress of the material is given by:  $\sigma = 1200\varepsilon^{0.28}$  MPa. The semi die angle is  $6^\circ$  and  $\mu=0.1$ . Calculate the draw stress and the power required for the deformation if the wire moves at a speed of 2.5 m/s.

Solution:

Given:  $r = 0.24 = (A_0 - A_f)/A_0$

We can calculate the strain from the expression:  $\varepsilon = \ln \frac{1}{1-r} = 0.274$

The average flow stress is given by:  $\bar{Y}' = k \frac{\varepsilon^n}{1+n} = 1200(0.274)^{0.28} / 1.28 = 652.73 \text{ MPa}$

We can use equation 9 for calculating the draw stress:

$$p = \bar{Y}' \ln \left( \frac{A_o}{A_f} \right) (1 + \mu \cot \alpha) = 349.86 \text{ MPa}$$

Now, the final area of cross-section of the wire,  $A_f = A_o(1-r) = 59.66 \text{ sq.mm}$

To determine power required, we can use the formula:

$$\text{Power} = \text{Draw force} \times \text{speed of drawing} = \text{Draw stress} \times A_f \times \text{speed} = 52180.93 \text{ W}$$

Source:

<http://nptel.ac.in/courses/112106153/27>