Application of Monte Carlo Technique for Analysis of Tolerance & Allocation of Reciprocating Compressor Assembly

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Abstract
The variation in part dimensions is one of the main causes of variation in product quality. The allowable range in which the dimension can vary is called tolerance. Tolerance can be expressed in either of two ways, 1). Bilatera 2). Unilateral tolerance. In bilateral tolerance, it is specified as plus or minus deviation from the basic size. In unilateral tolerance, variation is permitted only in one side direction from the basic size. If a part size and shape are not within tolerance limit, the part is not acceptable. The assignment of actual values to the tolerance limits has major influence on the overall cost and quality of an assembly or product. If the tolerances are too small (tight), the individual parts will cost more to make. If the tolerances are too large (loose), an unacceptable percentage of assemblies may be scrapped (rejected) or require rework. In view of the above in this thesis two aspects have been investigated: (1). Tolerance analysis of a compressor assembly by Monte Carlo simulation method. (2). Tolerance allocation by proportional scaling (worst case & roots sum square methods). In this thesis, Monte Carlo simulation is proposed for tolerance analysis of final assembly. In tolerance analysis the upper and lower boundary tolerances of the final assembly is determined. The component tolerances are all known or specified and the resulting assembly variation is calculated. The adherence of the assembly tolerance is a measure of the quality level. The quality is expressed in terms of percentage of the assemblies which meet the engineering tolerance limits. For high quality levels, the rejection may be expressed in parts-per million (ppm), that is, the number of rejection per million This thesis pays attention to the tolerance analysis for low volume, large variety production, in which parts with certain common characteristics are typically interchangeable. In the present study Monte Carlo Simulation technique is applied to predict assembly tolerances with different statistical distributions for parts and components assemblies. In tolerance allocation, actually the tolerances of parts were determined through assembly tolerance. Tolerance allocation of parts has been determined by proportional scaling by worst case method and roots sum square methods.

Keywords
Monte Carlo Technique, Tolerance, Compressor, Matlab, Reciprocating

I. Introduction

To account for variability of dimensions (due to manufacturing) at the design phase, we assign a tolerance or range of acceptable values to each suitable (not every dimension requires a tolerance) dimension of the part. Tolerance is the total amount by which a dimension may vary. It is used to determine the permissible limits (maximum and minimum) of the dimensions.

In addition to manufacturing cost considerations, tolerances are usually specified to meet functional requirements of assemblies. In order for mating feature (faces) of parts to fit together and operate properly, each part must be manufactured within these limits. The key, gears on the shaft, and other similar members mounted by press or shrink-fit are toleranced so that the desired interference is maintained without being so large as to make the assembly impossible or the resulting stress too high. Tolerancing is essential element of mass production and interchangeable manufacturing, by which parts can be made in widely separated location and then brought together for assembly. Tolerancing makes it also possible for spare parts to replace broken or worn ones in existing assemblies successfully. In essence, without interchangeable manufacturing, modern industry could not exist, and without effective size control by the engineer, interchangeable manufacturing could not be achieved. Tolerancing information is essential for part process planning, assembly operations part inspection, and for other design and production activities. Design engineers need tolerance analysis to distribute allowances among related design dimensions, to check design results and then design assemblies. Production engineers need tolerance analysis to transform design coordinates into manufacturing coordinates and to perform tolerance calculations and distributions in process planning. Tolerance analysis is critical in resolving tradeoffs between the demands of part interchangeability and product quality. The quality characteristics of parts need to be subject to less stringent requirements, in order to meet the product requirements of part interchangeability and variety, so the assembly process will be easier and the assembly tolerance predictions will be more precise. Tolerance analysis mainly applied to mass production environments and is necessary incompatible with the future needs. Tolerance analysis customarily takes place during the design phase, when it is used to forecast input parameters (parts tolerances and their means and standard deviations) for statistical process control and/or other quality control functions during the production phase. The forecast values depend heavily on the following assumptions regarding the statistical distributions of part variations that are valid only in mass production.

Tolerance analysis depends on the

1. Statistical distributions (normal, uniform & mixed) of part tolerances.
2. They usually assume that the part dimensions are symmetrically distributed.
3. To forecast the assembly tolerances and through them, determine the product rejection rate.

II Problem formulation

Tolerance Analysis can be either worst-case method or statistical method. Statistical tolerancing is a more practical and economical way of looking at tolerances and works on setting the tolerances so as to assure a desired yield.

Statistical tolerance analysis uses a relationship of the form:

$$y = f(x1,\ldots.., xn) \quad (1)$$

 ISSN : 2249-5762 (Online) | ISSN : 2249-5770 (Print)

Where, y is the response of the assembly and x1…, xn are the values of some characteristics of the individual parts or subassemblies making up the assembly. We call f the assembly response function (ARF). The relationship can exist in any form for which it is possible to compute a value for y given values of x1…, xn. The input variables x1…, xn are continuous random variables.

Once the moments of y are determined, one can compute a tolerance range for y that would envelope a given fraction of the assembly yield. Here Monte Carlo simulation method is very useful. Monte Carlo Simulation is a powerful tool for tolerance analysis of mechanical assemblies, for both nonlinear assembly functions and non-Normal distributions. It is based on the use of a random number generator to simulate the effects of manufacturing variations on assemblies.

A. The Monte Carlo simulation procedure

1. The statistical distribution is specified for the variation in each component dimension. The distribution may be described algebraically or empirically.
2. Generate the random number r1.
3. Find X1 corresponding to r1.
4. Generate the random number r2.
5. Find X2 corresponding to r2.
6. Generate the random number r3.
7. Find X3 corresponding to r3.
8. Calculate the Assembly function x0=X1+X2+X3.
9. Calculate: Mean, standard deviation, median, variance, skewness, kurtosis, Max. Value, Min. value, etc.
10. Draw the histogram.

B. Modeling of tolerance allocation

In tolerance allocation, the assembly tolerance is known from design requirements, whereas the magnitude of the component tolerances to meet these requirements are unknown. The available assembly tolerance must be distributed or allocated among the components in some rational way.

Tolerance Allocation Methods

1. Allocation by Proportional Scaling
2. Allocation by Weight Factors

C. Proposed Solution

Statistics tool from MATLAB:

For normal distribution,

Inverse cdf is x=icdf ('Normal', p, μ, σ)

Where, p= Cumulative density function,

P=rn

rn=Random number

μ= Mean, σ= Standard deviation

Step5. Write the program in a Mat Lab.

Step6. Ten thousands simulations run for each part and module.

Step7. Calculate the Module (M_1), Module (M_2), and Module (M_3).

$M_1=X_1+X_2+X_3$ (1)

$M_2= X_4 - X_5 + X_6 - X_7 + X_8$ (2)

$M_3= X_9 + X_{10} + X_{11}$ (3)

Step 8. Calculate the compression chamber height

$X_0= - M_1 - M_2 + M_3 + X_{12} + X_{13}$ (4)

Step 9. Calculate: Mean (μ) and Standard deviation (σ), usl, lsl, Median, Variance, Skewness, kurtosis, maximum dimension, minimum dimension and Rate of rejection etc.

Step10. Draw the histogram.

III. Modeling for tolerance analysis

In tolerance analysis the assembly yield is unknown. It is calculated by summing the component tolerances to determine the assembly variation, then applying the upper and lower spec limits to the calculated assembly distribution.

The assembly yield is the quality level. It is the percent of assemblies which meet the engineering tolerance requirements. It may be expressed as the percent of acceptable assemblies or the percent rejects. For high quality levels, the rejects may be expressed in parts-per-million (ppm), that is, the number of rejects per million assemblies.

A. Methods for Tolerance Analysis

There are a variety of methods and techniques available for the above computational problem. Essentially, the methods can be categorized into four classes.

1. Stack Tolerancing or Linear Propagation (Root Sum of Squares)
2. Non-linear propagation (Extended Taylor series)
3. Numerical integration (Quadrature technique)
4. Monte Carlo simulation

1. Stack Tolerancing or Linear Propagation

This is also called as stock tolerancing and uses the well-know root sum of squares (RSS) formula. The assembly response function here is of the form:

$y=a_0 + a_1X_1+a_2X_1+\ldots\ldots\ldots\ldots +a_nX_n$

(1)

where, a_0, a_1 …,a_n are constant and X_1…, X_n are assumed to be mutually independent. Many directional and gap-related measures fall into this category. Because of linear relationship and mutual independence, the mean and variance of y are given by:

$\mu_y=a_0+a_1\mu_1+a_2\mu_2+a_3\mu_3+\ldots\ldots..+a_n\mu_n$ (2)

$\sigma^2y=a_1^2\sigma^2+a_2^2\sigma^2+\ldots\ldots\ldots+a^2{}_n\sigma_n{}^2$ (3)

where μ_i is the mean and σ_i, the standard deviation of X_i, i=1,…,n. the nomenclature RSS arises because of the formula above for standard deviation. If the individual distributions are normal, then y is also normally distributed. Even if the individual distributions are not normal, y can safely be treated as normal, by invoking the central limit theorem.

If the linear relation for y above is only approximately true. Then one can expand $f(X_1\ldots X_n)$ as a Taylor series and drop all but the constant and linear terms. This is often used device in statistical tolerancing to handle approximately linear relationships.

In such a case,

$a_i= \delta f/\delta x_i$ evaluated at $x_i=\mu_i$, i=1,…, n.

And all of the constant terms are gathered into a_0. The computation of the above partial derivatives could be of two types. In the first case, the function f is known and the partial derivatives are known to exist. In second case, the functional relationship.

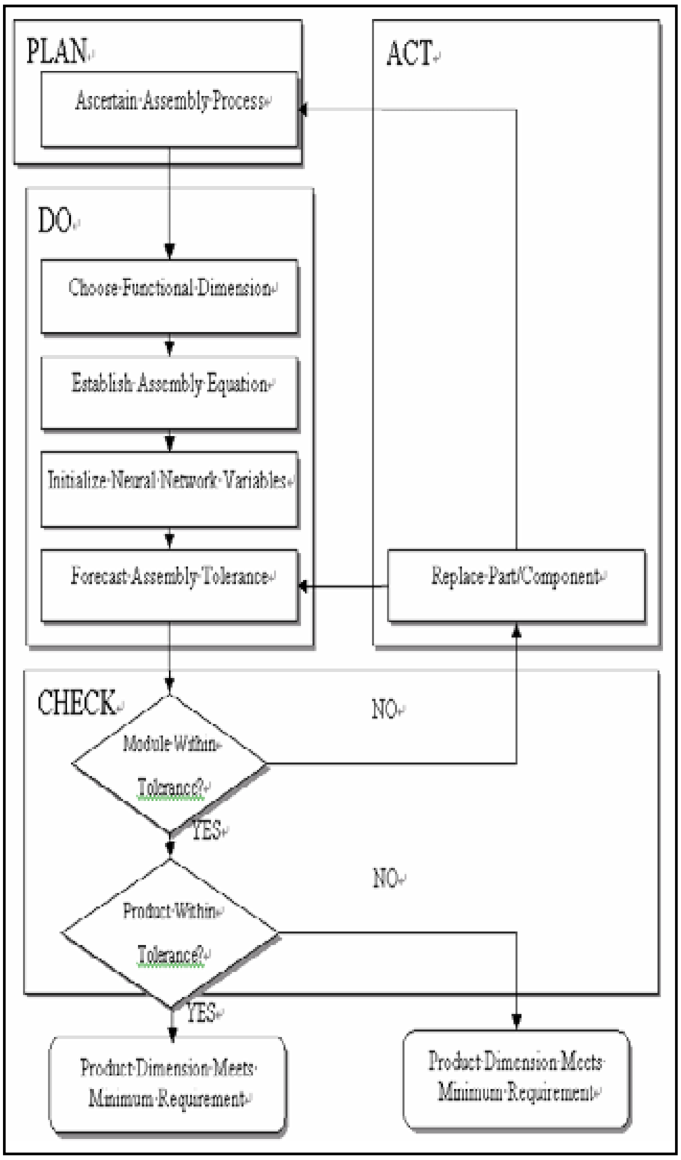

Fig. 1: Tolerance analysis procedure

Is either too intractable or not even available in analytic form. In such a case, numerical estimates have to be obtained for partial derivatives. The linear case is easily the simplest and the most efficient among all tolerance analysis approaches.

2. Non linear Propagation

If the assembly response function y is highly non-linear, application of the RSS method could lead to serious errors. In such a case, an extended Taylor series approximation for the relationship f can possibly be employed. For this, f needs to be available in analytic form. Usually, the expansion is considered up to sixth order. The expansion is possible only when all the appropriate partial derivatives exist. The main computational issue here is that of computing the partial derivatives. Tractable formulae for the first four moments of y are available and are ideally suited for tolerance analysis and synthesis. These formulae need only the first four moments of the distributions of $X_1 \ldots , X_n$. Most often, the partial derivatives are computed using analytic methods. However, numeric evaluation may need to contemplated in some cases (in such cases, the quadrature technique is more appropriate).

3. Numerical integration

If the function f is not available in analytic form and y can only be computed through numerical calculations or engineering methods or simulations, numerical methods have to be used. Quadrature methods are prominently used here. The basis of the numerical methods is that for any function $h(X_1,\ldots..,X_n)$ (different from f) of mutually independent random variables $(X_1,\ldots..,X_n)$ with probability density functions $wX_i(x_i)$, the expected value of h is given by the integral

$$\int_{-\infty}^{+\infty} .....\int h(x_1......,x_n)\prod_{i=1}^{n}(wx_i(x_i)dx_i) \qquad (5)$$

The above expression can be approximated by a quadrature expression [1, 2] that involves evaluations of h at $2n^2+1$ prescribed value. These evaluations involve only the first four moments of X1…, Xn. Given an assembly response function f, a corresponding function h as above can be defined and simple moment transfer relations can be used to compute the first four moments of f. The quadrature technique adapts well to statistical tolerancing problems since it can handle the iteration inherent in a tolerancing problem efficiently.

4. Monte Carlo Simulation

The appeal of Monte Carlo lies in its applicability under very general settings and the unlimited precision that can be achieved. In particular, Monte Carlo can be used in all situations in which the above three techniques (stack tolerancing, extended Taylor series, and numerical integration) can be used and can yield more precise estimates. For this reason, Monte Carlo technique is easily the most popular tool used in tolerancing problems. The caveat, however, is the large computational time. For situations where the above three techniques are adequate and have acceptable precision, the Monte Carlo technique is much more expensive in terms of computational time. Monte Carlo analysis proceeds as follows. Pseudo random number generators are used to generate a sample of numbers $X_1,\ldots..,X_n$, belonging to the random variables $X_1,\ldots..,X_n$ respectively. The value of y, say $y = f(X_1,\ldots..,X_n)$, corresponding to this sample is computed. This procedure is replicated a large number of times, say N times. This would yield a random sample, $fx_1 \ldots yN_g$ for y. Standard statistical estimation methods are then used to analyze the distribution of y. The precision of this statistical analysis increases proportional to $\sqrt{N}$ and therefore unlimited precision can be achieved through large number of replications. Special techniques are available for significantly enhancing the precision of the Monte Carlo method for a given N. These include: weighted sampling, reuse of samples, and use of approximation functions

5. Principles of Monte Carlo Simulation

Monte Carlo Simulation is a powerful tool for tolerance analysis of mechanical assemblies, for both nonlinear assembly functions and non-Normal distributions. It is based on the use of a random number generator to simulate the effects of manufacturing variations on assemblies. fig. 2 illustrates the method.

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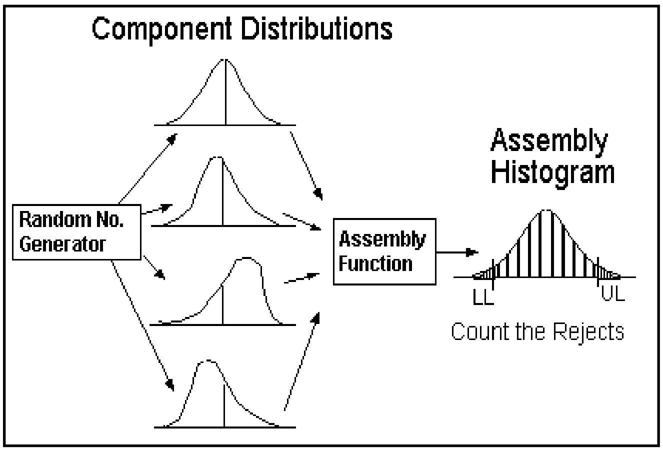

Fig. 2: Assembly tolerance analysis by Monte Carlo simulation

IV. Modeling of tolerance allocation

The analytical modeling of assemblies provides a quantitative basis for the evaluation of design variations and specification of tolerances. An important distinction in tolerance specification is that engineers are more commonly faced with the problem of tolerance allocation. In tolerance allocation, the assembly tolerance is known from design requirements, whereas the magnitude of the component tolerances to meet these requirements are unknown. The available assembly tolerance must be distributed or allocated among the components in some rational way. The influence of the tolerance accumulation model and the allocation rule chosen by the designer on the resulting tolerance allocation. In tolerance allocation, on the other hand, the assembly yield is specified as a design requirement. The component tolerances must then be set to assure that the resulting assembly yield meets the spec. The rational allocation of component tolerances requires the establishment of a rule for distributing the assembly tolerance among the components. The following sections present several examples of useful rules.

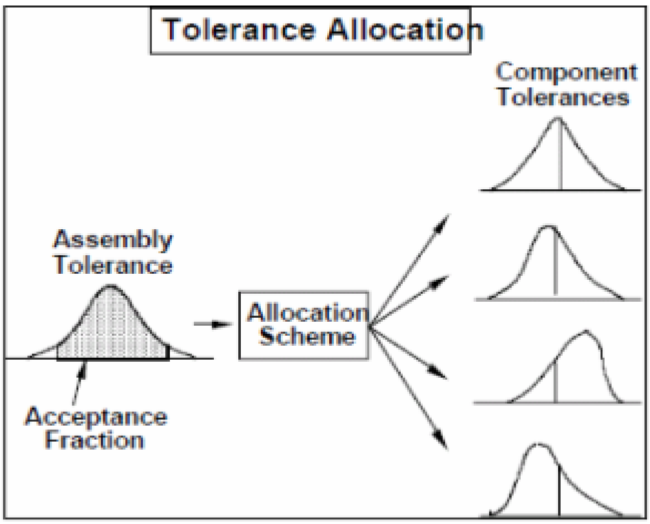

Fig. 3: Tolerance Allocation

Tolerance Allocation is a design tool. It provides a rational basis for assigning tolerances to dimensions. Two algorithms are described in this thesis for performing tolerance allocation, which is defined as the re-distribution of the "tolerance budget" within an assembly to reduce over-all cost of production, while meeting target levels for quality. There are several advanced tolerance techniques available to a designer to improve quality levels in assemblies. They are primarily fine-tuning operations which can be applied to an assembly distribution to reduce the number of rejected assemblies, sometimes dramatically.

V Simulation of tolerance analysis

In this work the Monte Carlo Simulation based Tolerance Analysis Model was used. A product called compressor was used. Bjorke. In the compressor assembly process, the height of the compression chamber is determined from the dimensions of several parts assembled inside the compressor cylinder. The problem is to keep the compressor's height from exceeding its predetermined range as a result of tolerance stack-up. For tolerance analysis, the normal, uniform, and mixed distributions are used.

A. Description of the Compressor Assembly

The Compressor Assembly consists of three independent assembly modules: M_1, M_2, and M_3. These three modules are sequentially assembled into the final product. The compression chamber height (compressor) variable is denoted X_0 and the independent variables representing the dimensions and tolerances of the parts are labeled X_1, $X_2 \ldots X_{13}$. Fig. 4, shows the dimensions of the compressor assembly. The definitions of variables are described below and these variables are chosen according to the relationship between parts and components generated by assembly process.

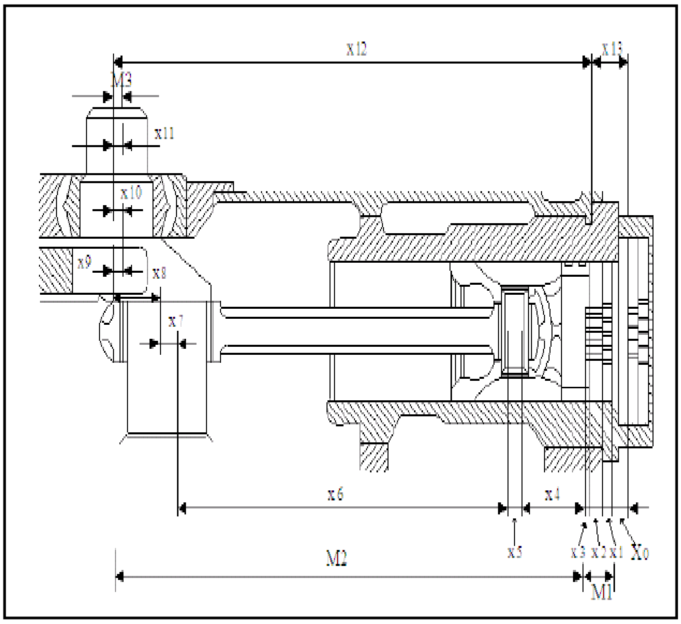

Fig. 4: Assembly dimension for compressor

X_0: The compression chamber height
Module 1(M_1): The thickness of the valve and gasket
X_1: The thickness of the valve sleeve
X_2: The thickness of the valve port plate
X_3: The thickness of the gasket

Fig. 5: Air compressor(Reciprocating Type)

VI Tolerance allocation

In tolerance allocation, the assembly tolerance is known from design requirements, whereas the magnitude of the component tolerances to meet these requirements are unknown. The available assembly tolerance must be distributed or allocated among the components in some rational way. The rational allocation of component tolerances requires the establishment of some rule upon which to base the allocation. There are two methods of tolerance allocation by proportional scaling described below based on assembly the Compressor Assembly.

A. One dimensional Assembly Worst Case Tolerance Allocation by Proportional Scaling

The following problem is based on assembly the Compressor Assembly shown in fig. 6. The Compressor Assembly consists of three independent assembly modules: M_1, M_2, and M_3. These three modules are sequentially assembled into the final product. The compression chamber height (compressor) variable is denoted X_0 and the independent variables representing the dimensions and tolerances of the parts are labeled X_1, X_2…X_{13}. Fig. 6, shows the dimensions of the compressor assembly. The definitions of variables are described below and these variables are chosen according to the relationship between parts and components generated by assembly process. This assembly dimension used for tolerance allocation described below.

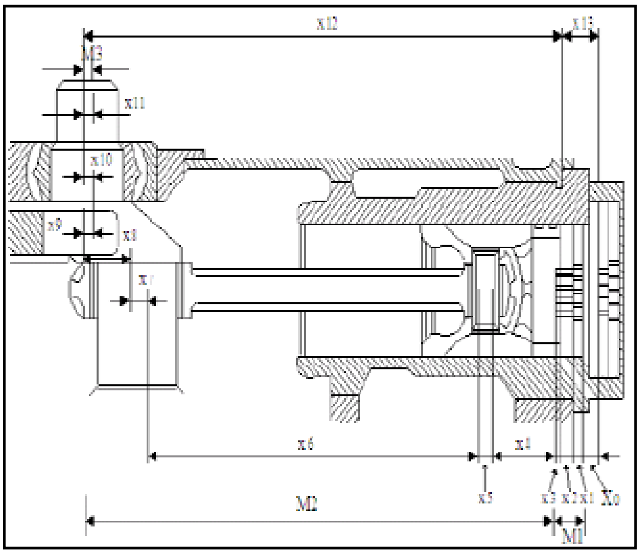

Fig. 6: Assembly dimension for compressor

X_0: The compression chamber height
Module 1(M_1): The thickness of the valve and gasket
X_1: The thickness of the valve sleeve
X_2: The thickness of the valve port plate
X_3: The thickness of the gasket
Module 2(M_2): The functions of piston, wrist pin, connecting rod, and the crank
X_4: The distance from the piston head to the wrist pin bore (on the piston)
X_5: The gap between the wrist pin bore and the wrist pin bearing
X_6: The center distance between the wrist pin bearings and the connecting rod bearing
X_7: The gap between the connecting rod bearing and the crank
X_8: The throw of the crank
Module 3 (M_3): The top dead centre
X_9: The gap between the crank shaft and the inner ring of the crank shaft bearing
X_{10}: The eccentricity of the crank shaft bearing
X_{11}: The gap between the outer ring of the crank shaft bearing and the bearing mount
Total Assembly:
X_{12}: The distance between the bearing mount bore in the cylinder block and the cylinder sleeve flange
X_{13}: The thickness of the cylinder sleeve flange and the space.

VII Conclusion

In the present study two aspects of tolerance of a compressor assembly are dealt here with namely tolerance analysis and tolerance allocation. In tolerance analysis by Monte Carlo simulation assuming normal distribution, it has been observed that the calculated skewness coefficient of all parts dimension, all modules dimension are less than zero, which shows that it is a negatively skewed distribution. As mean is equal to median, it is a symmetrical distribution. The calculated kurtosis coefficient is approximately equal to 3.0, which shows that it is a normal distribution. The rejection rate of final assembly is approximately equal to 0.21 percent. So the dimension of final assembly (X_0) is nearly in tolerance limits. The rejection rate allowed is only 1.00 percent. So this Monte Carlo simulation method is very good technique for tolerance analysis. In tolerance analysis by Monte Carlo simulation assuming uniform distribution, it has been observed that the calculated skewness coefficient of all parts dimension, all modules dimension are less than zero, which shows that it is a negatively skewed distribution. Mean is equal to median which shows that it is a symmetrical distribution. The calculated kurtosis coefficient is less than 3.0, which show that it is a platykurtic distribution. This distribution is less peaked than normal distribution. The rejection rate of final assembly is equal to zero percent. The rejection rate allowed is only 1.00 percent. So this Monte Carlo simulation method is also a very good technique for tolerance analysis.

In tolerance analysis assuming mixed distribution it has been observed that the calculated skewness coefficient of all parts dimension, all modules dimension are less than zero, which shows; it is a negatively skewed distribution. Mean is equal to median which shows that it is a symmetrical distribution. The calculated kurtosis coefficient is greater than 3.0, which show that it is a leptokurtic distribution. This distribution is more peaked than normal distribution. The rejection rate of final assembly is approximately equal to 2.050 percent. So the dimension of final assembly (X_0) is not in tolerance limits. The rejection rate allowed

 ISSN : 2249-5762 (Online) | ISSN : 2249-5770 (Print)

is only 1.00 percent (ppm). So this Monte Carlo simulation method is not a very good technique in the mixed distribution case and gives approximate results. In the second problem the tolerance allocation of the parts have been determined for a given tolerance of the compressor assembly. Here the tolerance allocation is done using two methods namely 1. Worst case tolerance allocation by proportional scaling and

2. Roots sum square tolerance allocation by proportional scaling.

The tolerances of parts are allocated and the results are found. It has been found that the design tolerances of the parts are meeting the assembly requirements. Thus it gives better design tolerance of the assembly parts

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